Breaking of Ensemble Equivalence in Complex Networks

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It is generally believed that for physical systems in the thermodynamic limit, the microcanonical description as a function of energy coincides with the canonical description as a function of temperature. However, various examples of systems for which the microcanonical and canonical ensembles are not equivalent have been identified. A complete theory of this intriguing phenomenon is still missing.

In this talk we show that ensemble nonequivalence can manifest itself also in random graphs with topological constraints. We find that, while graphs with a given number of links are ensemble-equivalent, graphs with a given degree sequence are not. This result holds irrespective of whether the "energy" associated with the constraint is nonadditive (as in unipartite graphs) or additive (as in bipartite graphs). In contrast with previous expectations, our results show that: (1) physically, nonequivalence can be induced by an extensive number of local constraints, and not necessarily by long-range interactions or nonadditivity; (2) mathematically, nonquivalence is determined by a different large-deviation behaviour of microcanonical and canonical probabilities for a single microstate, and not necessarily for almost all microstates. The latter criterion, which is entirely local, is not restricted to networks and holds in general.

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