# Fluctuations, Large Deviations and Rigidity in Superhomogeneous Systems

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Abstract: Superhomogeneous (a.k.a. hyperuniform) particle systems are point processes on R^d (or Z^d) that are translation invariant (or periodic) and for which the variance of the number of particles in a region V grows slower than the volume of V. Examples include Coulomb systems, determinantal processes with projection kernels and certain perturbed lattice models. I will first review some old work on superhomogeneous systems and and then describe some new work (with Subhro Ghosh) providing sufficient conditions (involving decay of pair correlations) for number rigidity in such systems in dimension d=1,2. A particle system is said to exhibit number rigidity if the probability distribution of the number of particles in a bounded region R, conditioned on the particle configuration in R^c, is concentrated on a single integer N. All known (to us) examples in which number rigidity has been established in d=1,2 satisfy our conditions, and we conjecture that superhomogeneity is also a necessary condition for number rigidity in all dimensions d. On the other hand, it follows from the work of Peres and Sly on perturbed lattice systems that in d>2 there are no such sufficiency conditions involving the decay of correlations.