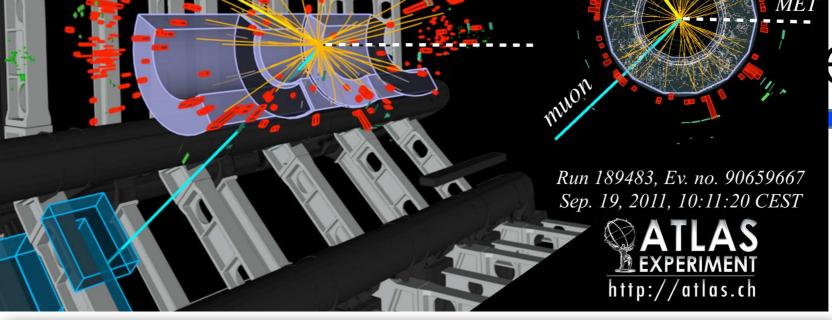
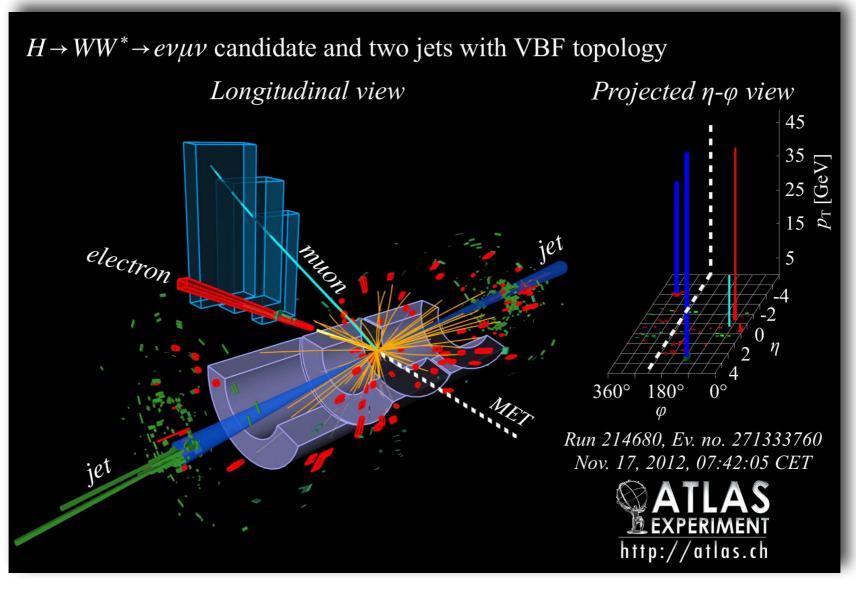
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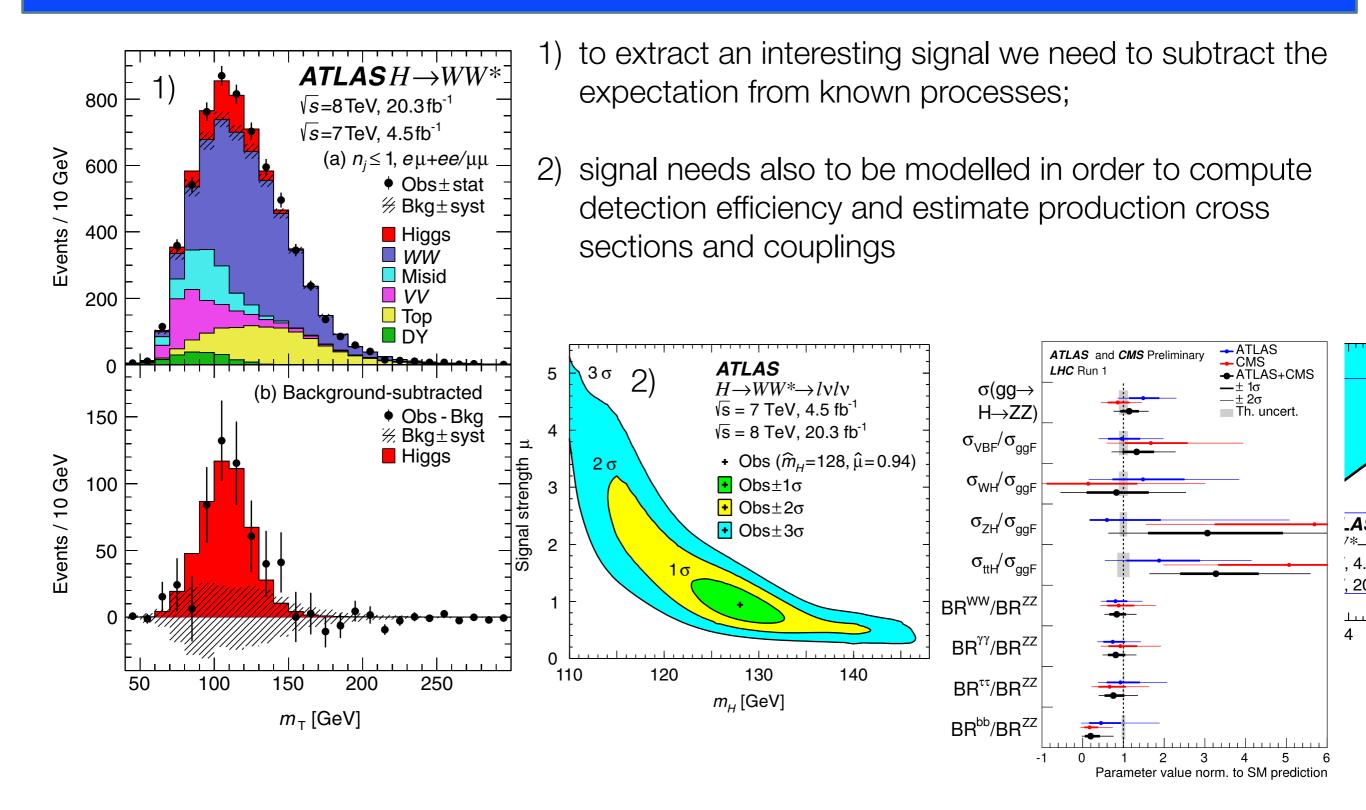




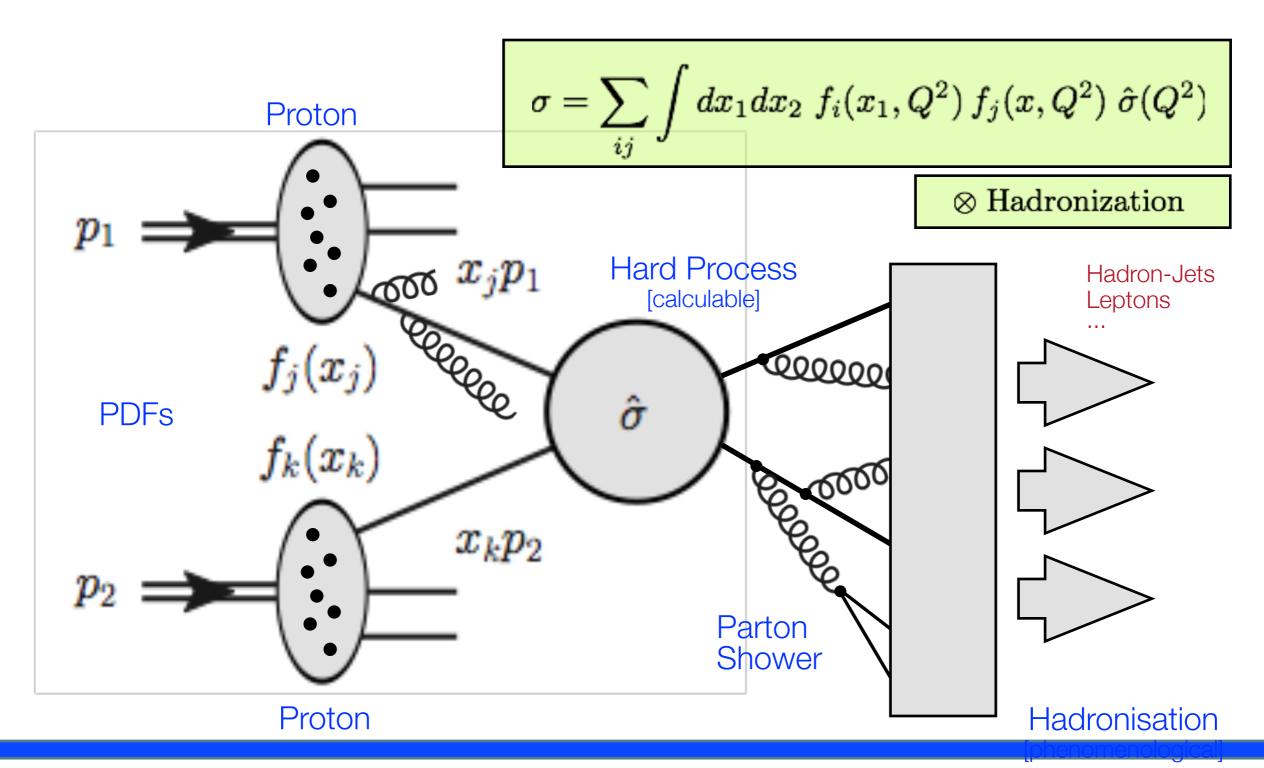
Acknowledgements

These slides have been prepared using heavily the material prepared by Christian Schultz Coulon and clarifying few points plus updating to recent developments in proton-proton physics.

Why MC simulation?



The simulation chain



MC simulations in particle physics

How Monte Carlo simulation works

- Numerical process generation based on random numbers
- Very powerful method in particle physics

Event generation programs:

Pythia Herwig, Sherpa...

Hard partonic subprocess + fragmentation and hadronisation ...

Detector simulation:

Geant4
Fluka low energy hadron interactions...

interaction & response of all produced particles ...

Event Generator

simulate physics process (quantum mechanics: probabilities!)



Detector Simulation

simulate interaction with detector material



Digitisation

translate interactions with detector into realistic signals

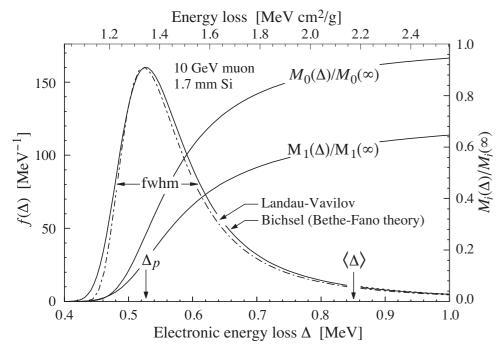


Reconstruction/Analysis

as for real data

Baseline of the simulation process

Typically, we need to generate a continuous variable following some distribution i.e. energy loss of a particle in a given material segment; angle of a photon in the h reference frame for the $h \rightarrow \gamma \gamma$ decay



$$dP = f(x,..)dx$$

$$\longrightarrow \text{ distribution formula}$$

$$\text{probability to get an } x_0 \text{ value between } x \text{ and } x + dx$$

if we want to simulate flat angular in 3D, that is uniform distribution on a sphere of radius 1:

$$dP = kdS = kd\Omega = ksin(\theta)d\theta d\phi$$

$$dP = f(\theta, \phi)d\theta d\phi = ksin(\theta)d\theta d\phi$$

$$f(\theta, \phi) = k \sin(\theta)$$

flat distribution in φ not flat in θ

Distribution function transformation properties

- 1) software libraries provide basic functions to produce flat distributed random numbers in the interval [0,1] (ex. root TRandom3 class), they are typically fast and accurate uniform random numbers generators;
- 2) starting from uniform distributed random numbers, it is possible to generate numbers following any distribution using different techniques

$$dP_x = f(x)dx y = g(x)$$

$$x \in [x_a, x_b]$$

$$dP_y = h(y)dy = h(y)g'(x)dx$$

g(x) is a monotonic function of x How "y" distributes in $[g(x_a), g(x_b)]$?

Because y is a monotonic function of x the probability to have y between g(x) and g(x+dx) is equal to the probability to have x between x and x+dx

$$dP_x = dP_y$$

$$h(y)g'(x) = f(x) \Rightarrow h(y) = \frac{f(x)}{g'(x)} = \frac{f(g^{-1}(y))}{g'(g^{-1}(y))}$$

Ex1.: range map

$$[0,1] \to [a,b]$$
 $y = (b-a)x + a$

$$f(x) = 1 \quad g'(x) = b - a \quad h(y) = \frac{1}{b - a}$$
 uniform

y is uniformly distributed in [a,b]

Distribution function transformation properties

Ex. 2: integration method: we want to generate a distribution f(x) knowing that y is flat distributed, we want to find the transformation g⁻¹(y) that allows it:

$$y = g(x) = \frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' \quad g'(x) = \frac{f(x)}{\int_a^b f(x')dx'}$$

$$h(y) = \frac{f(x)}{g'(x)} = \frac{f[g^{-1}(y)]}{f[g^{-1}(y)]} \cdot \int_a^b f(x')dx' = \int_a^b f(x')dx'$$

y is flat distributed:

- 1) generate y flat in [f_{min}, f_{max}];
- 2) compute $x = g^{-1}(y)$, x will be distributed in $g^{-1}(f_{min})$, $g^{-1}(f_{max})$

Finding g⁻¹(y) is equivalent to solve the equation:

$$\frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' = y \qquad \text{often it can be done analytically in general it can be solved numerically}$$

Hit or miss method.

- 1) generate x flat in x_{min}, x_{max}
- 2) generate y flat in 0, f_{max}
- 3) if y < f(x) accept the event, otherwise ignore it

for a given x_0 in x, x+dx the fraction of accepted events is proportional to f(x)dx -> dPx = f(x)dx



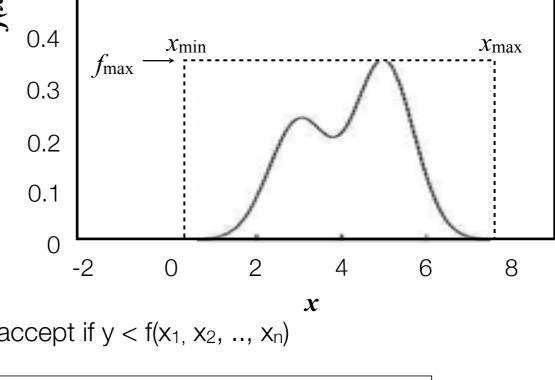
- can be used for all functions ...
- can be extended to N-dimension (generate $x_1, x_2, ..., x_n$), y accept if $y < f(x_1, x_2, ..., x_n)$

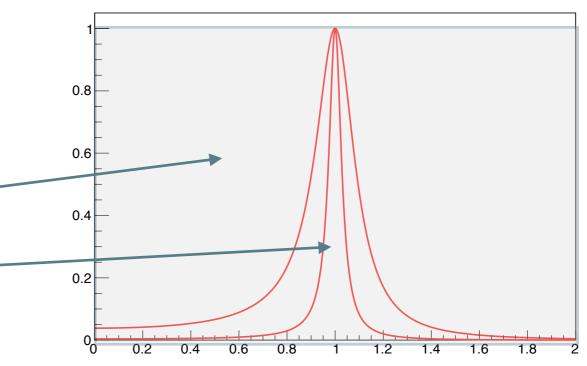


• can be extremely slow

points generated uniformly in the square points accepted only below the curve

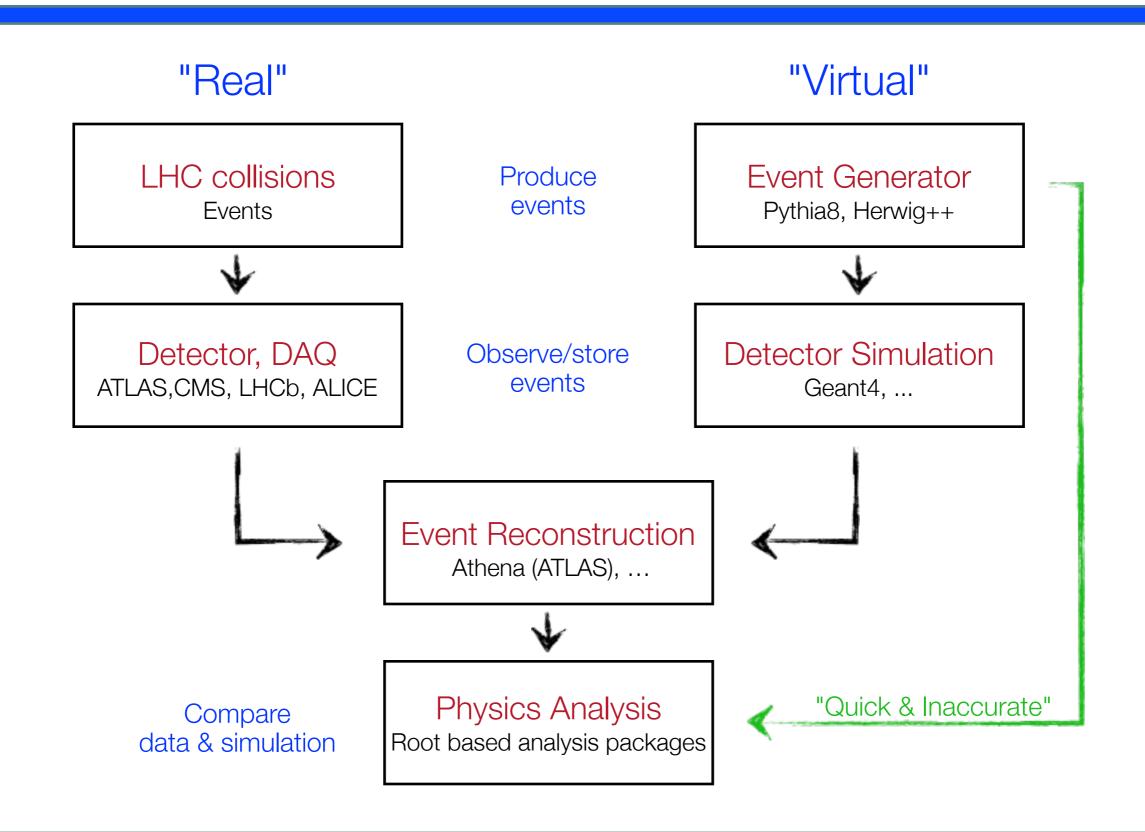
MC generators implement "smart" generation techniques to increase efficiencies



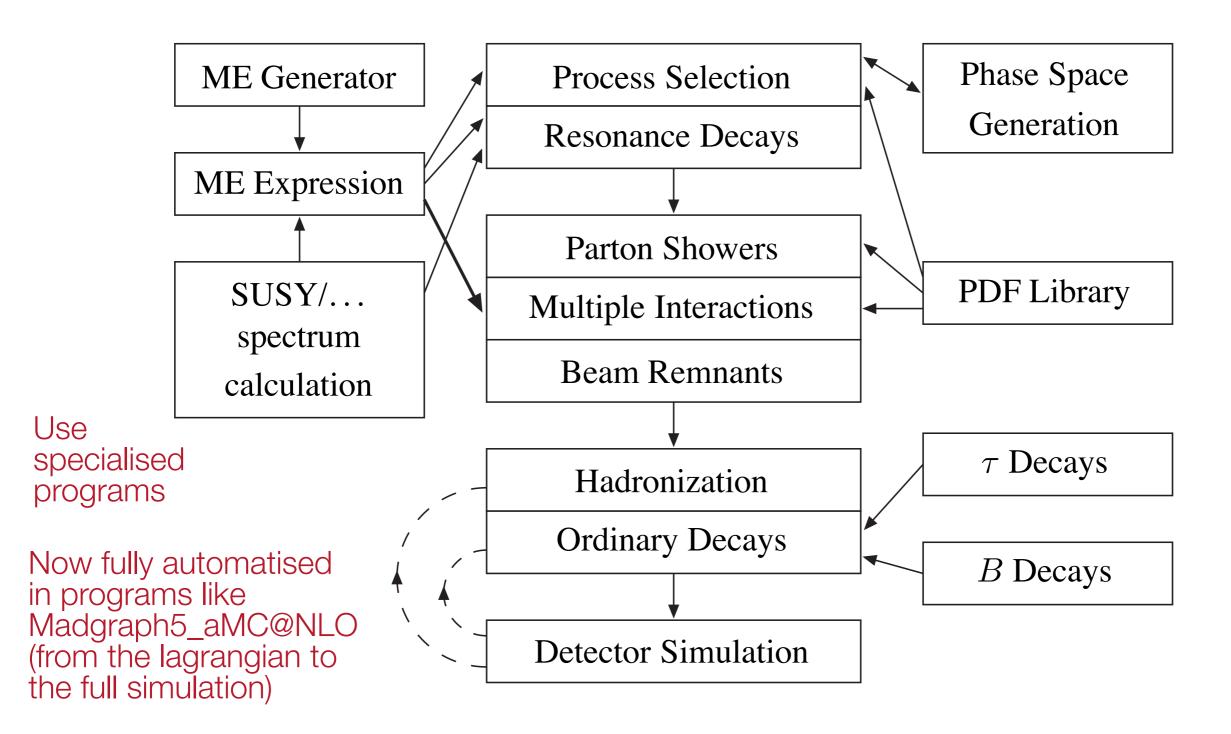


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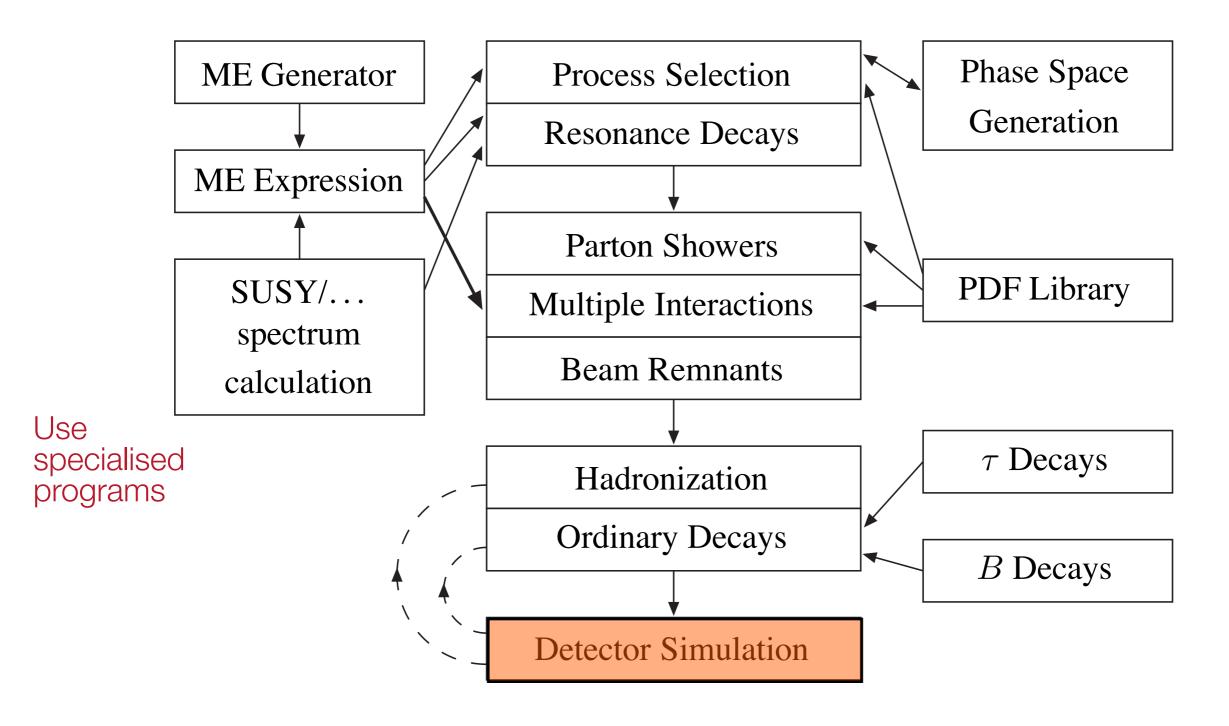
Comparison between real and simulated events



Simulation elements



Simulation elements



GEANT Geometry And Tracking

Detailed description of detector geometry [sensitive & insensitive volumes]

Tracking of all particles through detector material ...

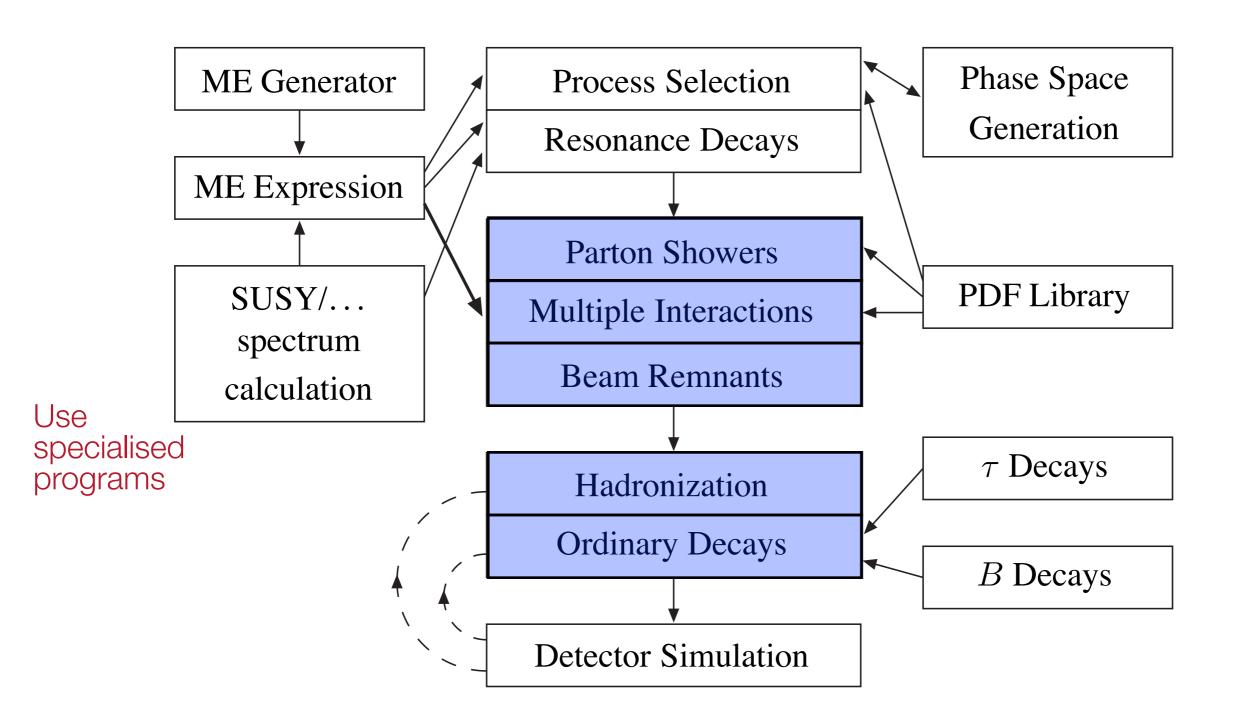
Geant4: ATLAS Geometry
[cut-away view]

[geant4.kek.jp/~tanaka/GEANT4/ATLAS G4 GIFFIG/]

Detector response

Developed at CERN since 1974 (FORTRAN)

[Today: Geant4; programmed in C++]



Strong interactions:

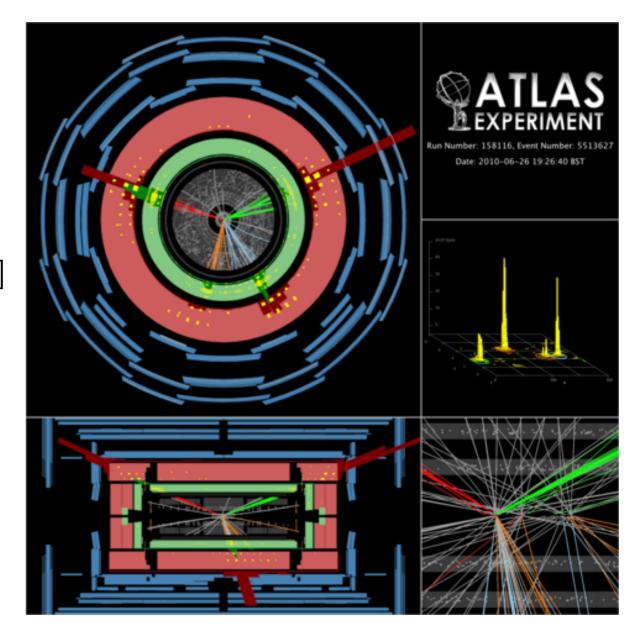
No free Quarks

Expect jets

i.e. bundles of particles at high energies [hadron p_T range limited w.r.t. initial parton]

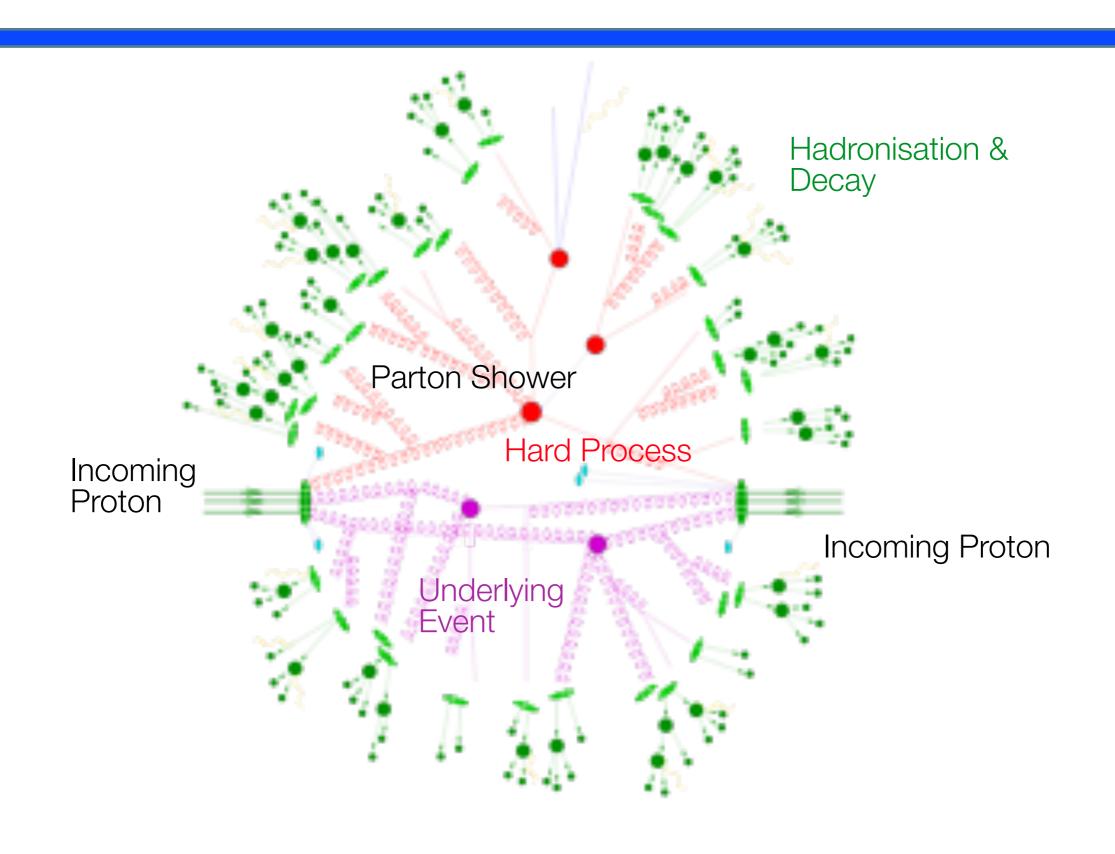
First observation of jets in e⁺e⁻ collisions @ $E_{CMS} > 6$ GeV [SPEAR, SLAC, 1975]

Later also observed in hadron-hadron collisions [e.g. @ CERN ISR]



An event with 4 jets @ LHC

Goal: Infer parton properties from jet properties [need to calculate and/or model fragmentation & hadronisation process]



Possible simulation approaches

Pure matrix element (ME) simulation:

MC integration of cross section & PDFs, no hadronisation (recall: cross section = |matrix| element |2| |2| phase space)

Useful for theoretical studies, no exclusive events generated

[Example: MCFM (http://mcfm.fnal.gov); many LHC processes up to NLO, HNNLO (http://theory.fi.infn.it/grazzini/codes.html) Higgs production at NNLO]

Event generators:

Combination of ME and parton showers ...

Typical: generator for leading order ME combined with leading log (LL) parton shower MC (see later)

Exclusive events → useful for experimentalists ...

Parton showers

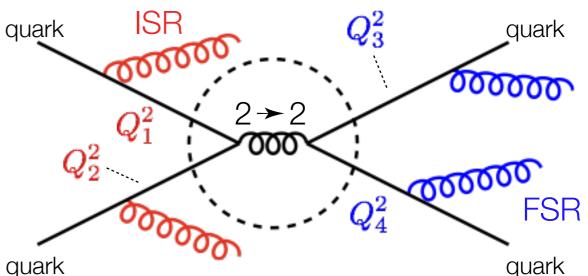
A realistic simulation needs many particles in the final state, it is quite difficult (sometimes impossible) to compute a pp $(2) \rightarrow$ many particles process

$$(2 \rightarrow n) = \dots$$

 $\dots = (2 \rightarrow 2) \oplus ISR \oplus FSR$

FSR: Final state radiation

Q² ~ p_{quark in²} ~ m² > 0 decreasing [time-like shower] $p_{\rm quark \, in} = p_{\rm quark \, out} + p_{\rm gluon}$



Calculable

ISR: Initial state radiation

 $Q^2 \sim p_{quark out}^2 \sim -m^2 > 0$ increasing

[space-like shower]

$$p_{\text{quark out}} = p_{\text{quark in}} - p_{\text{gluon}}$$

Hard process $[2 \rightarrow 2]$:

$$\sigma = \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\widehat{t} \, f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\widehat{\sigma}_{ij}}{\mathrm{d}\widehat{t}}$$

Shower evolution:

Viewed as probabilistic process, which occurs with unit total probability; cross section not directly affected; only indirectly via changed event shape.

Parton showers

$$p_1 + p_2 + p_3 = (E_{cm}, 0)$$
 $E_1 + E_2 + E_3 = E_{cm}$ $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$$Q^{2} = m_{13}^{2} = (p_{1} + p_{3})^{2} = (E_{1} + E_{3}, \vec{p}_{1} + \vec{p}_{3})^{2} = (E_{cm} - E_{2}, -\vec{p}_{2})^{2} = E_{cm}^{2} + E_{2}^{2} - 2E_{cm}E_{2} - |\vec{p}_{2}|^{2}$$

$$Q^{2} = m_{13}^{2} = (p_{1} + p_{3})^{2} = (E_{1} + E_{3}, \vec{p}_{1} + \vec{p}_{3})^{2} = (E_{cm} - E_{2}, -\vec{p}_{2})^{2} = E_{cm}^{2} + E_{2}^{2} - 2E_{cm}E_{2} - |\vec{p}_{2}|^{2}$$

$$= E_{\rm cm}^2 \left(1 - \frac{2E_2}{E_{\rm cm}} \right) + E_2^2 - |\vec{p}_2|^2 \right) \implies \frac{Q^2}{E_{\rm cm}^2} = 1 - x_2$$

gluon splitting

$$Q^{2} = m_{2}$$
 $Q^{2} = m_{2}$
 $Q^{2} = m_{2}$
 $Q^{2} = m_{2}$
 $Q^{2} = m_{2}$

$$1 - x_2 = rac{m_{13}^2}{E_{
m cm}^2} = rac{Q^2}{E_{
m cm}^2}$$

$$m_{13}^2=2E_1E_3(1-cos\theta)$$
 $x_2\to 1\Rightarrow m_{13}^2\to 0 \Rightarrow \theta\to 0$ collinear limit

$$dx_2 = -\frac{dQ^2}{E_{\rm cm}^2}$$

Rewrite for
$$x_2 \rightarrow 1$$
: [qg collinear limit]
$$z = \frac{E_1}{E_b} = \frac{2E_1}{E_{\rm cm}} = x_1$$
$$x_1 + x_2 + x_3 = 2$$

$$E_q = E_1 = zE_b$$

$$E_b = E_1 + E_3 = \frac{E_{cm}}{2}(x_1 + x_3) = \frac{E_{cm}}{2}(2 - x_2) = \frac{E_{cm}}{2}$$

$$E_g = E_3 = (1 - z)E_b$$

$$d\mathcal{P} = \frac{d\sigma_{\rm qqg}}{\sigma_0} = \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \frac{dx_2}{(1-x_2)} \cdot \frac{x_1^2 + x_2^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \cdot \frac{dQ^2}{Q^2} \cdot \frac{4}{3} \left[\frac{1+z^2}{1-z} \right] dz \qquad x_3 \approx 1-z$$

$$z \to 1 \Rightarrow E_g \to 0 \quad \text{soft divergence}$$

$$d\mathcal{P}_{a\to bc}=rac{lpha_s}{2\pi}rac{dQ^2}{Q^2}P_{a\to bc}(z)dz$$
 Splitting probability determined by splitting functions $P_{q imes qg}$

Splitting Function P_{q→qg} q

Analogous splitting functions used in PDF evolution

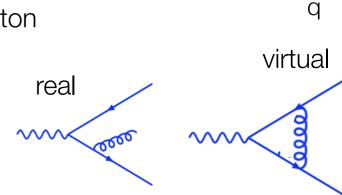
$$P_{ ext{q} o ext{qg}} = rac{4}{3} rac{1 + z^2}{1 - z}$$
 $P_{ ext{g} o ext{gg}} = 3 rac{(1 - z(1 - z))^2}{z(1 - z)}$

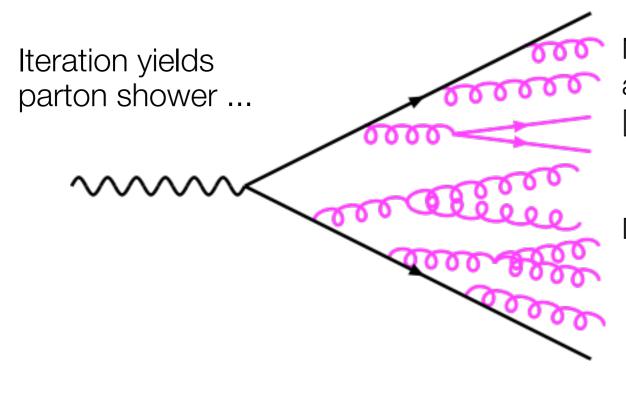
$$P_{\mathrm{g} o\mathrm{gg}} = 3 \overline{z(1-z)}$$
 $P_{\mathrm{g} o\mathrm{q}ar{\mathrm{q}}} = rac{n_f}{2} (z^2 + (1-z)^2)$

z: fractional momentum of radiated parton

n_f: number of quark flavours

In NLO calculations soft and $P_{
m g
ightarrow gg} = 3rac{(1-z(1-z))^2}{z(1-z)}$ In NLO calculations soft and collinear divergencies cancelled by virtual contributions: they persist in LO calculations.





Need soft/collinear cut-offs to avoid non-perturbative regions ... [divergencies!]

Details model-dependent

e.g.
$$Q > m_0 = min(m_{ij}) \approx 1$$
 GeV, $z_{min}(E,Q) < z < z_{max}(E,Q)$ or $p_{\perp} > p_{\perp min} \approx 0.5$ GeV

Parton shower evolution 1

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

Time evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

$$\mathcal{P}_{\text{nothing}}(0 < t \le T) = \lim_{n \to \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \le T_{i+1})$$

$$= \lim_{n \to \infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})) \qquad e^{-x} \approx 1 - x$$

$$= \exp\left(-\lim_{n \to \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right)$$

$$= \exp\left(-\int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt}dt\right)$$
[Taylor]

$$\rightarrow d\mathcal{P}_{first}(T) = d\mathcal{P}_{something}(T) \exp\left(-\int_0^T \frac{d\mathcal{P}_{something}(t)}{dt}dt\right)$$

Parton shower evolution 2

Instead of evolving to later and later times need to evolve to smaller and smaller Q² ... [Heisenberg: Q ~ 1/t]

Sudakov Form Factor

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) dz \exp\left(-\sum_{b,c} \int_{Q^2}^{Q_{\text{max}}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a\to bc}(z') dz'\right)$$

Probability to radiated with virtuality Q²

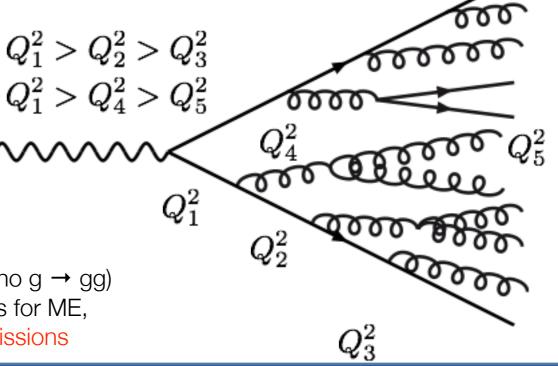
No radiation for higher virtualities i.e. for Q² ... Q²_{max}

Note that $\sum_{b,c} \iint dP_{a \to bc} = 1...$ [Convenient for Monte Carlo]

Sudakov form factor ...

- ... provides "time" ordering of shower ... [lower Q² \Leftrightarrow longer times]
- ... regulates singularity for first emission ...

But in the limit of repeated soft emissions $q \rightarrow qg$ (but no $g \rightarrow gg$) one obtains the same inclusive Q emission spectrum as for ME, i.e. divergent ME spectrum \Leftrightarrow infinite number of PS emissions



Sudakov picture of parton showers

Basic algorithm: Markov chain

[each step requires only knowledge of the previous step]

- (i) Start with virtuality Q1 and momentum fraction x1
- (ii) Generate target virtuality Q2 with random number RT uniform distributed in [0,1]

Probability to not have $Q_x > Q_2$

using:
$$\Delta(Q_i^2) = \exp\left(-\sum_{b,c} \int_{Q_i^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \to bc}(z') \, dz'\right) \quad \text{ solve the equation for } Q_2 \qquad R_t = \frac{\Delta(Q_2^2)}{\Delta(Q_1^2)}$$

[probability to evolve from t1 to t2 without radiation]

 γ 0 $Q^2 = m_{13}^2$

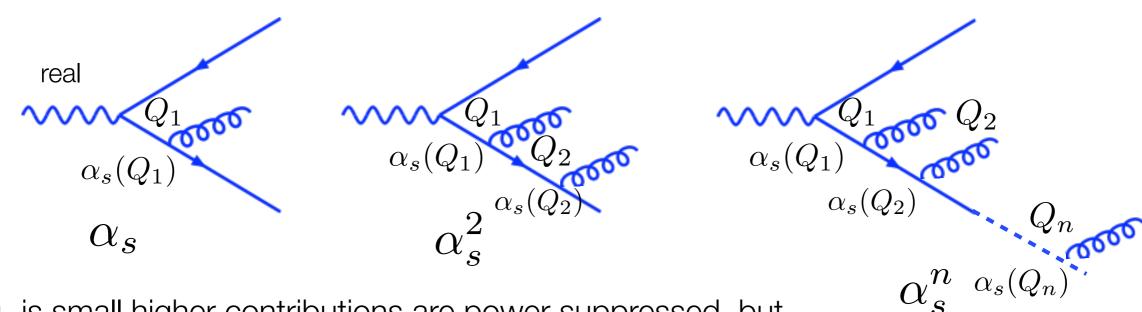
(iii) Q₂ known (x₂ known), need to compute x₁~z

$$P_{\rm q\to qg}=\frac{4}{3}\frac{1+z^2}{1-z} \qquad R_z=\frac{\int_0^z P(z')dz'}{\int_0^1 P(z')dz'} \qquad \text{flat distributed} \qquad R_z\in [0,1]$$

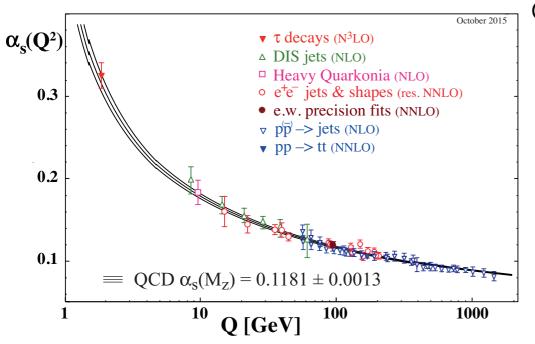
1 (iv) Generate random azimuthal angle Φ flat distributed

Process ends when partons are below threshold (p_T,Q)

Parton shower and logarithmic resummation



If as is small higher contributions are power suppressed, but...



as increases at small Q2

$$\alpha_s(Q_n) \sim \alpha_s(Q_1) ln(Q_1/Q_n)$$

$$\alpha_s(Q_1) + \alpha_s(Q_1)\alpha_s(Q_2) + \dots + \alpha_s(Q_1) \cdot \dots \cdot \alpha_s(Q_n)$$

$$\sim [\alpha_s(Q_1) ln(Q_1)]^2 \sim [\alpha_s(Q_1) ln(Q_1)]^n$$

if $\alpha_s(Q_1)ln(Q_1)$

is large, the expansion is broken, PS allows to sum up all the large contributions [Leading Log resummation]

Parton shower ordering

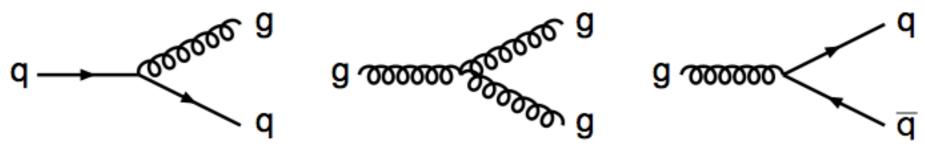
$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) dz \exp\left(-\sum_{b,c} \int_{Q^2}^{Q_{\text{max}}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a\to bc}(z') dz'\right)$$

In the splitting function appears only dQ^2/Q^2 , therefore if $P = f(z)Q^2 dP/P = dQ^2/Q^2$

Three main approaches to showering in use:

$$p_{\perp}^2 pprox z(1-z)m^2$$
 pt ordered showers $E^2\theta^2 pprox m^2/(z(1-z))$ angular ordered showers

Two are based on the standard shower language of a \rightarrow bc successive branchings:

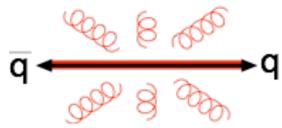


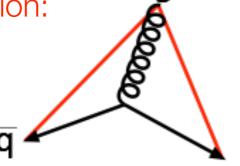
HERWIG, HERWIG++ : $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$

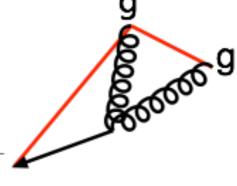
PYTHIA, 8 (basic) : $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

PYTHIA6, 8 (p_T oredered): mixture: collinear splitting but di-pole kinematic





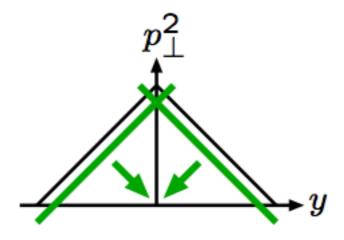




Ariadne : $Q^2 = p^2_{\perp}$; FSR mainly, ISR is primitive ... used also in sherpa and Pythia8

consider the full recoil and not only the branching

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) \sim \eta = -\ln \left[\left(\tan \frac{\theta}{2} \right) \right]$$

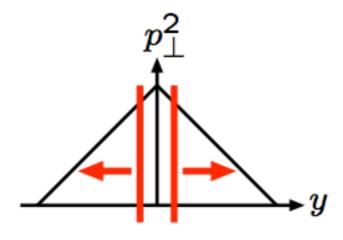


Large mass first ["hardness" ordered]

Covers phase space ME merging simple g → qq simple not Lorentz invariant no stop/restart

ISR: $m^2 \rightarrow -m^2$

PYTHIA: $Q^2 = m^2$ HERWIG/++: $Q^2 \sim E^2\theta^2$

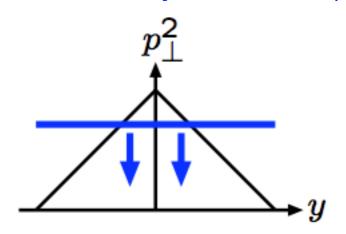


Large angle first [not "hardness" ordered]

Gaps in coverage ME merging messy g → qq simple not Lorentz invariant no stop/restart

ISR: $\theta \rightarrow \theta$

ARIADNE/Pythia8: $Q^2 = p^2_{\perp}$



Large p_⊥ first ["hardness" ordered]

Covers phase space ME merging simple g → qq messy Lorentz invariant can stop/restart

ISR: complicated

Color coherence

QED: Chudakov effect (mid-fifties)



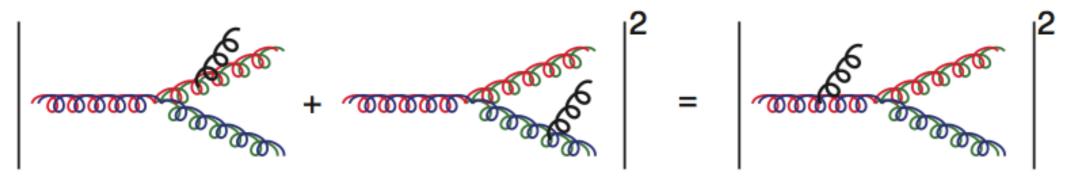
emulsion plate

reduced ionization

normal ionization

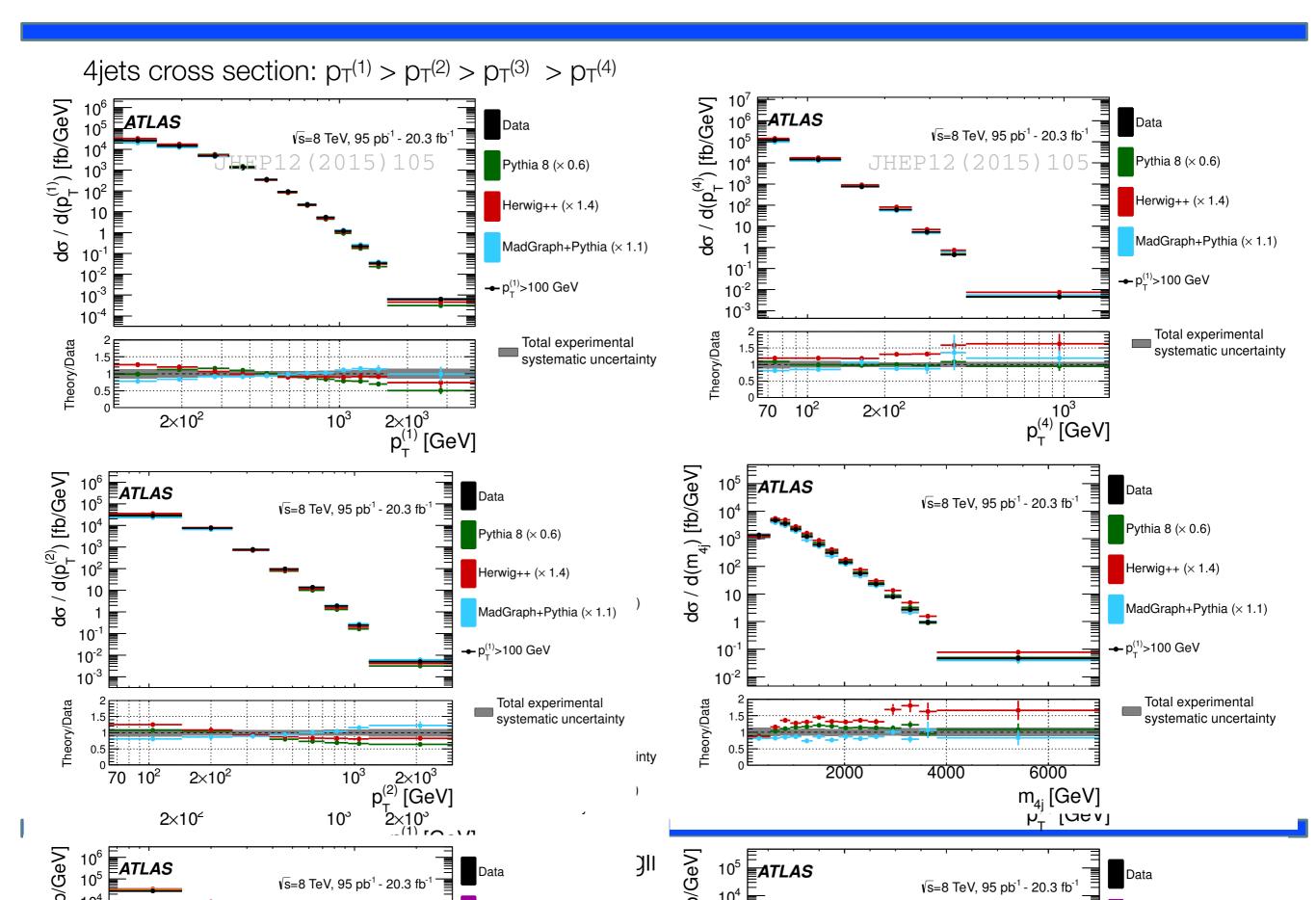
- 1. soft gluons see the pair of split gluons as a whole, color screening reduce their emission
- 2. angular ordered and p_T ordered PS reproduce the correct color coherence
- 3. Pythia Q² needs a-posteriori corrections

QCD: colour coherence for soft gluon emission



- solved by
- requiring emission angles to be decreasing
- or requiring transverse momenta to be decreasing

Compariosn to LHC data



Example of processes implemented in Pythia6

	No. Subprocess	No. Subprocess	No. Subprocess	No. Subprocess	No. Subprocess	No. Subprocess	No. Subprocess	
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$					169 $q_i \overline{q}_i \rightarrow e^{\pm} e^{*\mp}$	216 $f_i f_i \rightarrow \chi_1 \chi_1$		2
91 elastic scattering 29 single diffraction (AX) 32 single diffraction (AX) 33 single diffraction (AX) 39 single diffraction (AX) 30 $f_1 = f_2 = f_3 =$	00 00					217 $f_i f_i \rightarrow \chi_2 \chi_2$		
92 single diffraction (XB) 30 single diffraction (XB) 30 single diffraction (XB) 10 $f_1f_1 - f_2f_1 = f_3f_1 - f_3f_1 = f_3f_1 - f_3f_2 = f_3f_1 - f_3f_2 = f_3f_1 - f_3f_3 $		00				218 $f_i f_i \rightarrow \tilde{\chi}_3 \tilde{\chi}_3$	261 f.f f.f.	
93 single diffraction (AX) 94 double diffraction (AX) 95 dow- p_{\perp} production (AX) 95 (AX) 97 (AX) 97 (AX) 97 (AX) 97 (AX) 97 (AX) 97 (AX) 98 (AX) 99 (AX) 98 (AX) 98 (AX) 99 (AX) 98 (AX) 99 (AX) 98 (AX) 99		00 01			_ 1			
98 double diffraction 95 low-p_production Photon-induced: Show-p_production 199 $\gamma' q \rightarrow q$ Photon-induced: 185 Never production 190 $\gamma' q \rightarrow q$ 151 $if_i \rightarrow H^0$ 152 $gg \rightarrow h^0$ 152 $gg \rightarrow h^0$ 153 $\gamma \gamma \rightarrow H^0$ 153 $\gamma \gamma \rightarrow H^0$ 154 $gg \rightarrow h^0$ 155 $if_i \rightarrow H^0$ 156 $if_i \rightarrow P^0$ 157 $if_i \rightarrow P^0$ 158 $if_i \rightarrow P^0$ 159 $if_i \rightarrow P^0$ 190 $if_i \rightarrow P^0$ 191 $if_i \rightarrow P^0$ 192 $if_i \rightarrow P^0$ 193 $if_i \rightarrow P^0$ 194 $if_i \rightarrow P^0$ 195 $if_i \rightarrow P^0$ 196 $if_i \rightarrow P^0$ 197 $if_i \rightarrow P^0$ 197 $if_i \rightarrow P^0$ 198 $if_i \rightarrow P^0$ 199 $if_i \rightarrow P^0$ 199 $if_i \rightarrow P^0$ 190 $if_i \rightarrow P^0$ 190 $if_i \rightarrow P^0$ 190 $if_i \rightarrow P^0$ 191 $if_i \rightarrow P^0$ 192 $if_i \rightarrow P^0$ 193 $if_i \rightarrow P^0$ 195 $if_i \rightarrow P^0$ 196 $if_i \rightarrow P^0$ 197 $if_i \rightarrow P^0$ 197 $if_i \rightarrow P^0$ 198 $if_i \rightarrow P^0$ 199 $if_i \rightarrow P^0$			73 $Z_L W_L \rightarrow Z_L W_L$ 76 $W^+W^ Z^0Z^0$		_			L
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			44 44 44			222 $f_i f_i \rightarrow \tilde{\chi}_1 \tilde{\chi}_4$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				00 110		223 $f_i \bar{f}_i \rightarrow \tilde{\chi}_2 \tilde{\chi}_3$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						224 $f_i \bar{f}_i \rightarrow \tilde{\chi}_2 \tilde{\chi}_4$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						225 $f_i \bar{f}_i \rightarrow \tilde{\chi}_3 \tilde{\chi}_4$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						226 $f_i \bar{f}_i \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_1^{\mp}$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$						227 $f_i \bar{f}_i \rightarrow \tilde{\chi}_2^{\pm} \tilde{\chi}_2^{\mp}$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$,	195 $f_i \bar{f}_j \rightarrow f_k \bar{f}_l$		228 $f_i \bar{f}_i \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_2^{\mp}$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.22				229 $f_i\bar{f}_i \rightarrow \tilde{\gamma}_1\tilde{\gamma}_1^{\pm}$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				$362 f_i \bar{f}_i \rightarrow W_L^{\pm} \pi_{tc}^{\mp}$		230 fif $\rightarrow \tilde{v}_2 \tilde{v}_1^{\pm}$		
86 $gg \to \chi_0 eg$ 136 $gg \to f_1 ef_1$ 182 $g_1 ef_1 \to Q_0 ef_1$ 183 $f_1 ef_1 \to gg \to $	-			$363 f_i \bar{f}_i \rightarrow \pi_{tc}^+ \pi_{tc}^-$		231 $f_i \bar{f}_i \rightarrow \tilde{\chi}_3 \tilde{\chi}_1^{\pm}$., ., .,	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		_		$364 f_i \overline{f}_i \rightarrow \gamma \pi_{tc}^0$	340 $\epsilon_i \gamma \rightarrow \Pi_R \mu$ 347 $\epsilon_{\pm} \gamma \rightarrow \Pi_R \mu$	232 f.f V.V.	00 110 10	
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	440		157 gg \rightarrow A ⁰			230 $I_iI_j \rightarrow \chi_4\chi_2^-$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			158 $\gamma \gamma \rightarrow A^0$	_ ~ ~		$237 f_i f_i \rightarrow g \chi_1$	285 $b\overline{q}_i \rightarrow \tilde{b}_2\tilde{q}_i^*$	R
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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$								
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		24 $f_i \bar{f}_i \rightarrow Z^0 h^0$	178 $f_i f_j \rightarrow f_i f_j A^0$	373 $I_iI_j \rightarrow \pi_{tc}\pi_{tc}$	_			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$								
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$32 f_i g \rightarrow f_i h^0$	186 gg $\rightarrow Q_k \overline{Q}_k A^0$			242 $f_i f_j \rightarrow \tilde{g} \tilde{\chi}_2^{\pm}$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$102 \text{ gg} \rightarrow \text{h}^0$	187 $q_i \overline{q}_i \rightarrow Q_k \overline{Q}_k A^0$					
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		103 $\gamma \gamma \rightarrow h^0$	188 $f_i \bar{f}_i \rightarrow gA^0$			$244 gg \rightarrow \tilde{g}\tilde{g}$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	25 $f_i \bar{f}_i \rightarrow W^+W^-$	110 $f_i \bar{f}_i \rightarrow \gamma h^0$	189 $f_i g \rightarrow f_i A^0$,,	205 $f_i \bar{f}_i \rightarrow \tilde{\mu}_R \tilde{\mu}_R^*$			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	15 $f_i \bar{f}_i \rightarrow gZ^0$				206 $f_i \bar{f}_i \rightarrow \tilde{\mu}_L \tilde{\mu}_R^* +$			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			Charged Higgs:	,				
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$30 f_i g \rightarrow f_i Z^0$	113 $gg \rightarrow gh^0$				249 $f_i g \rightarrow \tilde{q}_{iR} \tilde{\chi}_2$	295 bg → b2g	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				00,	_		290 $DD \rightarrow D_1D_2^2 +$	†
20 $f_i \bar{f}_j \rightarrow \gamma W^{\pm}$ 123 $f_i f_j \rightarrow f_i f_j h^0$ 402 $q \bar{q} \rightarrow \bar{t} b H^+$ $\frac{387}{200} f_i f_i \rightarrow Q_k Q_k$								
			$402 q\overline{q} \rightarrow \overline{t}bH^{+}$					
		124 $f_i f_j \rightarrow f_k f_l h^0$		$388 \text{ gg} \rightarrow Q_kQ_k$				

Process simulation

Many specialised processes already available in Pythia8/Herwig++

but, processes usually only implemented at the lowest non-trivial order ...

Need external programs that ...

- 1. include higher order loop corrections or, alternatively, do kinematic dependent rescaling
- 3. allow matching of higher order ME generators [otherwise need to trust parton shower description ...]
- 5. provide correct spin correlations often absent in PS ...[e.g. top produced unpolarised, while t → bW → blv decay correct]
- 7. simulate newly available physics scenarios ...[appear quickly; need for many specialised generators]

Les Houches Accord ...

Specifies how parton-level information about the hard process and sequential decays can be encoded and passed on to a general-purpose generator.

Les Houches: regular annual meeting between theoreticians and experimentalists on MC generator developments.



Specialised Generators [some examples]

Specialized Generator

[→ Hard Process]



Les Houches Interface



Herwig, Pythia, Herwig++/7, Pythia8

[Resonance Decays]

Parton Showers

Underlying Event

Hadronization

Ordinary Decays

AcerMC: ttbb, .single top

ALPGEN : $W/Z + \leq 6i$,

 $nW + mZ + kH + \leq 3j$, ...

AMEGIC++ : generic LO

CompHEP: generic LO

GRACE: generic LO

[+Bases/Spring] [+ some NLO loops]

GR@PPA : bbbb

MadCUP : $W/Z+ \le 3j$, ttbb

HELAS & : generic LO

MadGraph

MCFM : NLO W/Z+ $\leq 2j$,

WZ, WH, $H+ \leq 1j$

O'Mega & WHIZARD

: generic LO

VECBOS : $W/Z+ \le 4j$

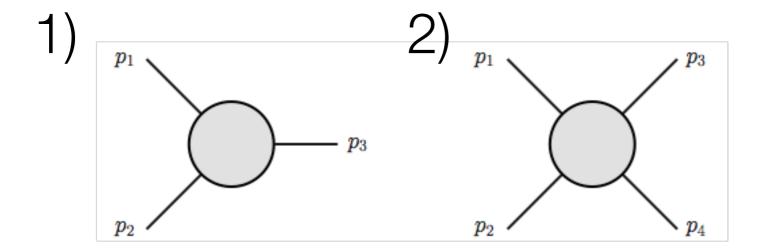
HRES : Higgs boson production

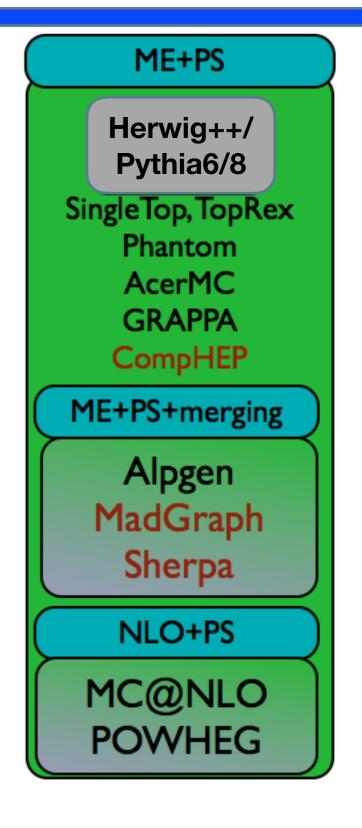
@NNLO

DYNNLO: W/Z production @NNLO

Type I: Leading order matrix element & leading log parton shower

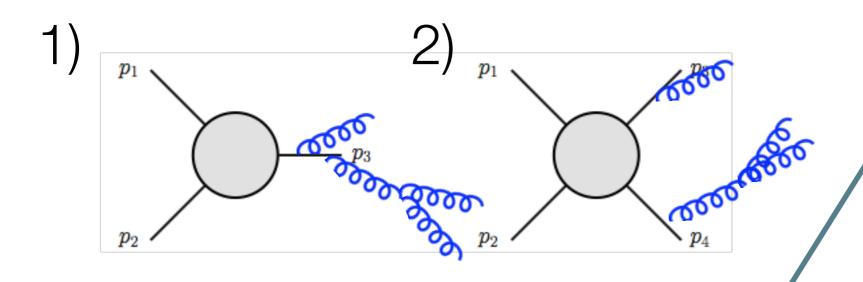
LO ME for hard processes $[2\rightarrow 1 \text{ or } 2\rightarrow 2]$



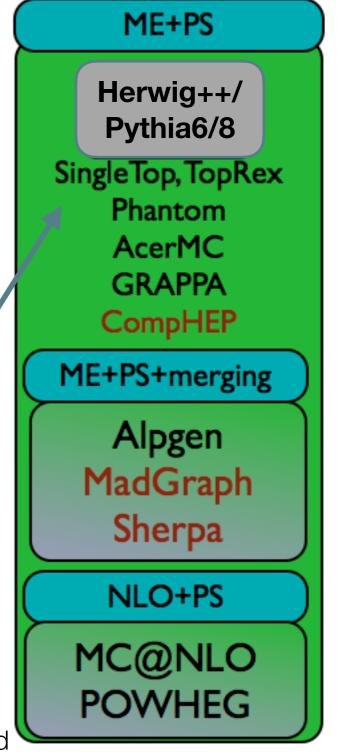


Type I: Leading order matrix element & leading log parton shower

LO ME for hard processes [2→1 or 2→2]

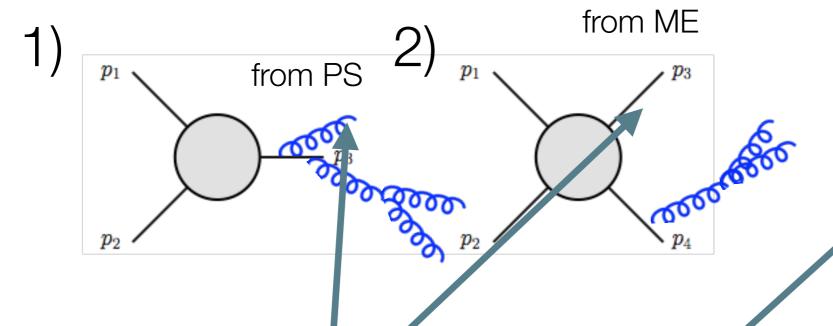


- Parton Shower: attaches gluons at each leg.
- only in soft-collinear approximation
- typically underestimate large angle/hard emission
- 1) or 2) at ME (different generations, different accuracy: cannot be combined

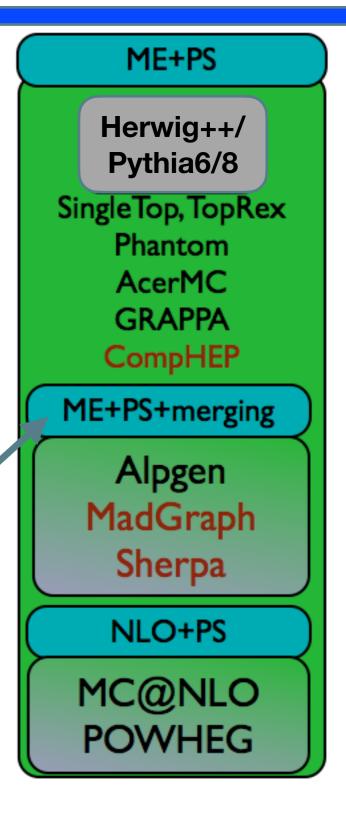


Type 2 : Leading order matrix element & leading log parton shower + merging

LO ME for hard processes [2→1 or 2→2]



- Type 1 can be improved using 1) + 2)
- use ME calculation for hard large angle jets
- but needs to remove double-counting: merging
 - CKKW: Catani, Krauss, Kuhn, Weber (Sherpa)
 - MLM
- very good description of high jet multiplicity kinematics



Merging @LO

MLM matching (simplified)

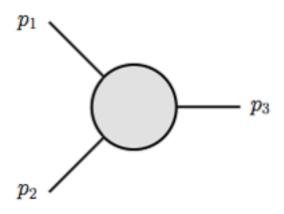
1) define matching cuts: for example $p_T^J > 20$ GeV, $\Delta R = 0.4$

B. Di Micco

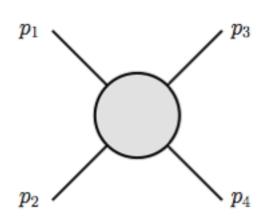
MLM matching (simplified)

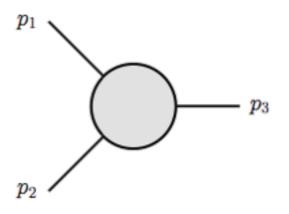
- 1) define matching cuts: for example $p_T^J > 20$ GeV, $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets

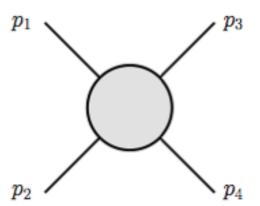
1 parton



2 partons





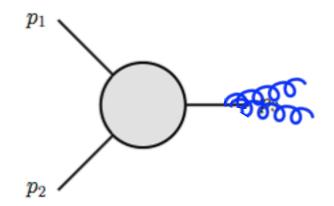


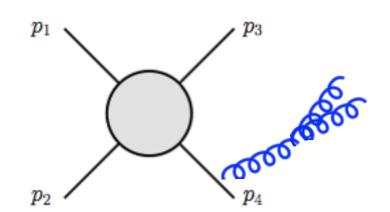
MLM matching (simplified)

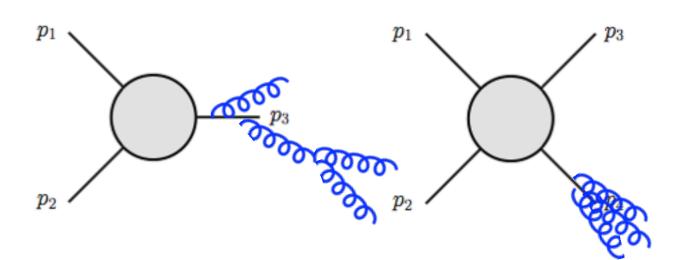
- 1) define matching cuts: for example $p_T^J > 20$ GeV, $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events





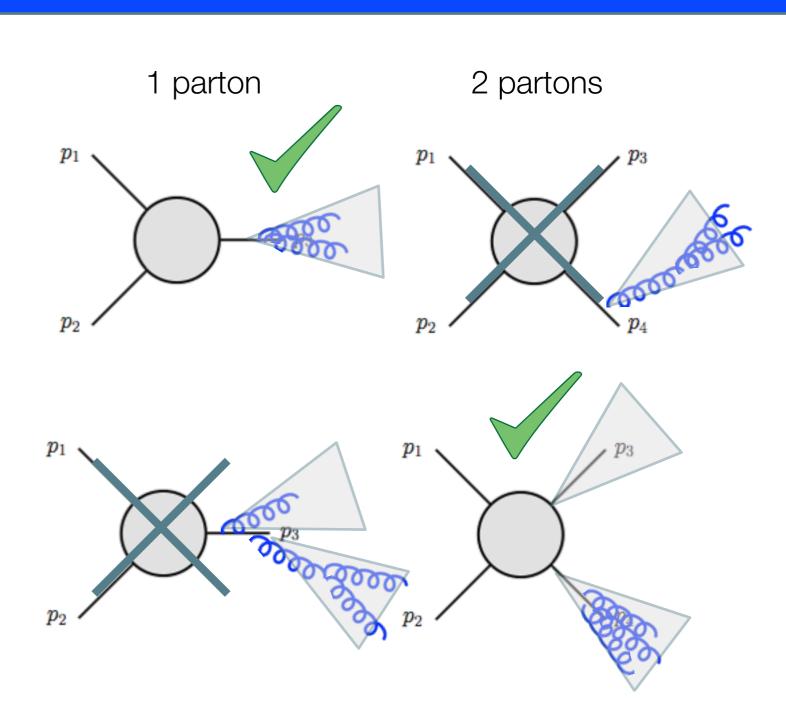






MLM matching (simplified)

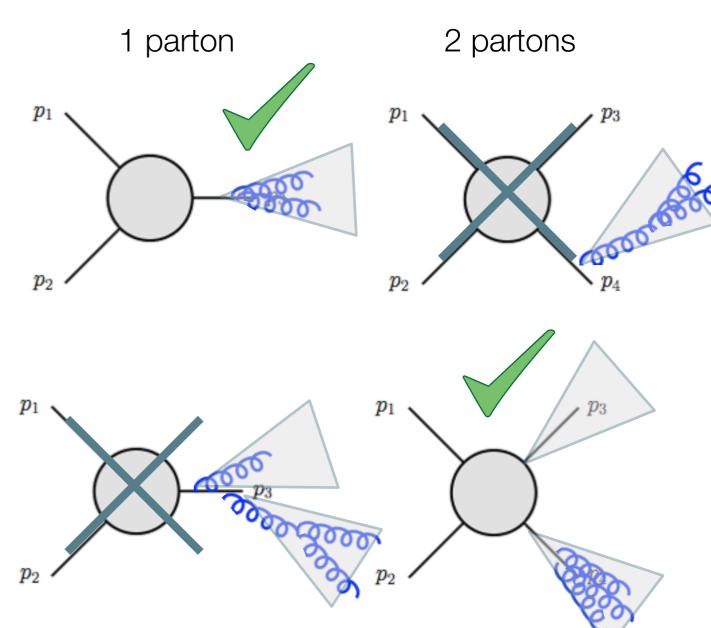
- 1) define matching cuts: for example $p_T^J > 20$ GeV, $\Delta R = 0.4$
- 2) generate ME with 1, 2, ... n jets
- 3) shower all events
- 4) select only events where jets above the p_T threshold match with final partons



B. Di Micco

MLM matching (simplified)

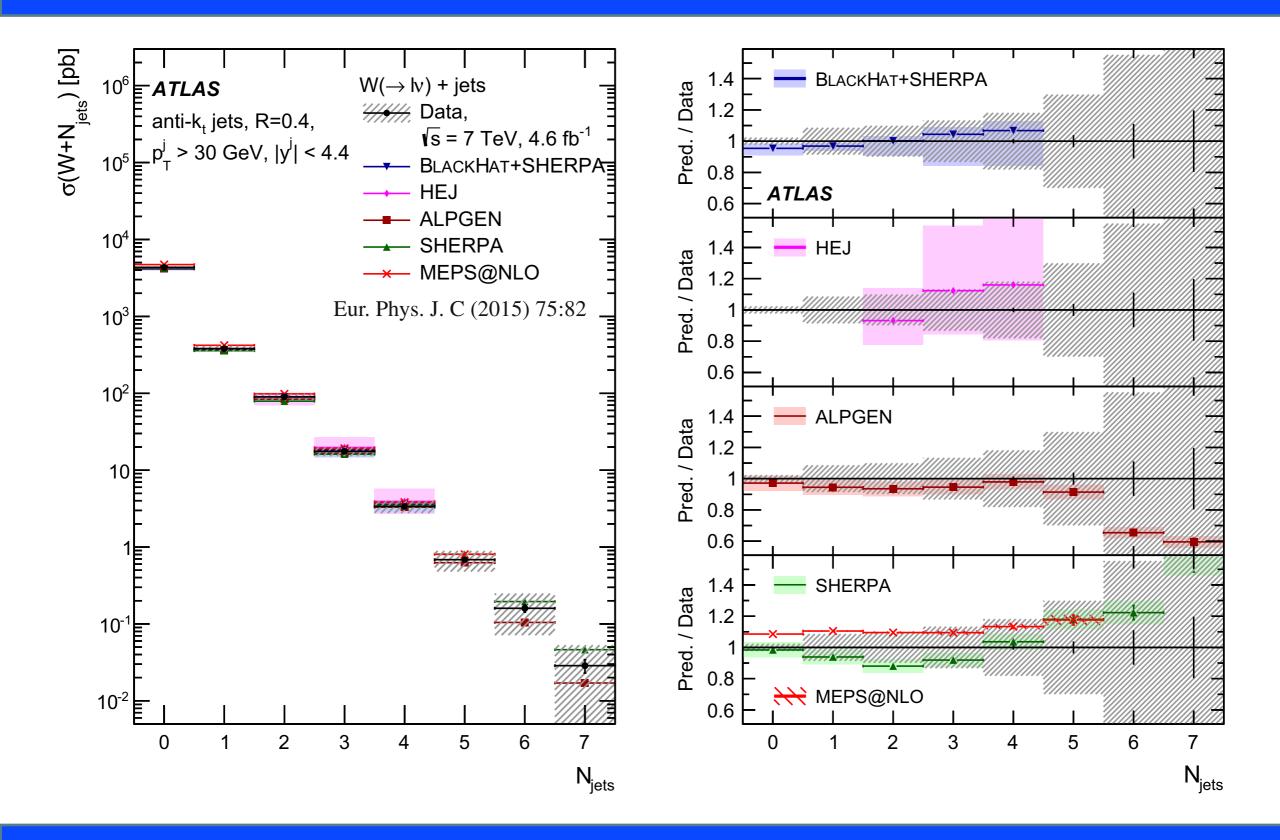
- 1) define matching cuts: for example $p_T^J > 20$ GeV, $\Delta R = 0.4$
- 2) generate ME with 1, 2, ... n jets
- 3) shower all events
- 4) select only events where jets above the p_T threshold and satisfying the ΔR cut match with final partons



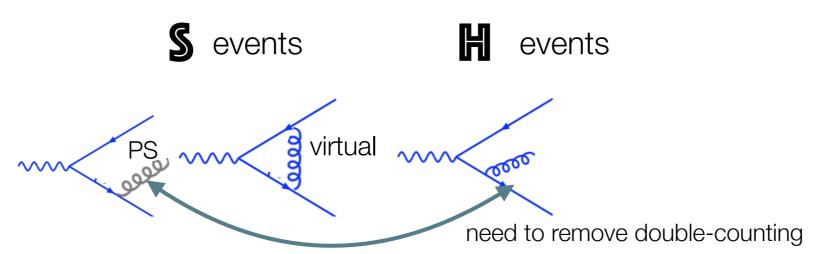
Consequences:

all jets with $p_T > 20$ GeV and $\Delta R > 0.4$ to other jets come from ME collinear and soft jets come from PS Use ME and PS where they perform better.

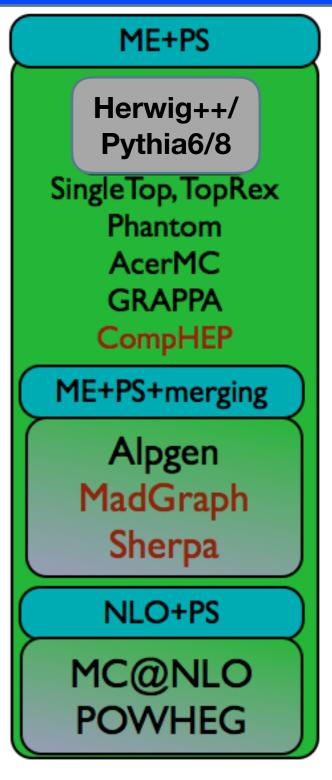
W+jets distributions



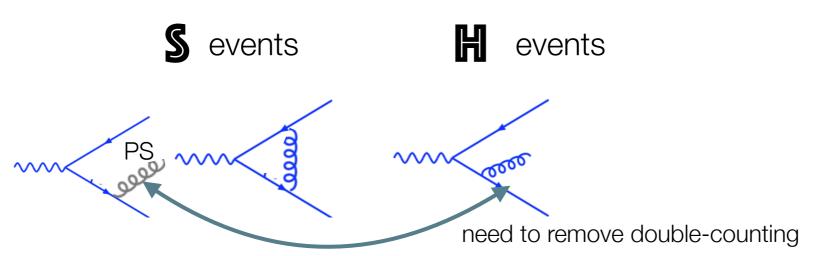
Type III: Next-to-leading order ME & leading-log parton shower



- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions



Type 3: Next-to-leading order ME & leading-log parton shower



- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions

Two matching methods:

Truncated showers:

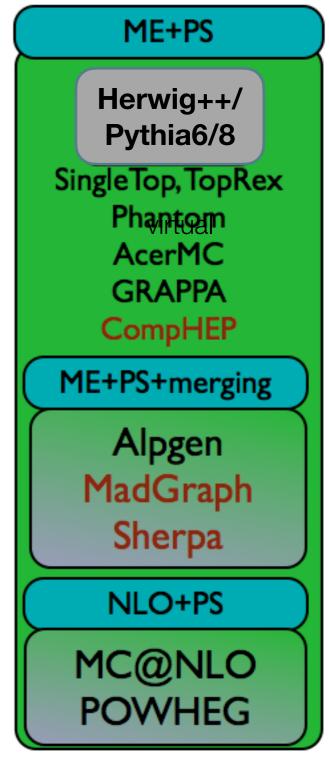
1. Powheg

- 1) first emission produced by the ME;
- don't allow the PS to produce patrons harder than the first emission;
- not exact at NLO (containes unbalanced higher order terms)

2. MC@NLO:

 $|ME|^2 = |ME + PS - PS(up to a_s^2)|^2$

- + Result is exact at NLO...
- produce some negative weights, it needs retuning for each PS

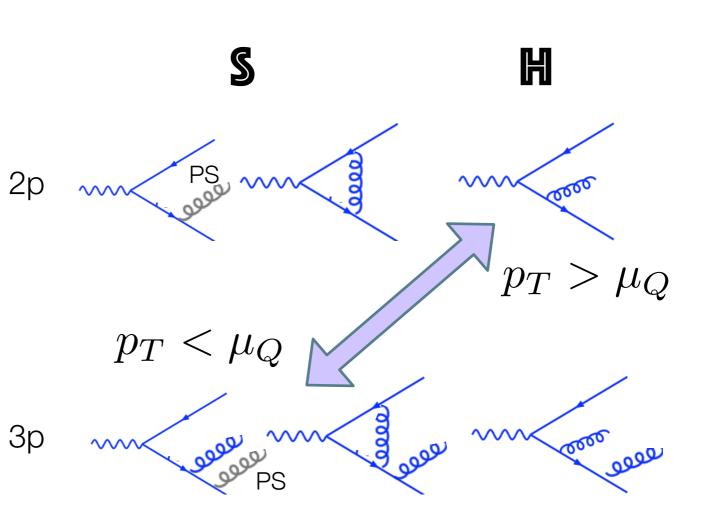


Merging @NLO (used starting from LHC at 13 TeV)

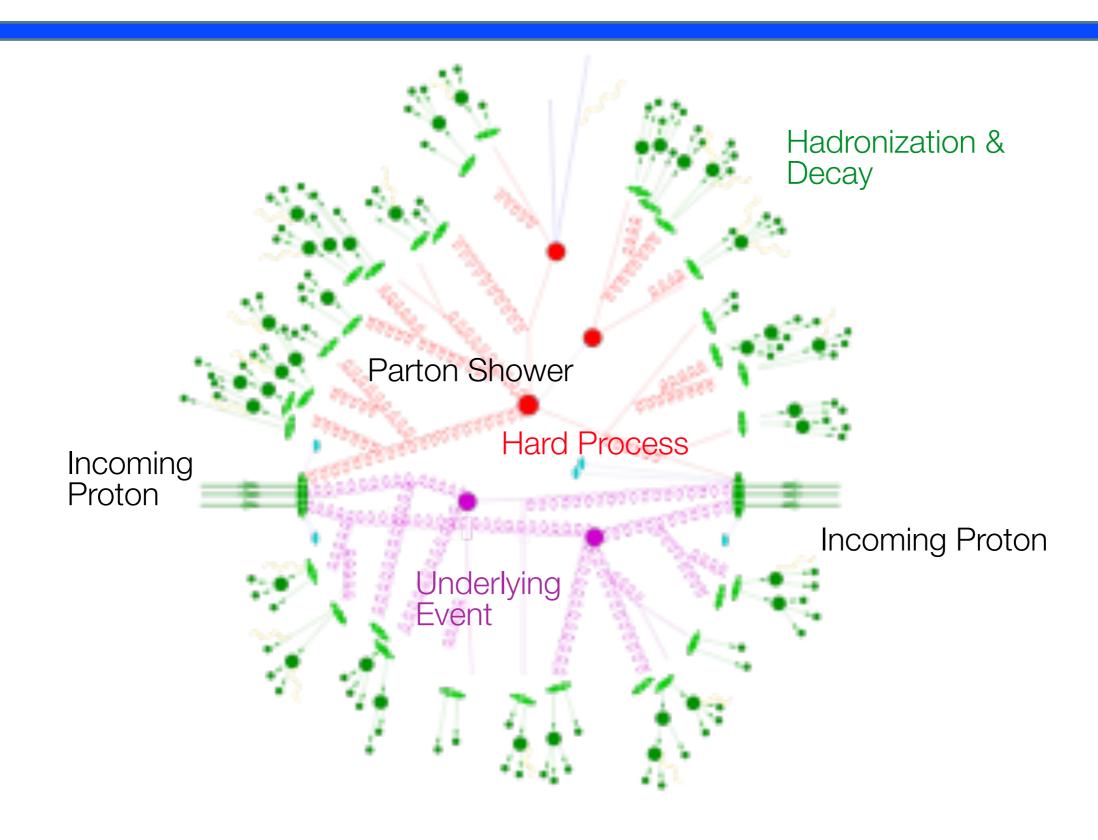
JHEP12 (2012) 061

FxFx (Frederix-Frixione) merging

- 1) define a matching scale μ_Q ;
- 2) don't allow **S** events with $p_T > \mu_Q$ (those will be provided by **H** events of n-1 partons NLO real emission); the restriction is imposed both at ME and on the shower starting scale $\mu < \mu_Q$
- 3) treat the obtained events as LO ones and apply an LO-style merging (this allow to produce smoother distributions)



Let's recap



From partons to color neutral hadrons

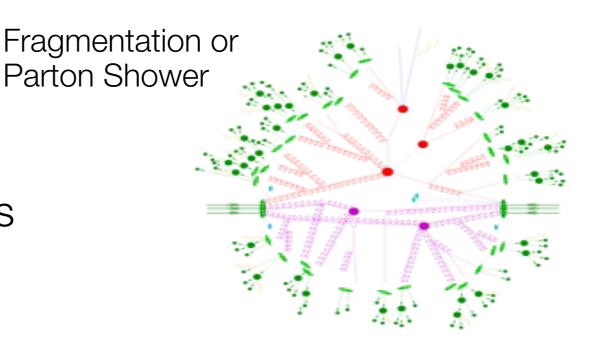
Fragmentation:

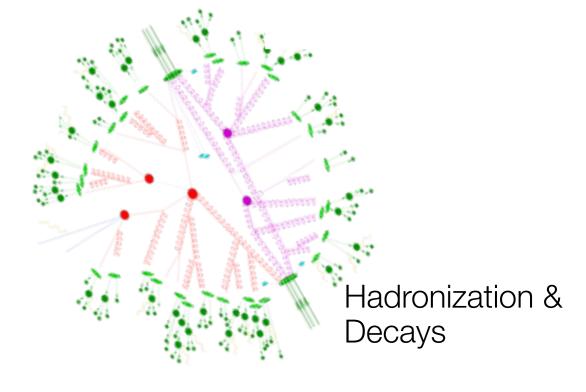
Parton splitting into other partons [QCD: re-summation of leading-logs] ["Parton shower"]

Hadronization:

Parton shower forms hadrons [non-perturbative, only models]

Decay of unstable hadrons [perturbative QCD, electroweak theory]





Non-perturbative transition from partons to hadrons ...

[Modelling relies on phenomenological models available]

Models based on MC simulations very successful:

Generation of complete final states ... [Needed by experimentalists in detector simulation]

Caveat: tunable ad-hoc parameters

Most popular MC models:

Pythia/8: Lund string model

Sherpa, Herwig/++: Cluster model

Independent fragmentation of each parton

Simplest approach: [Field, Feynman, Nucl. Phys. B136 (1978) 1]

Start with original quark
Generate quark-antiquark pairs
from vacuum

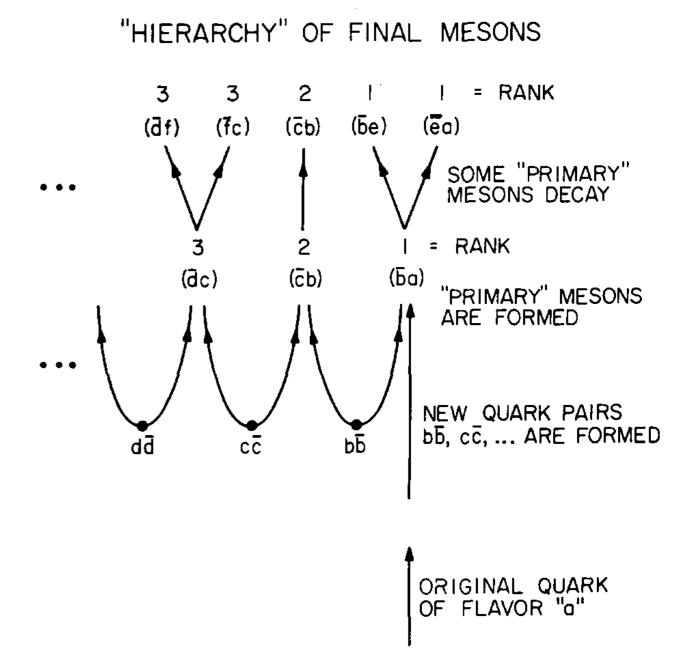
form "primary meson" with energy fraction z

Continue with leftover quark with energy fraction 1-z

Stop at low energies (cut-off)

Include flavour non-perturbative fragmentation functions D(z)

D(z): probability to find a meson/hadron with energy fraction z in jet ...



Lund String Model

[Andersson et al., Phys. Rep. 97 (1983) 31]

QCD potential:

$$V(r) = \underbrace{-\frac{4}{3}\frac{\alpha_s(1/r^2)}{r}} + kr$$
 neglected





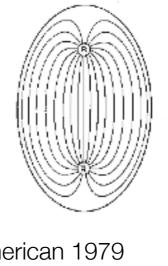
- String breaks up if potential energy is large enough to produce a new quark-antiquark pair
- Gluons = 'kinks' in string
- At low energy: hadron formation
- Very widely used ... [default in Pythia 6/8]

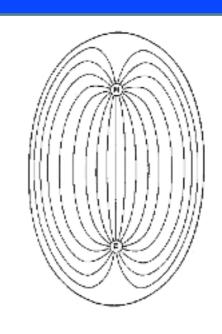
Lund String Model

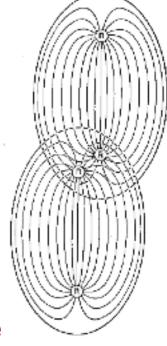
Repeated string breaks for large system with pure $V(r) = \kappa \cdot r$

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

Energy-momentum quantities can be read from space-time quantities ...







Scientific American 1979 Kenneth A. Johnson

Simple but powerful picture of hadron production

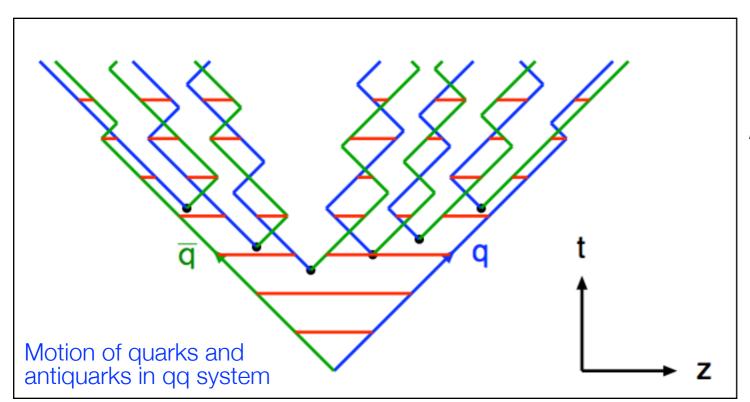
[with extensions to massive quarks, baryons, ...]

$$m_{\perp}^2 \equiv m_T^2 \equiv p_{\perp}^2 + m^2 = |\vec{p}_T|^2 + m^2$$

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right)$$

$$\propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

Yields: Common Gaussian p⊥ spectrum Heavy quark suppression



Cluster Model

[Webber, Nucl. Phys. B238 (1984) 492]

Color flow confined during hadronisation process

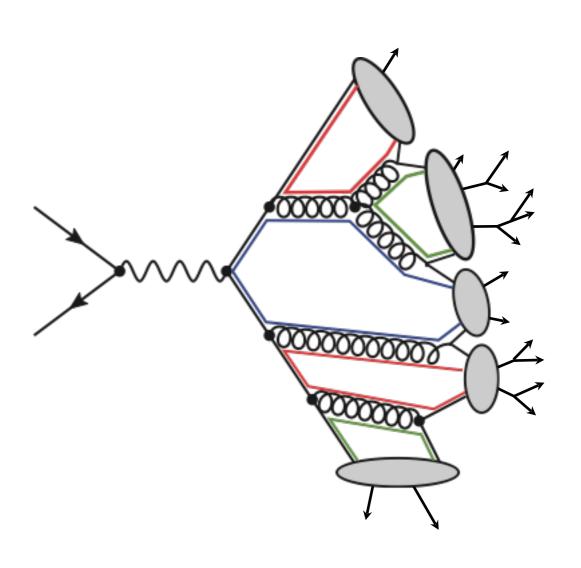
Formation of color-neutral parton clusters

Gluons (color-anticolor) split to quark-antiquark pairs

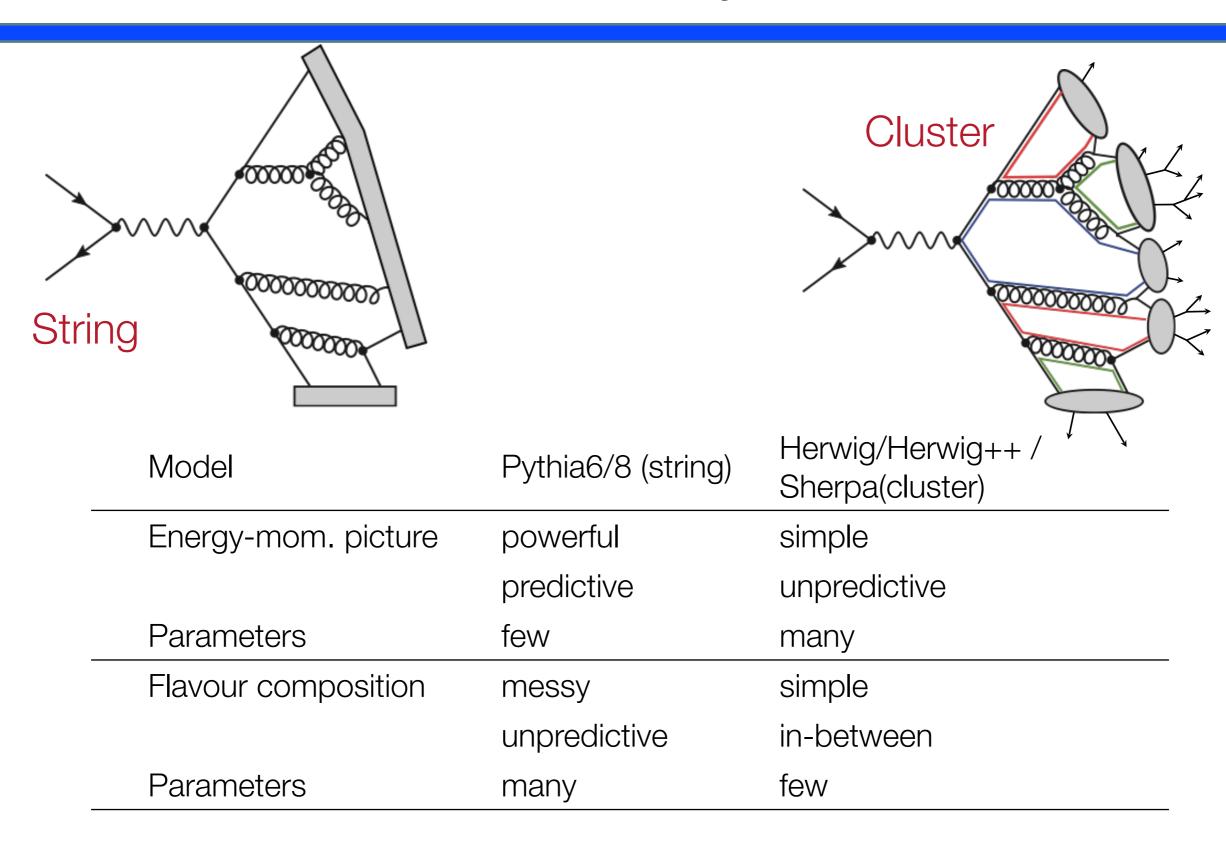
Clusters decay into 2 hadrons according to phase-space, i.e. isotropically

no free tuning parameters parton clusters

Very widely used ... [default in Herwig/Herwig++, Sherpa with some modifications]



Hadronisation models summary

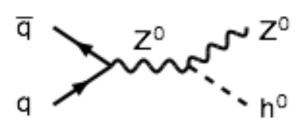


Structure of basic generator process [by order of consideration]

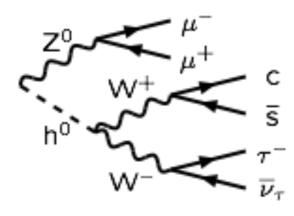
From the 'simple' to the 'complex' or from 'calculable' at large scales to 'modelled; at small

Matrix elements (ME)

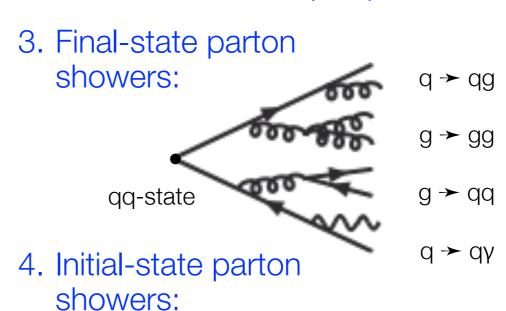
Hard subprocess:
 M|², Breit Wigners, PDFs

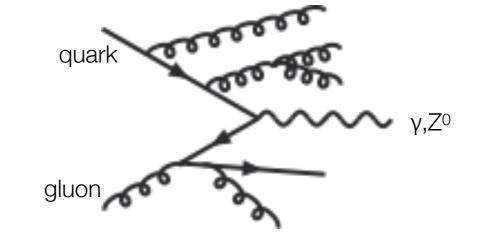


2. Resonance decays: Includes particle correlations



Parton Shower (PS)



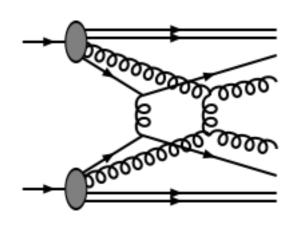


Conclusions: Structure of basic generator process

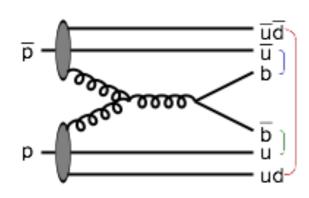
From the 'simple' to the 'complex' or from 'calculable' at large scales to 'modelled; at small

Underlying Event (UE)

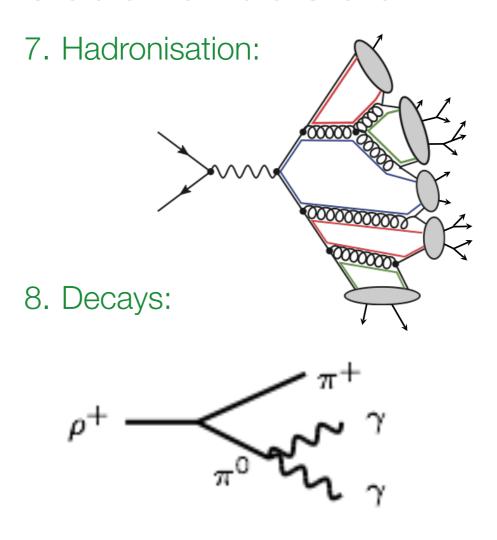
5. Multi-parton interaction:



6. Beam remnants:



Stable Particle State





The DGLAP evolution equation is said to resum large collinear logarithms. So where are these logarithsm, and where is the resummation?

Let's perform the integration of the DGLAP equation and expand the result:

$$\begin{split} f(x,t) &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \int_x^1 \frac{dz}{z} P(z) \, q\Big(\frac{x}{z},t'\Big) \\ &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \bigg\{ f_0\Big(\frac{x}{z}\Big) + \\ &+ \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') \, \Big[f_0\Big(\frac{x}{zz'}\Big) + ... \Big] \bigg\} \\ &= f_0(x) + \frac{\alpha_s}{2\pi} \ln \Big(\frac{t}{t_0}\Big) \int_x^1 \frac{dz}{z} P(z) \, f_0\Big(\frac{x}{z}\Big) + \\ &+ \frac{1}{2!} \, \Big[\frac{\alpha_s}{2\pi} \ln \Big(\frac{t}{t_0}\Big) \Big]^2 \int_x^1 \frac{dz}{z} P(z) \int_{x/z}^1 \frac{dz'}{z'} P(z') \, f_0\Big(\frac{x}{zz'}\Big) + ... \end{split}$$

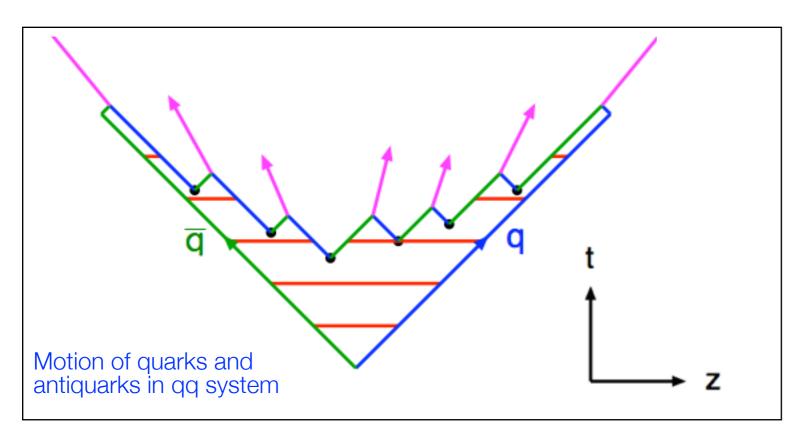
As suggested by the last step, it is indeed a resummation of all terms proportional to $\left[\frac{\alpha_1}{2\pi} \ln \left(\frac{t}{t_0}\right)\right]^n$.

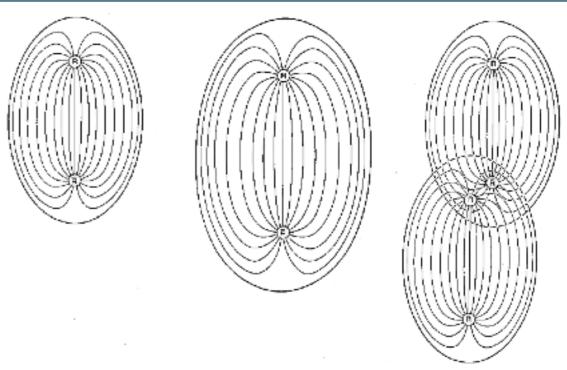
Lund String Model

Repeated string breaks for large system with pure $V(r) = \kappa \cdot r$

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

Energy-momentum quantities can be read from space-time quantities ...





Kenneth A. Johnson

Simple but powerful picture of hadron production

[with extensions to massive quarks, baryons, ...]

$$\mathcal{P} \propto \exp\left(-\frac{\pi \, m_{\perp q}^2}{\kappa}\right)$$

$$\propto \exp\left(-\frac{\pi \, p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi \, m_q^2}{\kappa}\right)$$

Yields: Common Gaussian p⊥ spectrum Heavy quark suppression