

Two-dimensional problems with critical nonlinearity

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This course aims to study a class of nonlinear problems with exponential nonlinearity.



Liouville

Those problems have first been considered by Liouville in relation with the question of finding Riemannian metrics having constant Gauss curvature on a domain of the plane. Aside differential geometry, it has been observed more recently that this class of problem arises also in statistical mechanics and in some chemotaxis models in biology.

In order to understand the existence and uniqueness of the solutions for this class of problem, one of the difficulty is obtaining a priori bound on the set of solutions. This lack of compactness is an obstacle in using classical variational method to detect solutions, but as will be discussed in this lecture there different ways to overcome this main difficulty.

I will also present some uniqueness results, and as we will see those one are closely related to some isoperimetric inequality on surfaces.