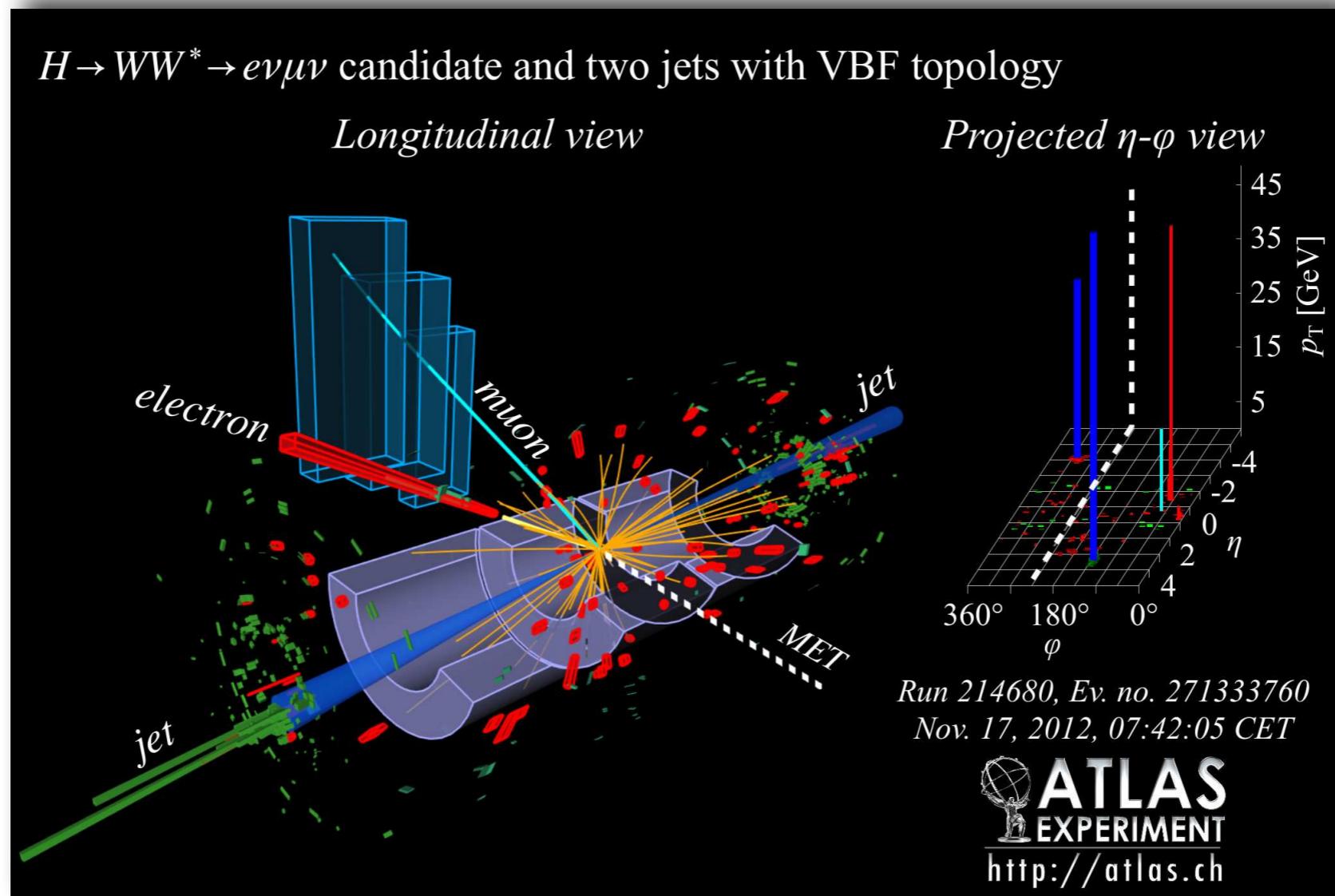


# Monte Carlo Generators at colliders

## High energy physics simulation

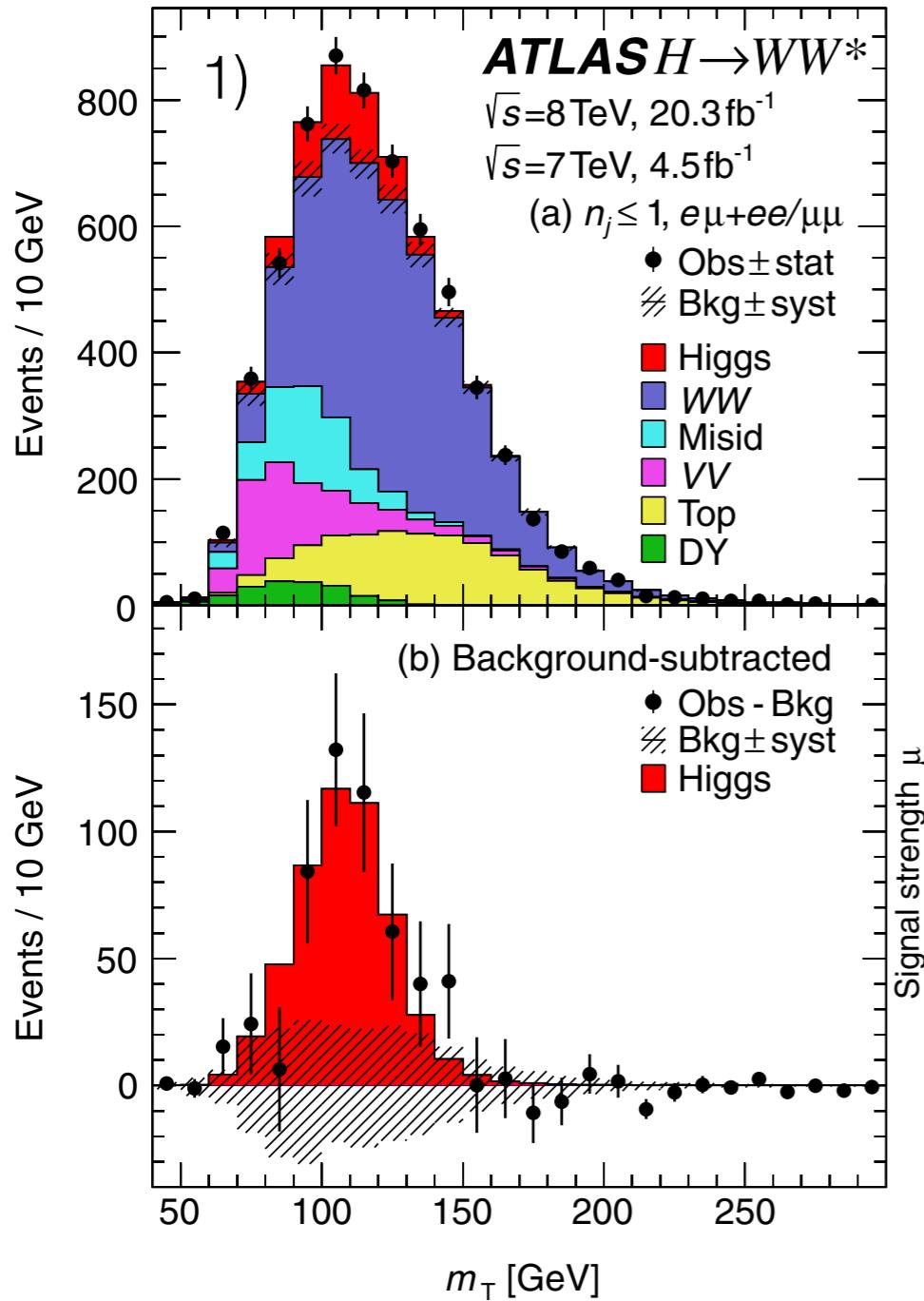


# Acknowledgements

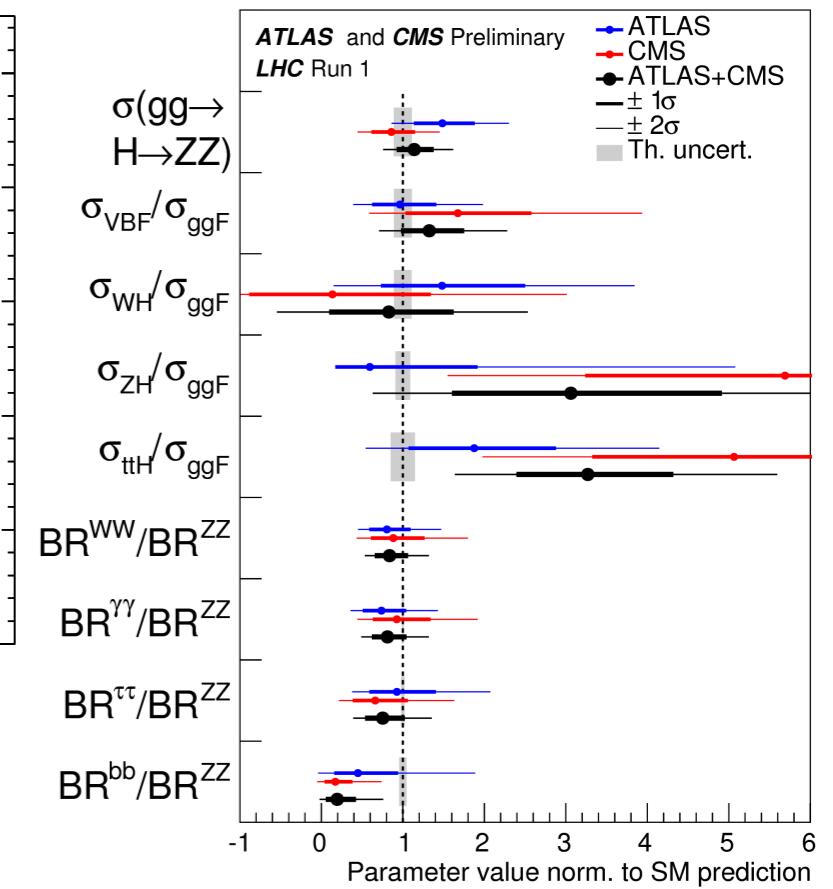
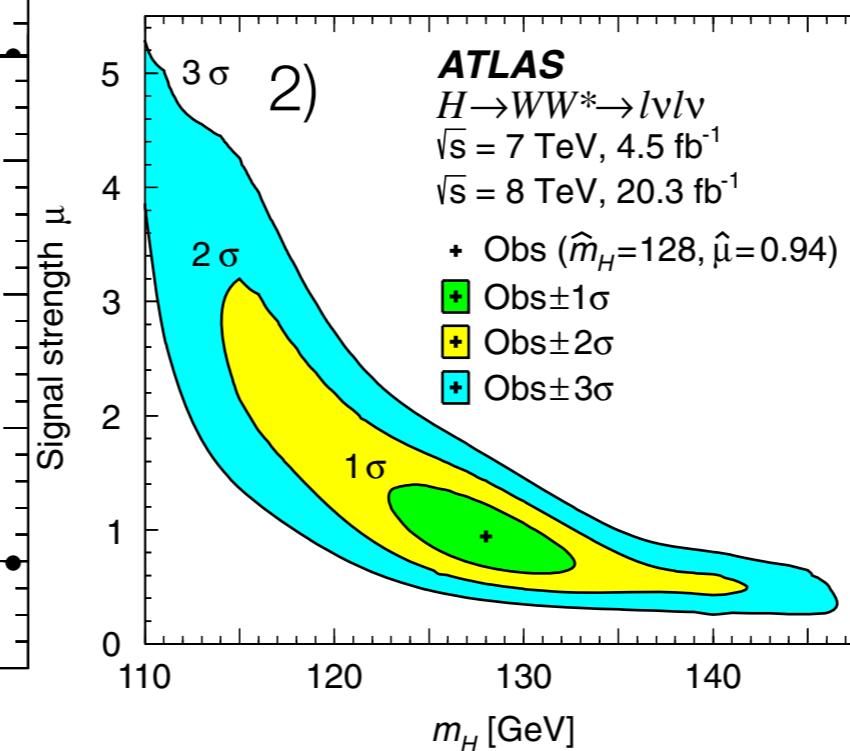
---

These slides have been prepared using heavily the material prepared by Christian Schultz Coulon and clarifying few points plus updating to recent developments in proton-proton physics.

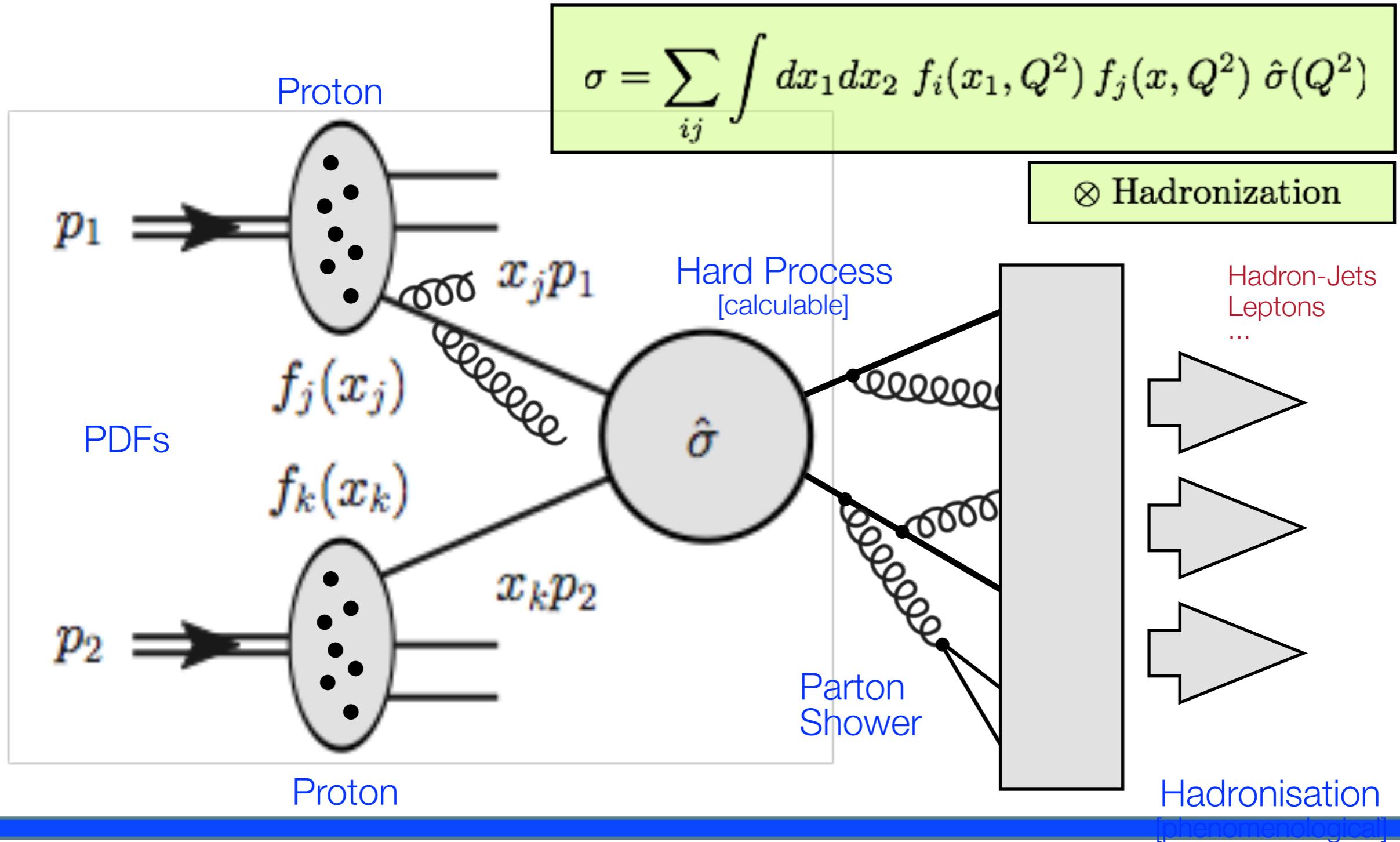
# Why MC simulation?



- 1) to extract an interesting signal we need to subtract the expectation from known processes;
- 2) signal needs also to be modelled in order to compute detection efficiency and estimate production cross sections and couplings



# The simulation chain



# MC simulations in particle physics

How Monte Carlo simulation works

- Numerical process generation based on random numbers
- Very powerful method in particle physics

**Event generation programs:**

**Pythia6, Pythia8, Herwig, Herwig7, Sherpa ...**

Hard partonic subprocess + fragmentation and hadronisation ...

**Detector simulation:**

**Geant4**

**Fluka low energy hadron interactions...**

interaction & response of all produced particles ...

**Event Generator**

simulate physics process  
(quantum mechanics: probabilities!)

**Detector Simulation**

simulate interaction with detector material

**Digitisation**

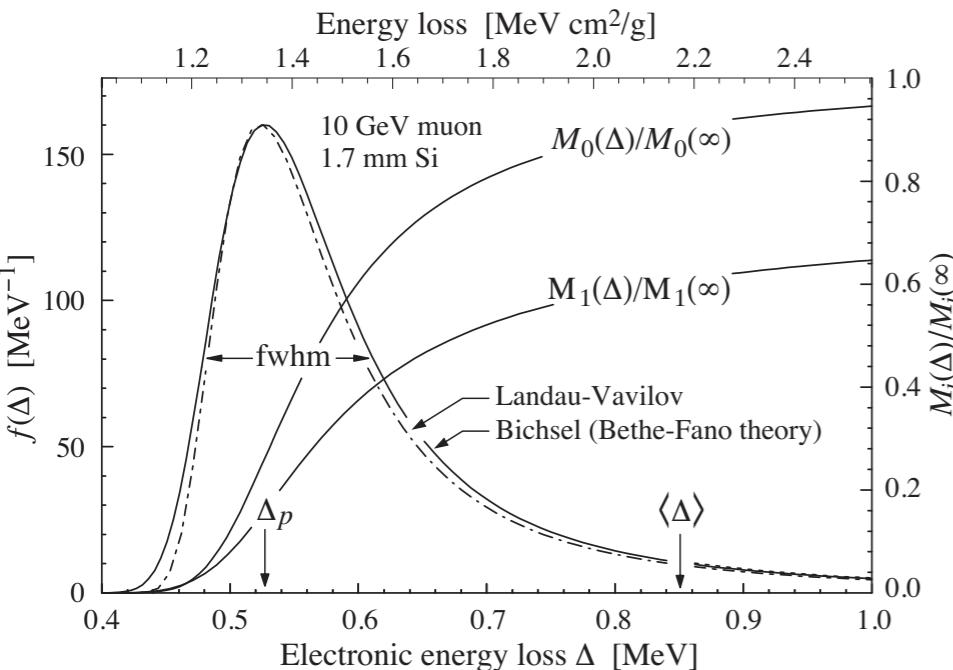
translate interactions with detector into realistic signals

**Reconstruction/Analysis**

as for real data

# Baseline of the simulation process

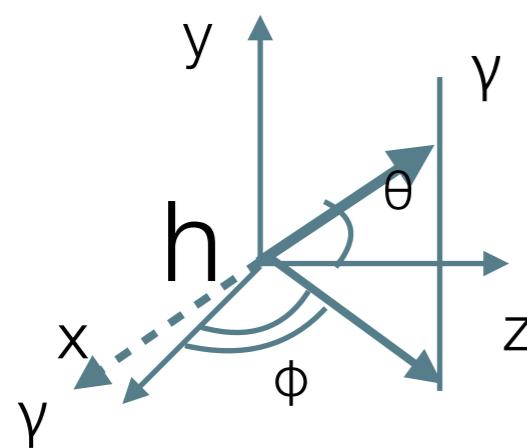
Typically, we need to generate a continuous variable following some distribution  
i.e. energy loss of a particle in a given material segment;  
angle of a photon in the h reference frame for the  $h \rightarrow \gamma\gamma$  decay



$$dP = f(x, \dots) dx$$

↳ distribution formula

probability to get an  $x_0$  value between  $x$  and  $x+dx$



if we want to simulate flat angular distributions, we can start from the azimuthal and polar angles

$$dP = f(\theta, \phi) d\theta d\phi = \sin\theta d\theta d\phi$$

flat distribution in  $\phi$   
non flat in  $\theta$

# Distribution function transformation properties

- 1) software libraries provide basic functions to produce flat distributed random numbers in the interval [0,1] (ex. root TRandom3 class), they are typically fast and accurate uniform random numbers generators;
- 2) starting from uniform distributed random numbers, it is possible to generate numbers following any distribution using different techniques

$$dP_x = f(x)dx \quad y = g(x)$$

$x \in [x_a, x_b]$

$$dP_y = h(y)dy = h(y)g'(x)dx$$

$g(x)$  is a monotonic function of  $x$   
How "y" distributes in  $[g(x_a), g(x_b)]$ ?

Because  $y$  is a monotonic function of  $x$  the probability to have  $y$  between  $g(x)$  and  $g(x+dx)$  is equal to the probability to have  $x$  between  $x$  and  $x+dx$

$$h(y)g'(x) = f(x) \Rightarrow h(y) = \frac{f(x)}{g'(x)} = \frac{f(g^{-1}(y))}{g'(g^{-1}(y))}$$

Ex.: range map

$$[0, 1] \rightarrow [a, b] \quad y = (b - a)x + a$$

$$f(x) = 1 \quad \text{uniform} \quad g'(x) = b - a \quad h(y) = \frac{1}{b - a} \quad y \text{ is uniformly distributed in } [a, b]$$

# Distribution function transformation properties

Ex. 2: integration method:

$$y = g(x) = \frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' \quad g'(x) = \frac{f(x)}{\int_a^b f(x')dx'}$$

$$h(y) = \frac{f(x)}{g'(x)} = \frac{f[g^{-1}(y)]}{f[g^{-1}(y)]} \cdot \int_a^b f(x')dx' = \int_a^b f(x')dx'$$

y is uniformly distributed:

- 1) generate y flat in  $[f_{\min}, f_{\max}]$ ;
- 2) compute  $x = g^{-1}(y)$ , x will be distributed in  $g^{-1}(f_{\min}), g^{-1}(f_{\max})$

Finding  $g^{-1}(y)$  is equivalent to solve the equation:

$$\frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' = y$$

# Hit or miss method.

- 1) generate  $x$  flat in  $x_{\min}, x_{\max}$
- 2) generate  $y$  flat in  $0, f_{\max}$
- 3) if  $y < f(x)$  accept the event, otherwise ignore it

for a given  $x$  in  $x, x+dx$  the fraction of accepted events is proportional to  $f(x)dx \rightarrow dP_x = f(x)dx$

- 1) advantages:

- can be used for all functions, even non continuous ...
- can be extended to N-dimension (generate  $x_1, x_2, \dots, x_n$ ),  $y$  accept if  $y < f(x_1, x_2, \dots, x_n)$

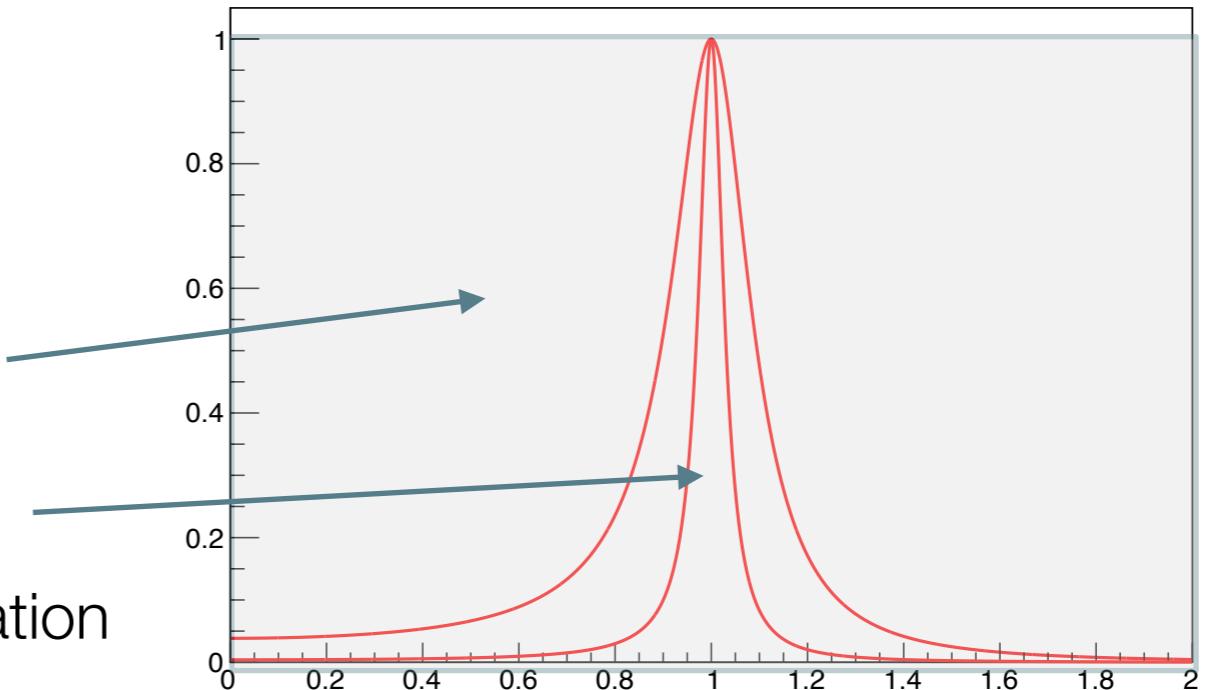
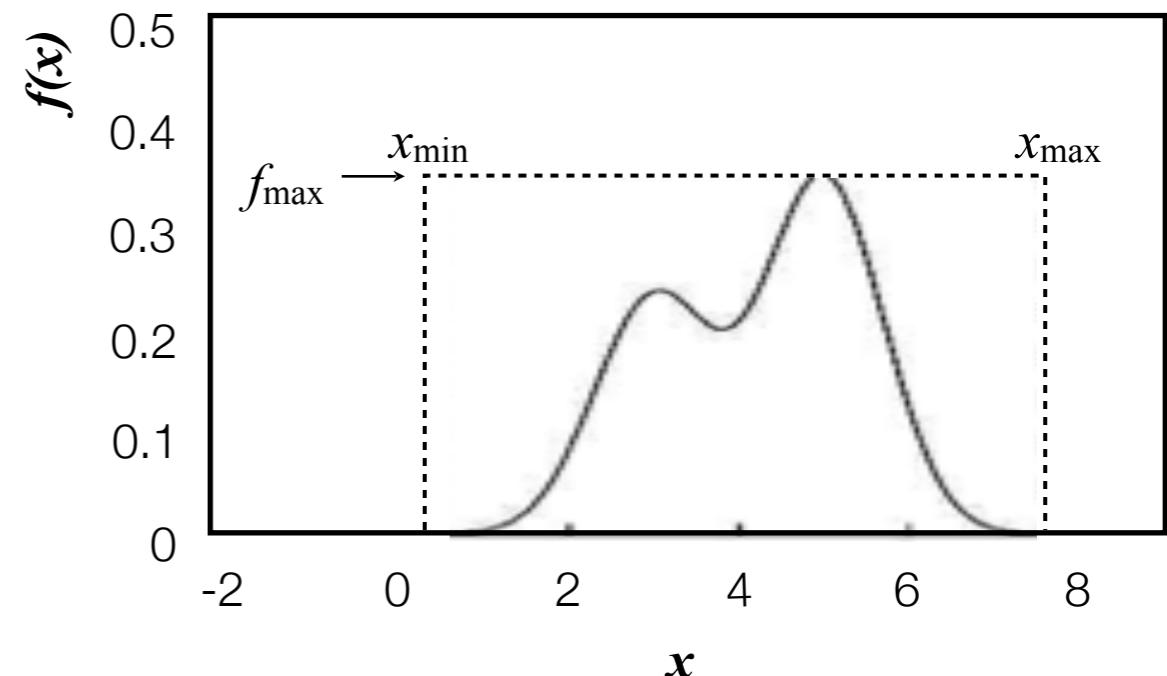
- 2) disadvantages

- can be extremely slow

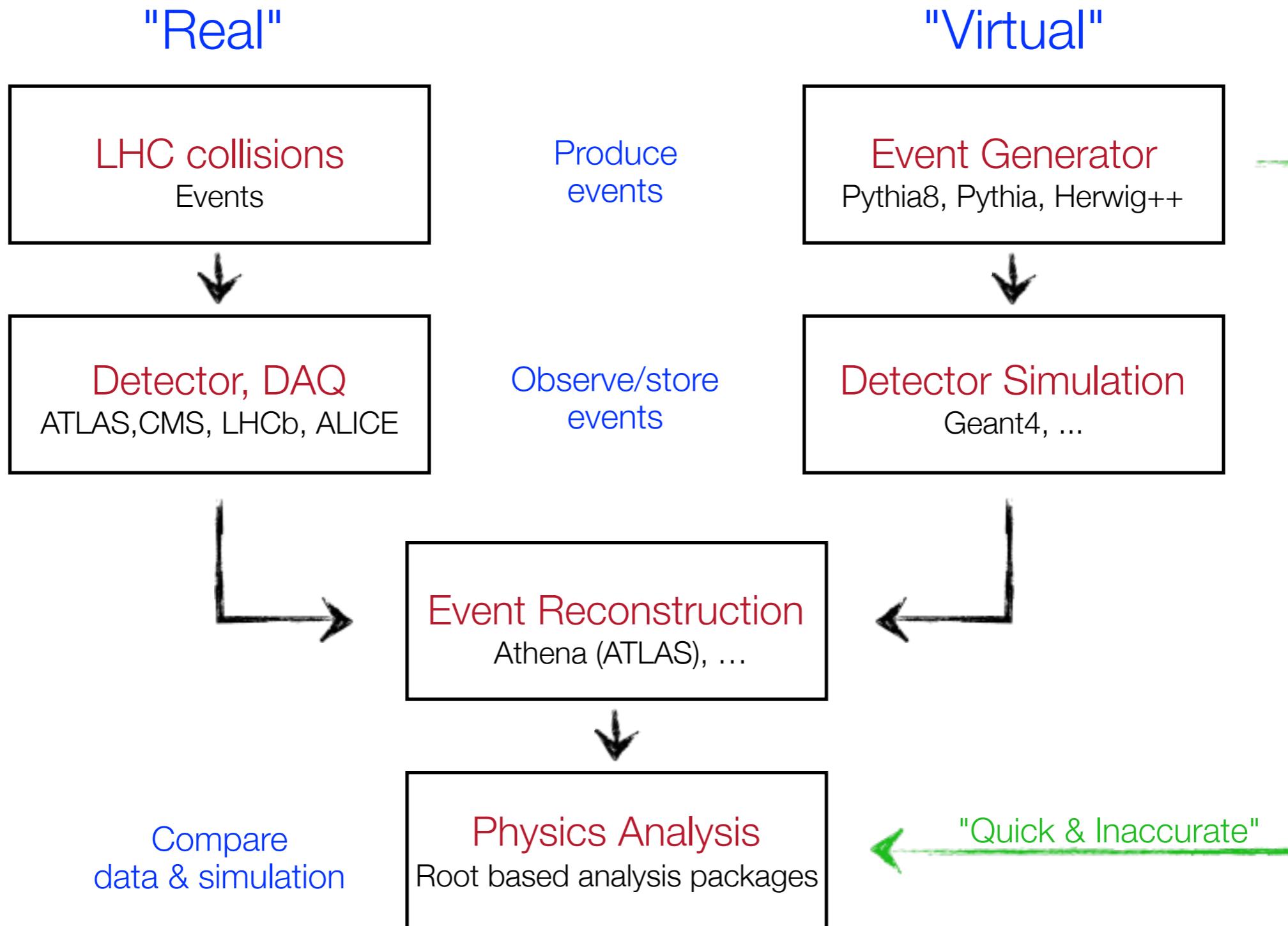
points generated uniformly in the square

points accepted only below the curve

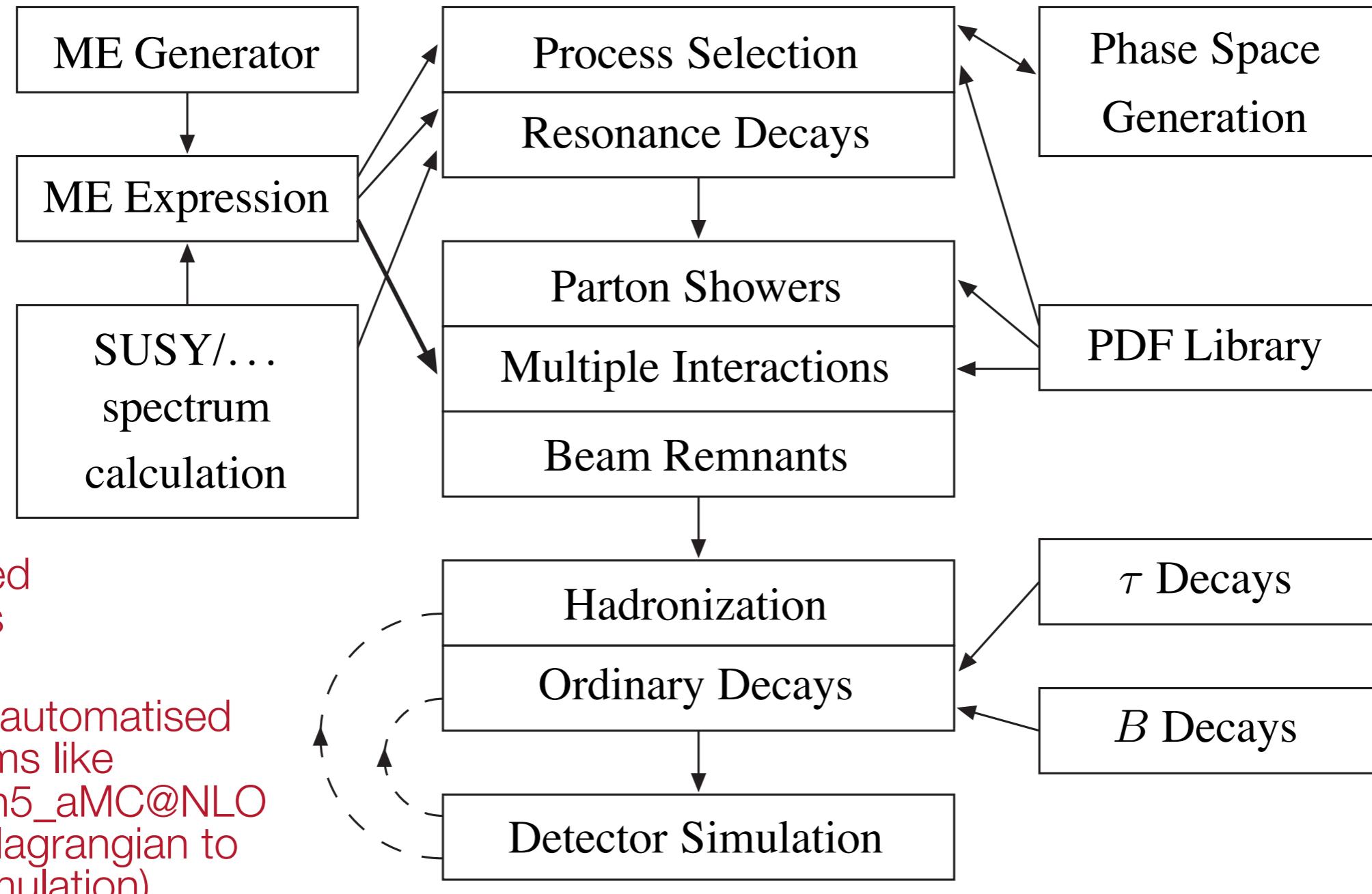
MC generators implement “smart” generation techniques to increase efficiencies



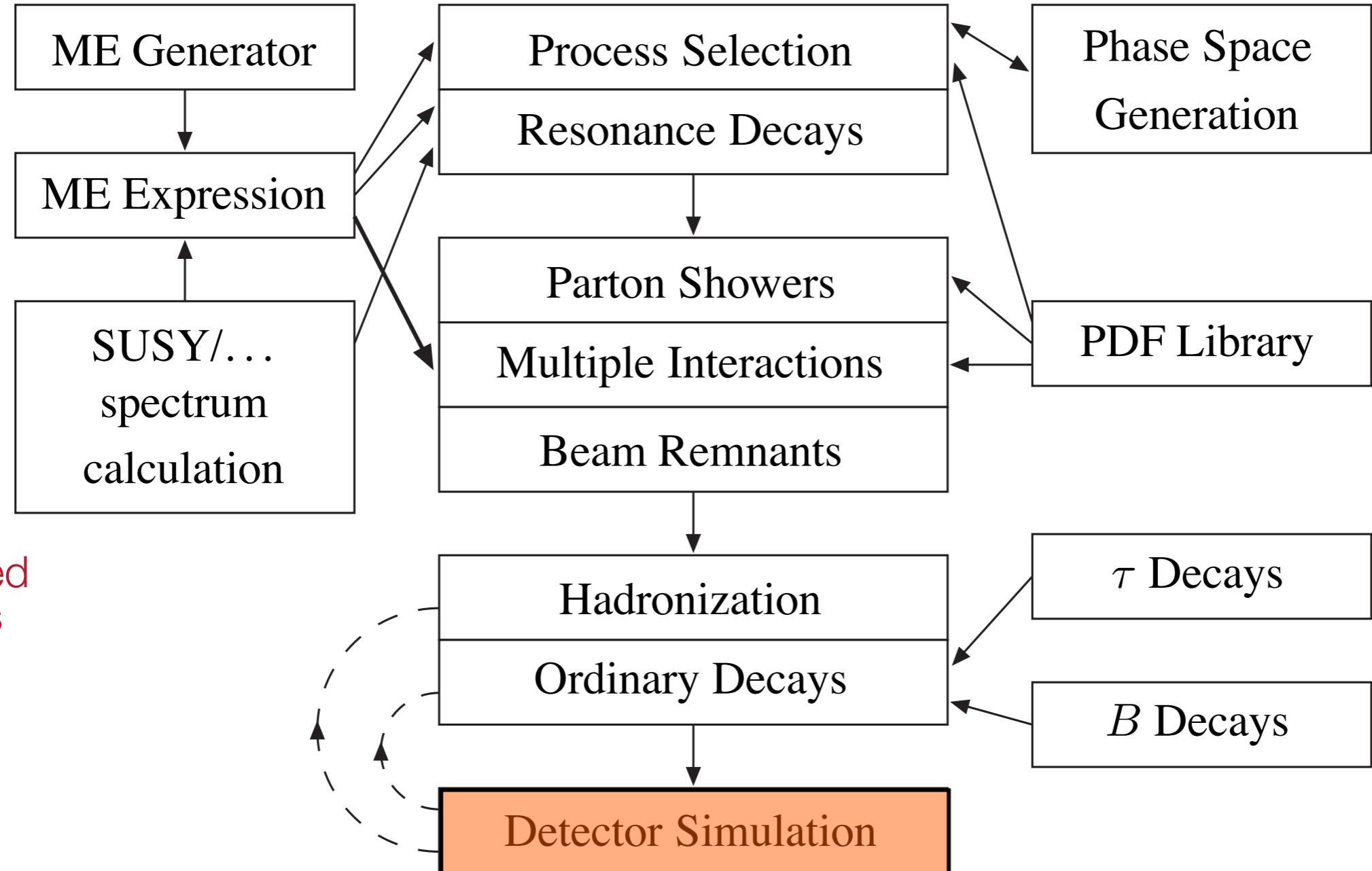
# Comparison between real and simulated events



# Simulation elements



# Simulation elements



# GEANT Geometry And Tracking

Detailed description of detector **geometry**

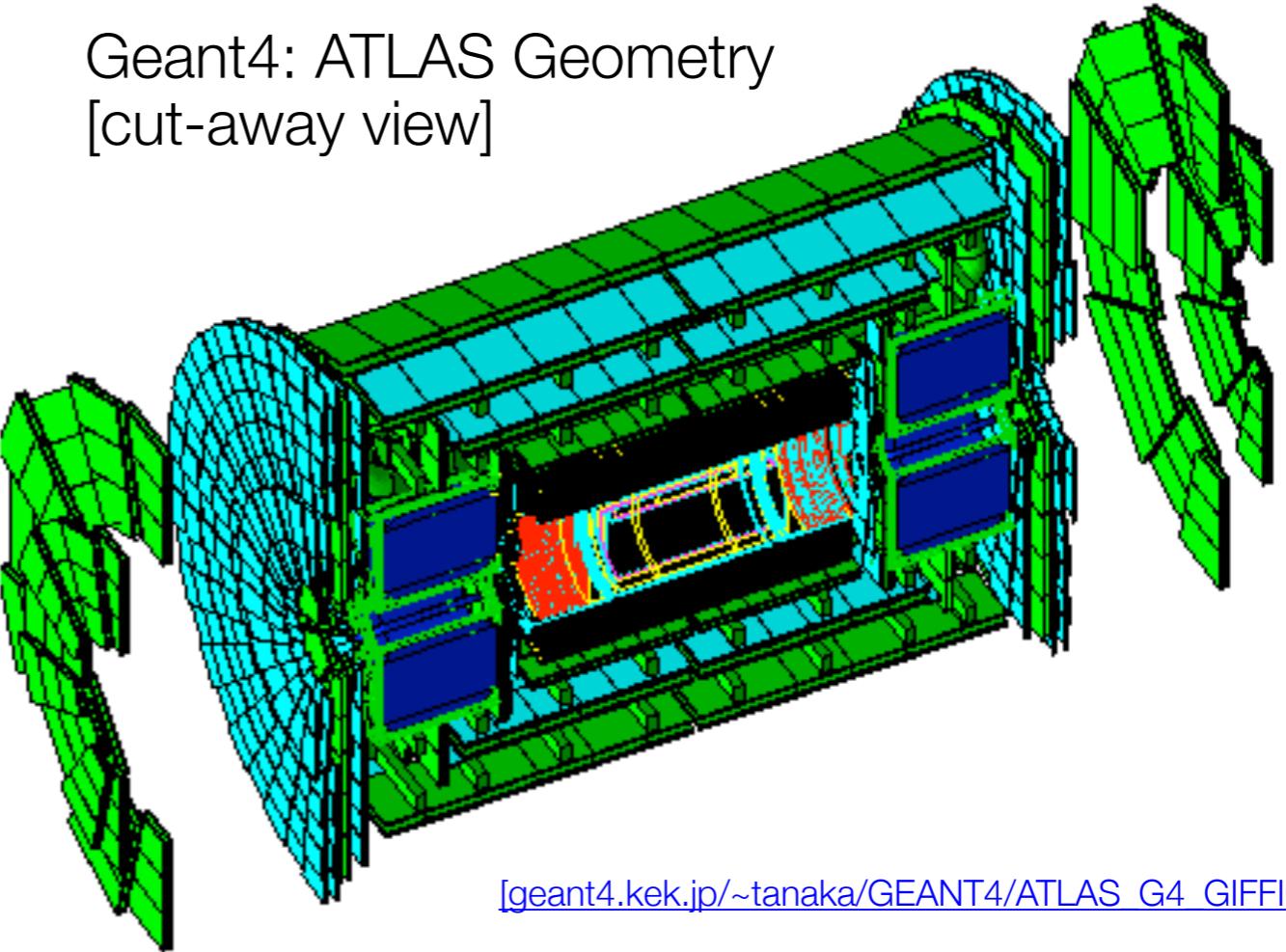
[sensitive & insensitive volumes]

**Tracking** of all particles through detector material ...

→ Detector response

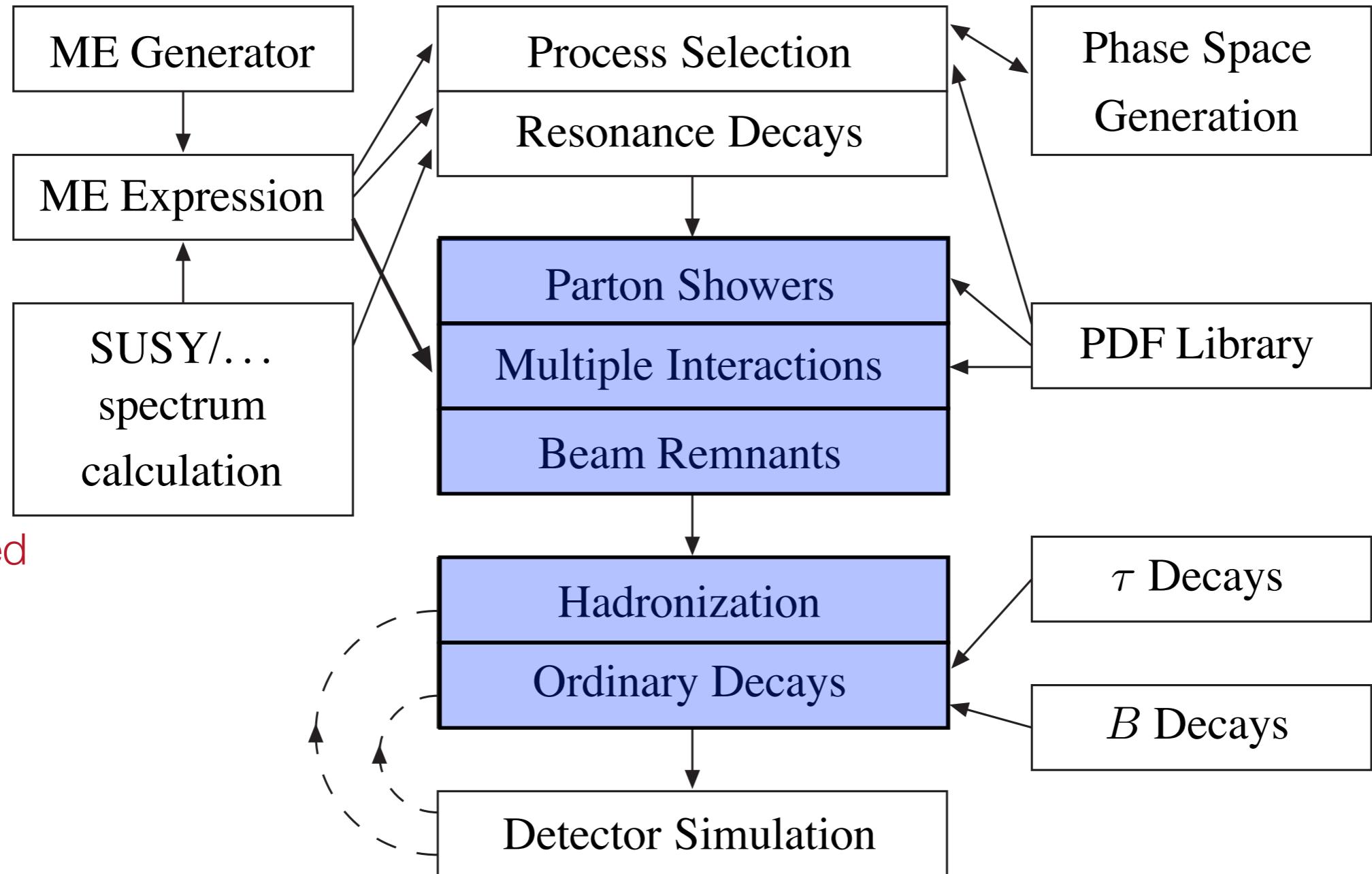
Developed at CERN since 1974 (FORTRAN)

[Today: Geant4; programmed in C++]



[\[geant4.kek.jp/~tanaka/GEANT4/ATLAS\\_G4\\_GIFFIG/\]](http://geant4.kek.jp/~tanaka/GEANT4/ATLAS_G4_GIFFIG/)

Use  
specialised  
programs



Strong interactions:

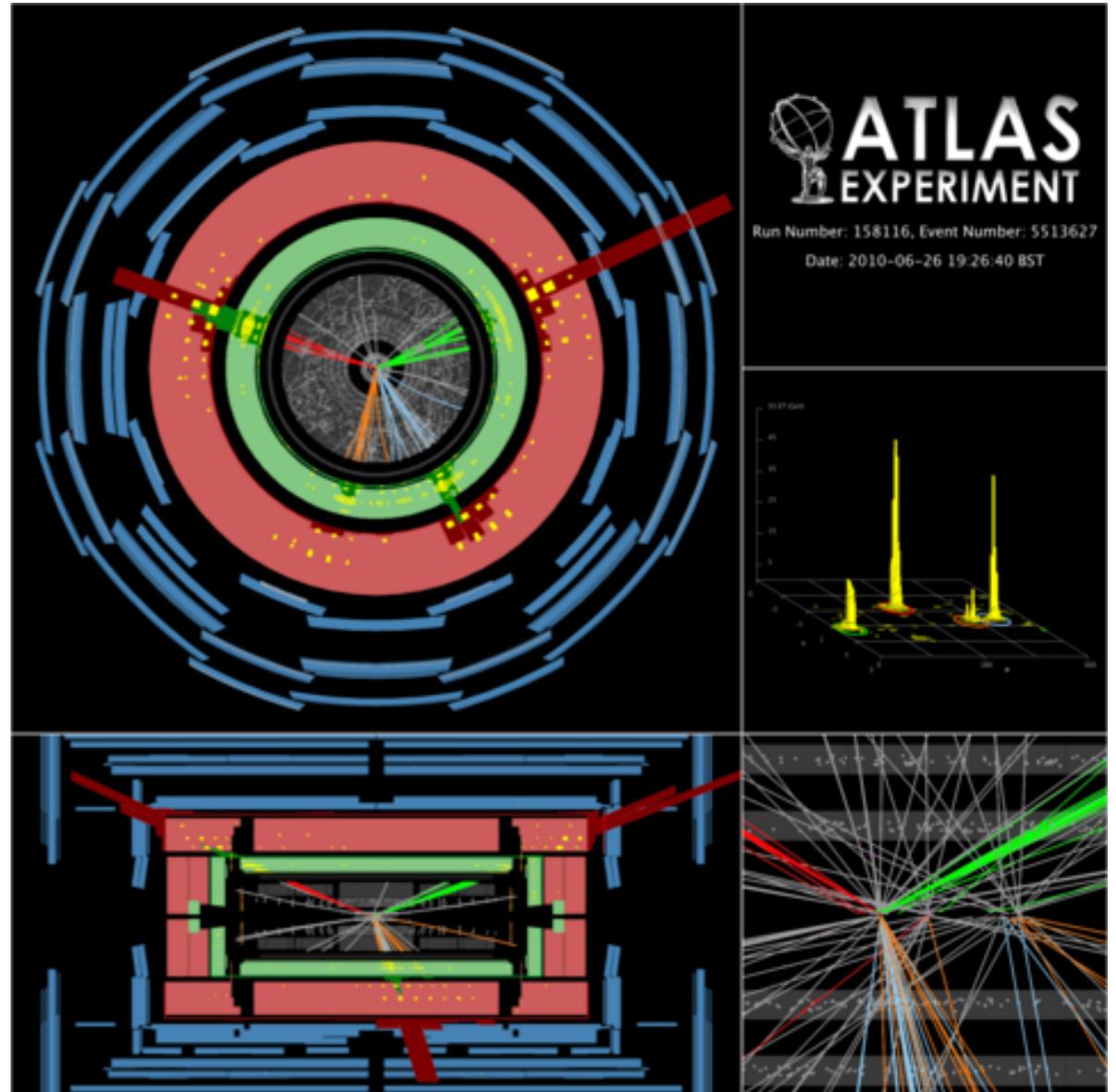
No free Quarks

Expect jets

i.e. bundles of particles at high energies  
[hadron  $p_T$  range limited w.r.t. initial parton]

First observation of jets  
in  $e^+e^-$  collisions @  $E_{\text{CMS}} > 6 \text{ GeV}$   
[SPEAR, SLAC, 1975]

Later also observed in  
hadron-hadron collisions  
[e.g. @ CERN ISR]



An event with 4 jets @ LHC

Goal: Infer parton properties from jet properties  
[need to calculate and/or model fragmentation & hadronisation process]



## Pure matrix element (ME) simulation:

MC integration of cross section & PDFs, no hadronisation  
(recall: cross section =  $|\text{matrix element}|^2 \otimes \text{phase space}$ )

Useful for theoretical studies, no exclusive events generated

[Example: MCFM (<http://mcfm.fnal.gov>); many LHC processes up to NLO,  
HNNLO (<http://theory.fi.infn.it/grazzini/codes.html>) Higgs production at NNLO]

## Event generators:

Combination of ME and parton showers ...

Typical: generator for leading order ME  
combined with leading log (LL) parton shower MC (see later)

Exclusive events → useful for experimentalists ...

# Parton showers

A realistic simulation needs many particles in the final state, it is quite difficult (sometimes impossible) to compute a  $p\bar{p}$  ( $2 \rightarrow$  many particles) process

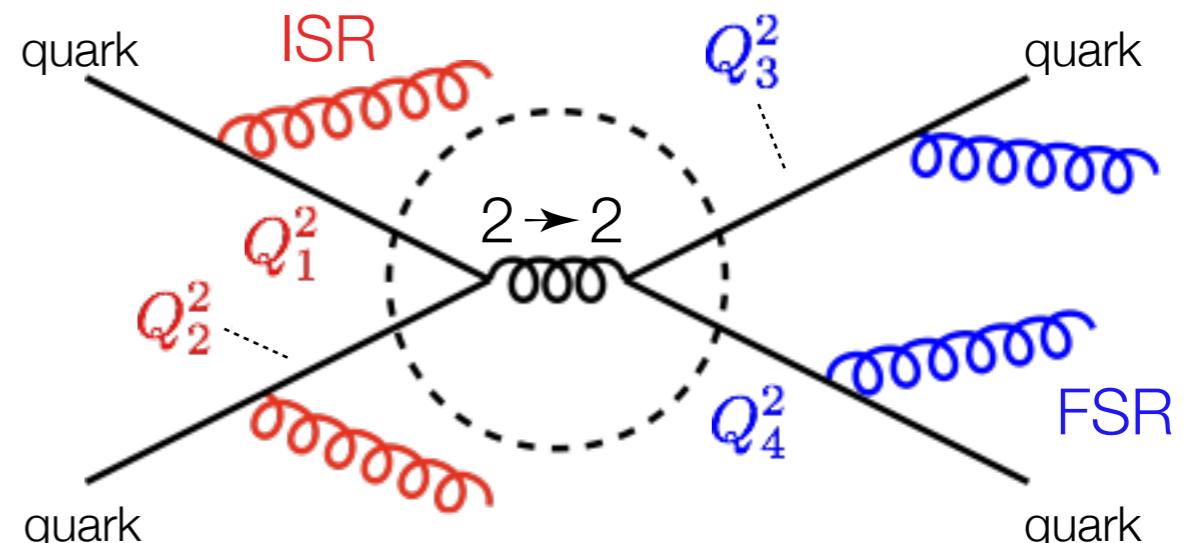
$$(2 \rightarrow n) = \dots$$

$$\dots = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR}$$

FSR: Final state radiation

$Q^2 \sim p_{\text{quark in}}^2 \sim m^2 > 0$  decreasing  
[time-like shower]

$$p_{\text{quark in}} = p_{\text{quark out}} + p_{\text{gluon}}$$



ISR: Initial state radiation

$Q^2 \sim p_{\text{quark out}}^2 \sim -m^2 > 0$  increasing  
[space-like shower]

$$p_{\text{quark out}} = p_{\text{quark in}} - p_{\text{gluon}}$$

Hard process [ $2 \rightarrow 2$ ]:

$$\sigma = \iiint dx_1 dx_2 d\hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

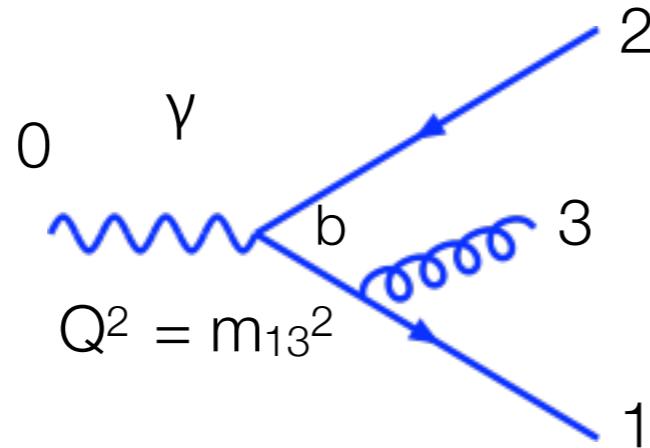
Calculable

Shower evolution:

Viewed as probabilistic process, which occurs with unit total probability;  
cross section not directly affected; only indirectly via changed event shape.

# Parton showers

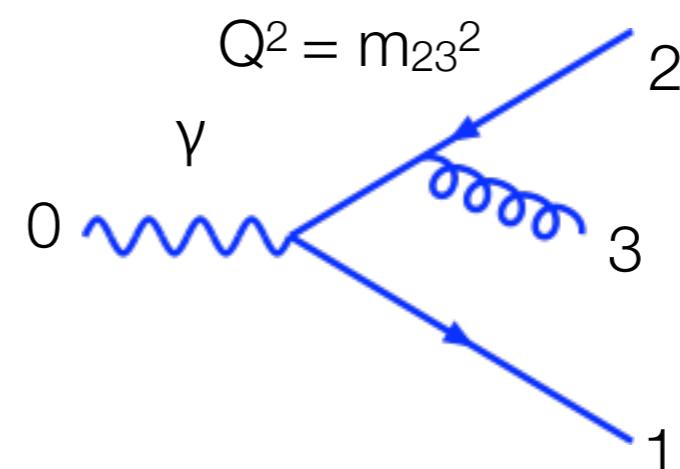
$e^+e^- \rightarrow qqq$



$$x_i = \frac{2E_i}{E_{\text{cm}}} \quad x_1 + x_2 + x_3 = 2$$

Cross Section:  $\frac{d\sigma_{\text{qqg}}}{dx_1 dx_2} = \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \sigma_0 \cdot \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$

Cross section has large contributions for  $x_1, x_2 \rightarrow 1$



[ $m_q = 0$ ; see e.g. Halzen/Martin]

from  $p_T$  balance  $1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2}$   $m_{13}^2 \sim 2E_1 E_2 (1 - \cos\theta)$   $x_2 \rightarrow 1 \Rightarrow m_{13}^2 \rightarrow 0 \Rightarrow \theta \rightarrow 0$  collinear limit

$$dx_2 = -\frac{dQ^2}{E_{\text{cm}}^2}$$

Rewrite for  $x_2 \rightarrow 1$ :  
[qg collinear limit]

$$x_1 \approx z \quad dx_1 \approx dz \\ x_3 \approx 1 - z$$

$$d\mathcal{P} = \frac{d\sigma_{\text{qqg}}}{\sigma_0} = \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \frac{dx_2}{(1-x_2)} \cdot \frac{x_1^2 + x_2^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \cdot \frac{dQ^2}{Q^2} \cdot \frac{4}{3} \left[ \frac{1+z^2}{1-z} \right] dz$$

$z \rightarrow 1 \Rightarrow E_g \rightarrow 0$  soft divergence

$$dP_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz$$

Splitting probability determined by splitting functions  $P_{q \rightarrow qg}$

Analogous splitting functions used in PDF evolution

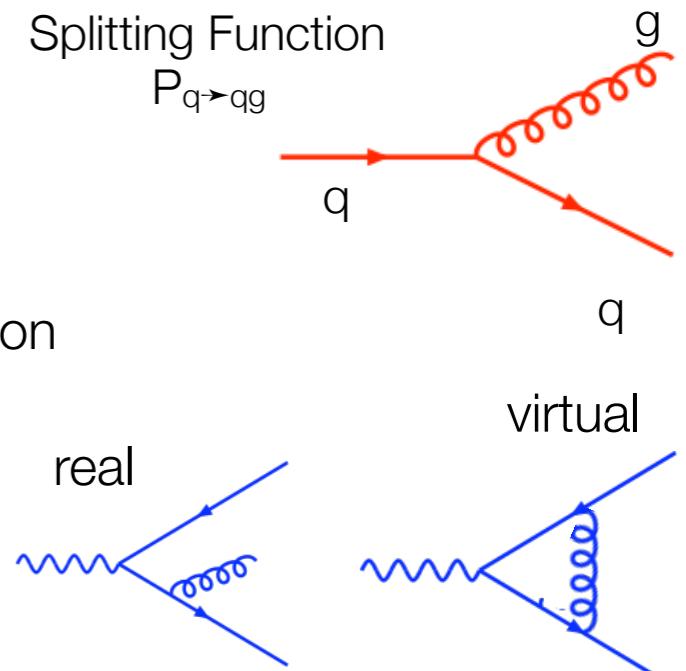
$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$z$  : fractional momentum of radiated parton  
 $n_f$  : number of quark flavours

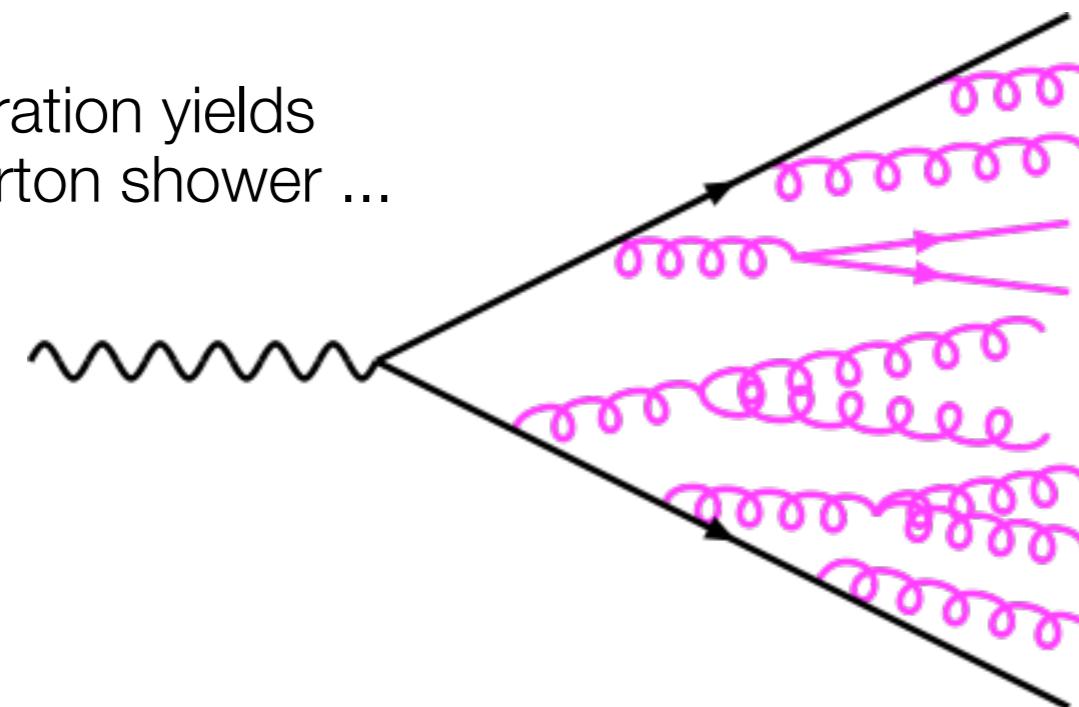
$$P_{g \rightarrow gg} = 3 \frac{(1-z)(1-z)^2}{z(1-z)}$$

In NLO calculations soft and collinear divergencies cancelled by virtual contributions: they persist in LO calculations.

$$P_{g \rightarrow q\bar{q}} = \frac{n_f}{2} (z^2 + (1-z)^2)$$



Iteration yields parton shower ...



Need soft/collinear cut-offs to avoid non-perturbative regions ... [divergencies!]

Details model-dependent

e.g.  $Q > m_0 = \min(m_{ij}) \approx 1 \text{ GeV}$ ,  
 $z_{\min}(E, Q) < z < z_{\max}(E, Q)$  or  
 $p_\perp > p_{\perp\min} \approx 0.5 \text{ GeV}$

# Parton shower evolution 1

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

Time evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

$$\begin{aligned} \mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\ &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})) \\ &= \exp \left( - \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \end{aligned}$$

$e^{-x} \approx 1 - x$

[Taylor]

→  $d\mathcal{P}_{\text{first}}(T) = d\mathcal{P}_{\text{something}}(T) \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right)$

# Parton shower evolution 2

Instead of evolving to later and later times  
need to evolve to smaller and smaller  $Q^2$  ...  
[Heisenberg:  $Q \sim 1/t$ ]

$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \exp \left( - \sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz' \right)$$

Probability to radiate  
with virtuality  $Q^2$

No radiation for higher  
virtualities i.e. for  $Q^2 \dots Q_{\max}^2$

Note that  $\sum_{b,c} \iint d\mathcal{P}_{a \rightarrow bc} = 1 \dots$

[Convenient for Monte Carlo]

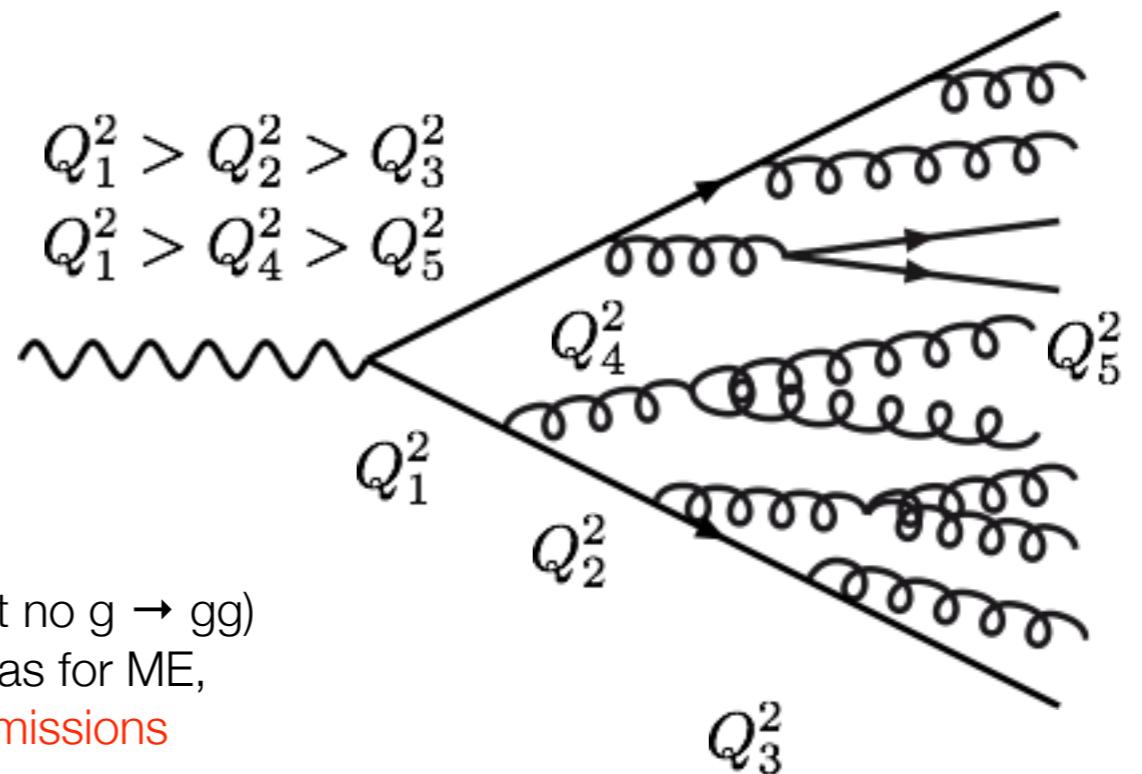
Sudakov form factor ...

... provides “time” ordering of shower ...  
[lower  $Q^2 \Leftrightarrow$  longer times]

... regulates singularity for first emission ...

But in the limit of repeated soft emissions  $q \rightarrow qg$  (but no  $g \rightarrow gg$ )  
one obtains the same inclusive  $Q$  emission spectrum as for ME,  
i.e. divergent ME spectrum  $\Leftrightarrow$  infinite number of PS emissions

Sudakov  
Form Factor



# Sudakov picture of parton showers

Basic algorithm: Markov chain

[each step requires only knowledge of the previous step]

(i) Start with virtuality  $Q_1$  and momentum fraction  $x_1$

(ii) Generate target virtuality  $Q_2$  with random number  $R_T$  uniform distributed in  $[0, 1]$

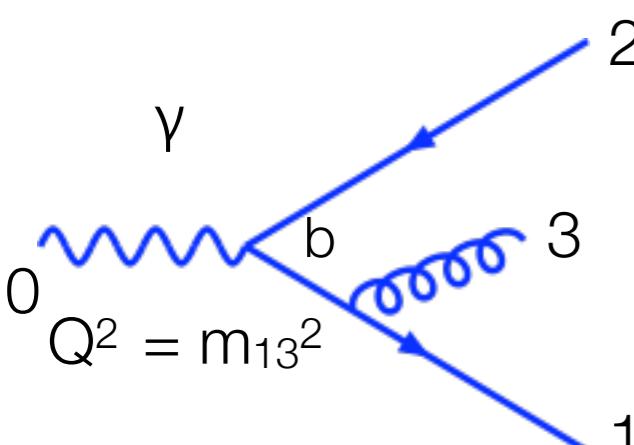
Probability to not have  $Q_x > Q_2$

using:

$$\Delta(Q_i^2) = \exp \left( - \sum_{b,c} \int_{Q_i^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz' \right)$$

$$R_t = \frac{\Delta(Q_2^2)}{\Delta(Q_1^2)}$$

[probability to evolve from  $t_1$  to  $t_2$  without radiation]



(iii)  $Q_2$  known ( $x_2$  known), need to compute  $x_1 \sim z$

$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$$R_z = \frac{\int_0^z P(z') dz'}{\int_0^1 P(z') dz'}$$

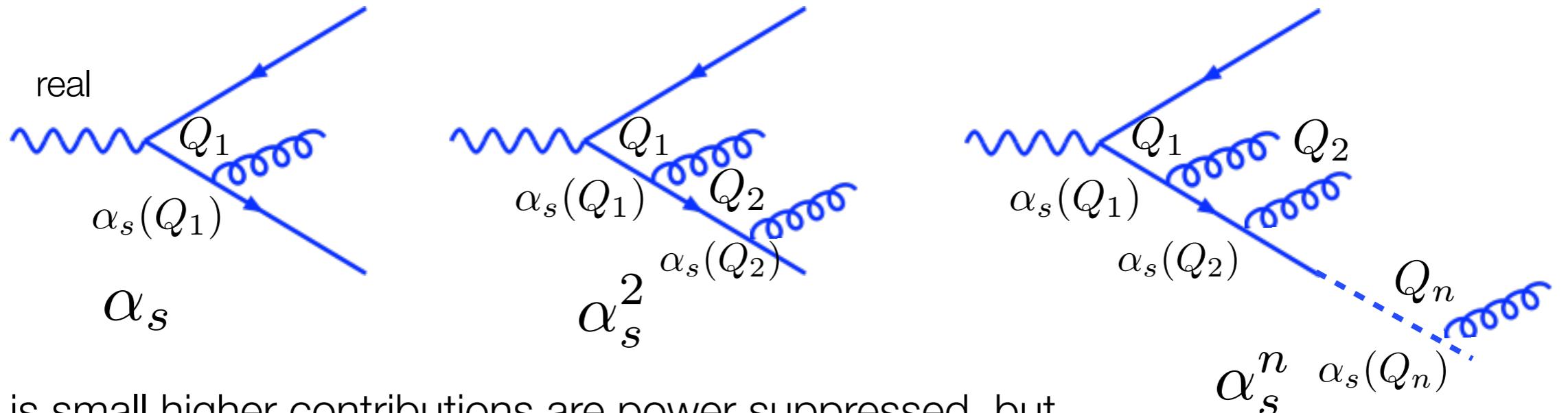
flat distributed

$$R_z \in [0, 1]$$

(iv) Generate random azimuthal angle  $\Phi$  flat distributed

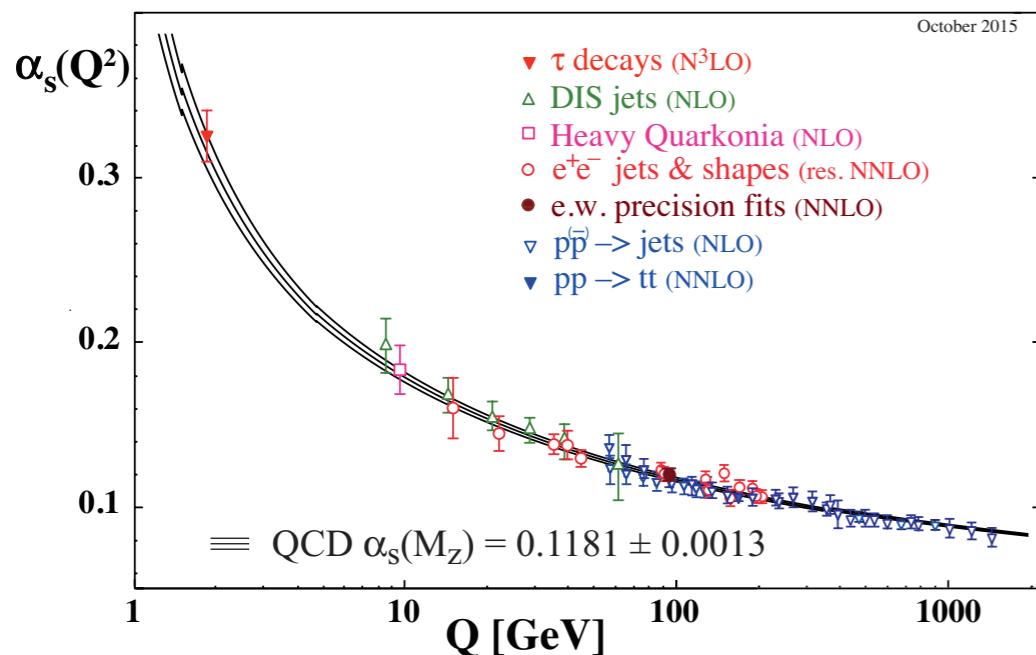
Process ends when partons are below threshold ( $p_T, Q$ )

# Parton shower and logarithmic resummation



If  $\alpha_s$  is small higher contributions are power suppressed, but...

$\alpha_s$  increases at small  $Q^2$



$$\alpha_s(Q_n) \sim \alpha_s(Q_1) \ln(Q_1/Q_n)$$

$$\begin{aligned} \alpha_s(Q_1) + \alpha_s(Q_1)\alpha_s(Q_2) + \dots + \alpha_s(Q_1) \cdot \dots \cdot \alpha_s(Q_n) \\ \sim [\alpha_s(Q_1) \ln(Q_1)]^2 \sim [\alpha_s(Q_1) \ln(Q_1)]^n \end{aligned}$$

$$\text{if } \alpha_s(Q_1) \ln(Q_1)$$

is large, the expansion is broken,  
PS allows to sum up all the  
large contributions [Leading Log  
resummation]

# Parton shower ordering

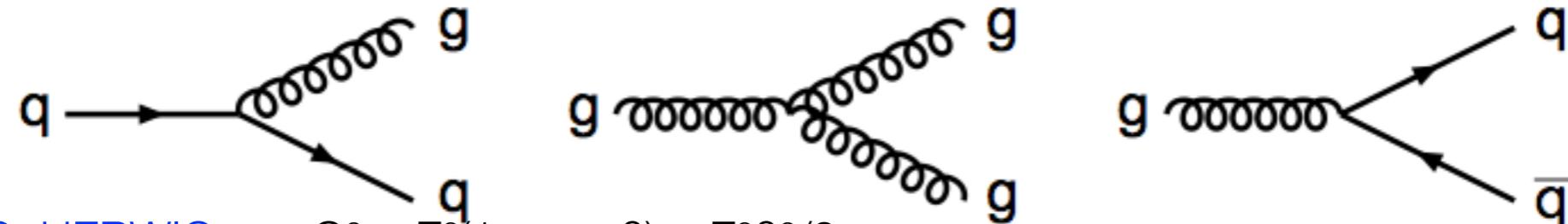
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \exp \left( - \sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz' \right)$$

In the splitting function appears only  $dQ^2/Q^2$ , therefore if  $P = f(z)Q^2$   $dP/P = dQ^2/Q^2$

Three main approaches to showering in use:

$$p_\perp^2 \approx z(1-z)m^2 \text{ p}_T \text{ ordered showers} \quad E^2\theta^2 \approx m^2/(z(1-z)) \text{ angular ordered showers}$$

Two are based on the standard shower language  
of a  $\rightarrow$  bc successive branchings:

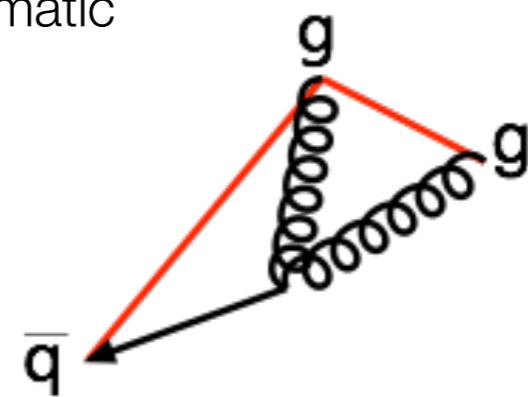
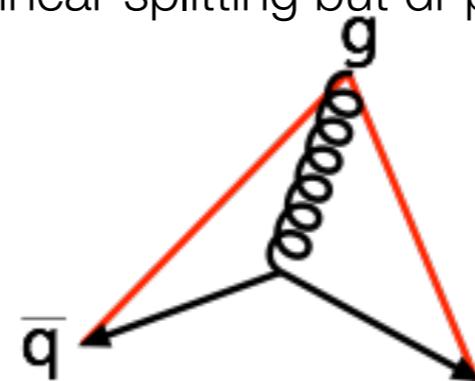
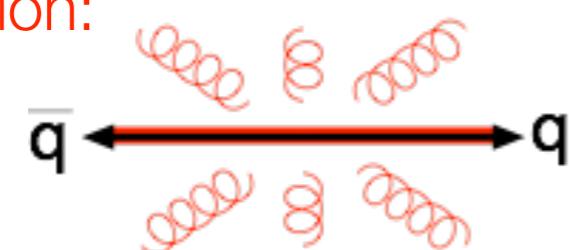


HERWIG, HERWIG++ :  $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$

PYTHIA, 8 (basic) :  $Q^2 = m^2$  (timelike) or  $= -m^2$  (spacelike)

PYTHIA6, 8 (p<sub>T</sub> ordered) : mixture: collinear splitting but di-pole kinematic

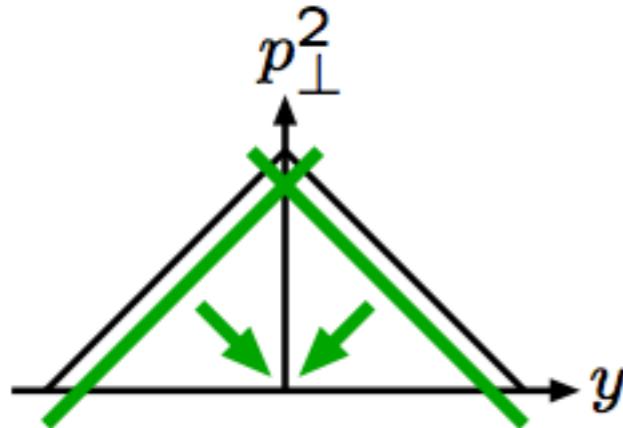
One is based on a picture of  
dipole emission:



Ariadne :  $Q^2 = p_\perp^2$ ; FSR mainly, ISR is primitive ...

consider the full recoil and not only the branching

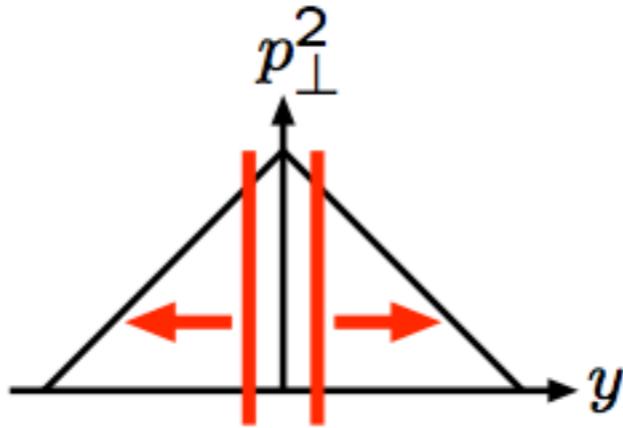
PYTHIA:  $Q^2 = m^2$  HERWIG/++:  $Q^2 \sim E^2 \theta^2$  ARIADNE/Pythia8:  $Q^2 = p_{\perp}^2$



Large mass first  
["hardness" ordered]

Covers phase space  
ME merging simple  
 $g \rightarrow qq$  simple  
not Lorentz invariant  
no stop/restart

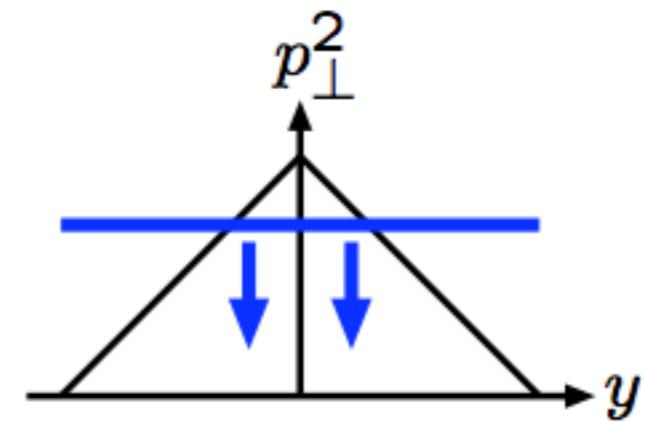
ISR:  $m^2 \rightarrow -m^2$



Large angle first  
[not "hardness" ordered]

Gaps in coverage  
ME merging messy  
 $g \rightarrow qq$  simple  
not Lorentz invariant  
no stop/restart

ISR:  $\theta \rightarrow \theta$



Large  $p_{\perp}$  first  
["hardness" ordered]

Covers phase space  
ME merging simple  
 $g \rightarrow qq$  messy  
Lorentz invariant  
can stop/restart

ISR: complicated

# Color coherence

QED: Chudakov effect (mid-fifties)

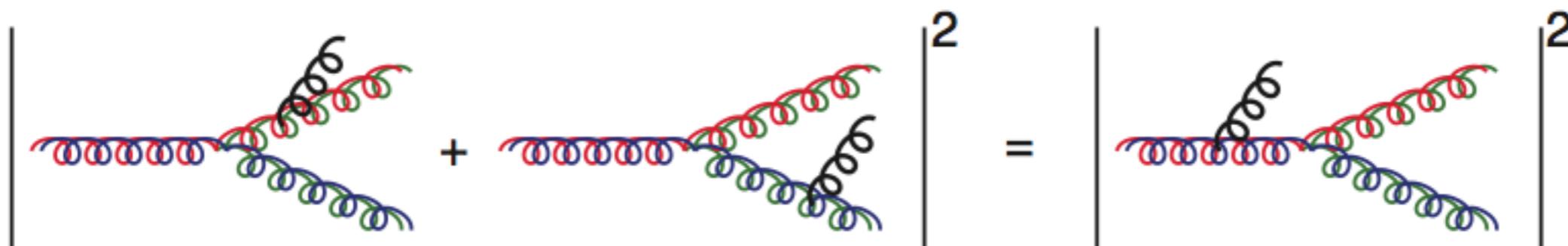


emulsion plate

reduced  
ionization

normal  
ionization

QCD: colour coherence for soft gluon emission



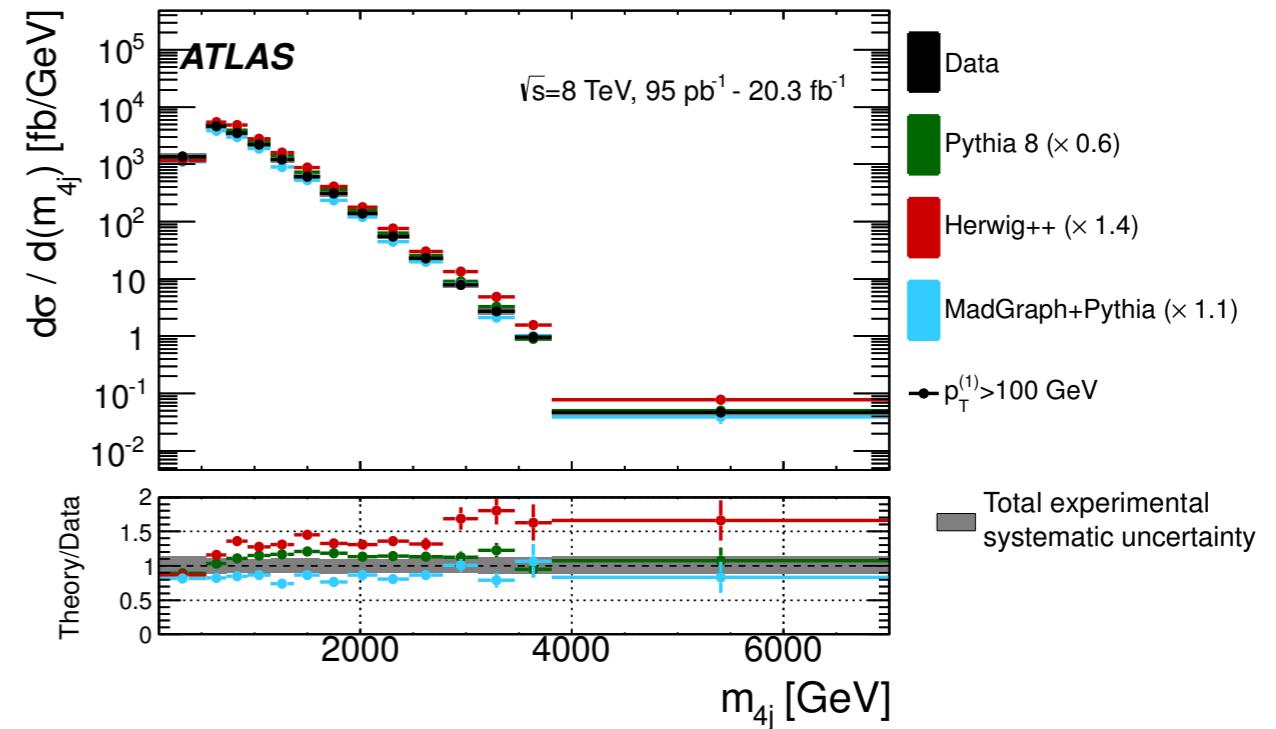
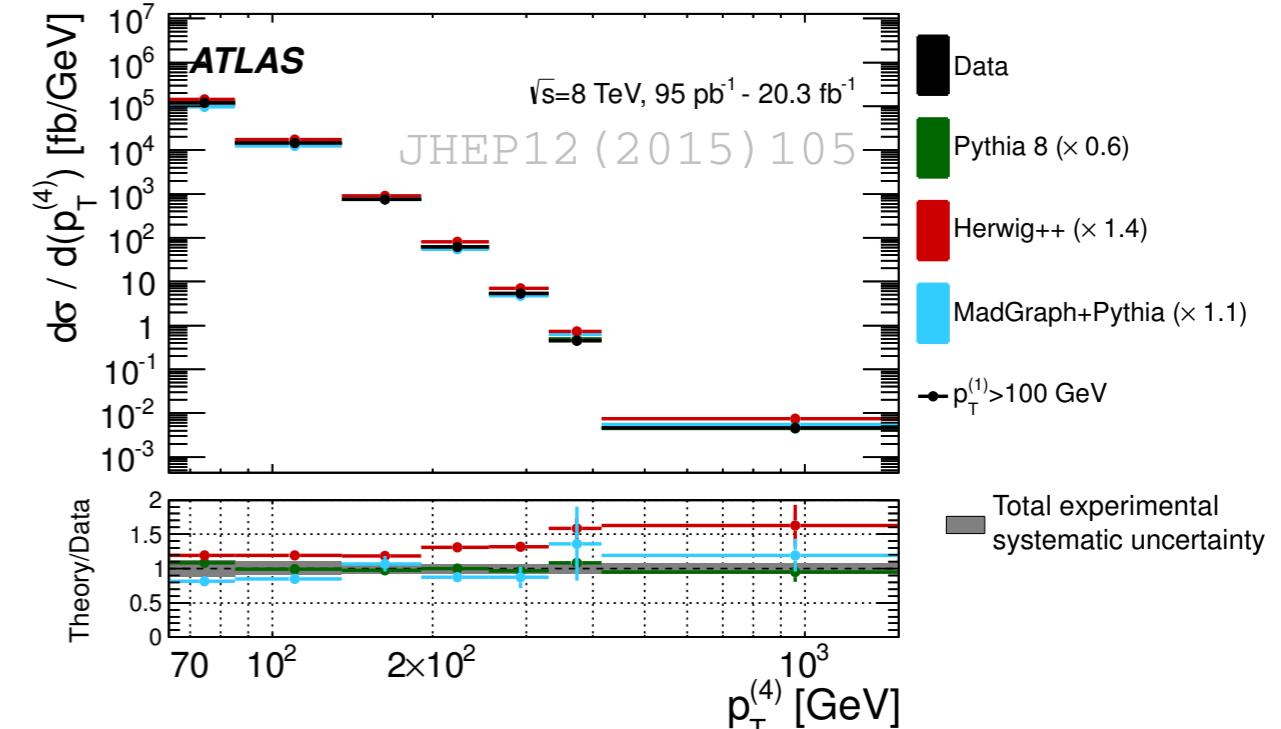
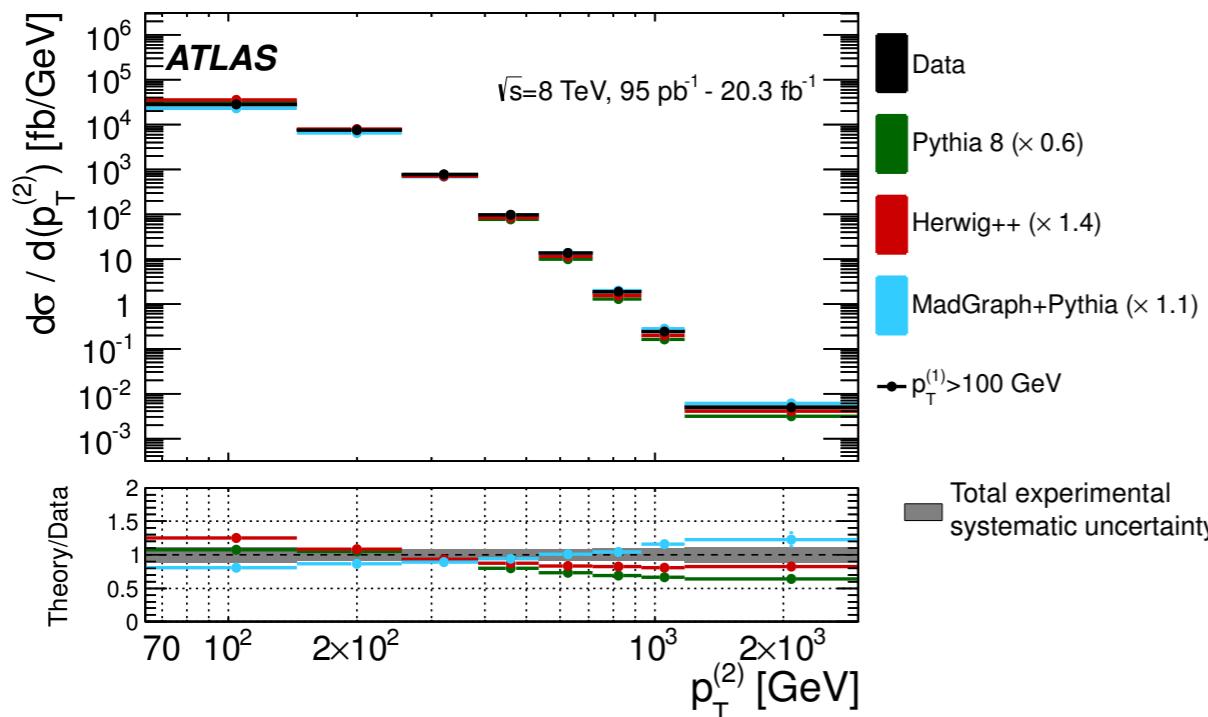
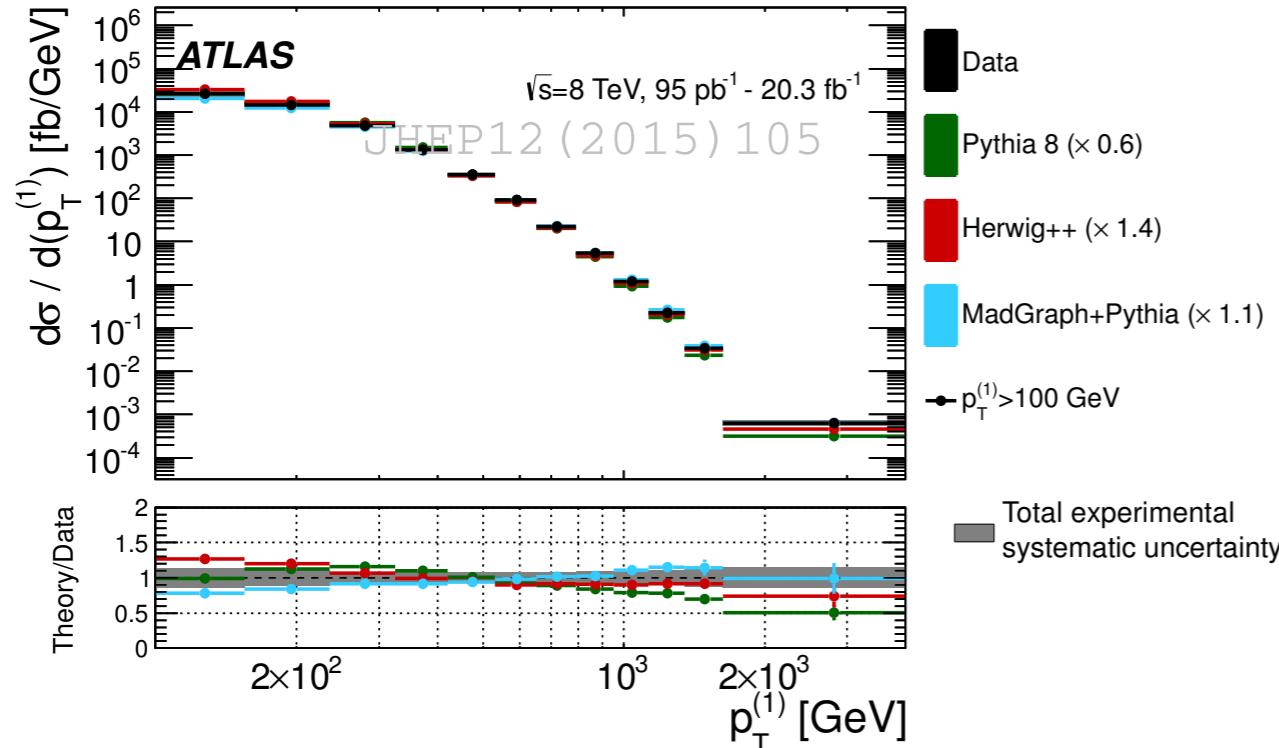
- solved by
- requiring emission angles to be decreasing

or

  - requiring transverse momenta to be decreasing

# Compariosn to LHC data

4jets cross section:  $p_T^{(1)} > p_T^{(2)} > p_T^{(3)} > p_T^{(4)}$



# Example of processes implemented in Pythia6

No.	Subprocess	No.	Subprocess	No.	Subprocess	No.	Subprocess	No.	Subprocess	No.	Subprocess
	Hard QCD processes:	36	$f_i \gamma \rightarrow f_k W^\pm$		New gauge bosons:	297	$f_i \bar{f}_j \rightarrow H^\pm h^0$		Higgs pairs:	146	$e \gamma \rightarrow e^*$
11	$f_i f_j \rightarrow f_i f_j$	69	$\gamma\gamma \rightarrow W^+ W^-$	141	$f_i \bar{f}_i \rightarrow \gamma/Z^0/Z'$	298	$f_i \bar{f}_j \rightarrow H^\pm H^0$	147	$d g \rightarrow d^*$	210	$f_i \bar{f}_j \rightarrow \tilde{\ell}_L \tilde{\nu}_\ell^* +$
12	$f_i \bar{f}_i \rightarrow k_f \bar{k}_f$	70	$\gamma W^\pm \rightarrow Z^0 W^\pm$	142	$f_i \bar{f}_j \rightarrow W^\pm$	299	$f_i \bar{f}_i \rightarrow A^0 h^0$	148	$u g \rightarrow u^*$	211	$f_i \bar{f}_j \rightarrow \tilde{\tau}_1 \tilde{\nu}_\tau^* +$
13	$f_i \bar{f}_i \rightarrow gg$		Prompt photons:	144	$f_i f_j \rightarrow R$	300	$f_i \bar{f}_i \rightarrow A^0 H^0$	167	$q_i q_j \rightarrow d^* q_k$	212	$f_i \bar{f}_j \rightarrow \tilde{\tau}_2 \tilde{\nu}_\tau^* +$
28	$f_i g \rightarrow f_i g$	14	$f_i \bar{f}_i \rightarrow g\gamma$	29	$f_i g \rightarrow f_i \gamma$	301	$f_i \bar{f}_i \rightarrow H^+ H^-$	168	$q_i q_j \rightarrow u^* q_k$	213	$f_i \bar{f}_i \rightarrow \tilde{\nu}_\ell \tilde{\nu}_\ell^*$
53	$gg \rightarrow f_k \bar{f}_k$	18	$f_i \bar{f}_i \rightarrow \gamma\gamma$		Heavy SM Higgs:	169	$q_i \bar{q}_i \rightarrow e^\pm e^\mp$	214	$f_i \bar{f}_i \rightarrow \tilde{\nu}_\tau \tilde{\nu}_\tau^*$		
68	$gg \rightarrow gg$	29	$f_i g \rightarrow f_i \gamma$	5	$Z^0 Z^0 \rightarrow h^0$	165	$f_i \bar{f}_i (\rightarrow \gamma^*/Z^0) \rightarrow f_k \bar{f}_k$	216	$f_i \bar{f}_i \rightarrow \tilde{\chi}_1 \tilde{\chi}_1$		
	Soft QCD processes:	114	$gg \rightarrow \gamma\gamma$	8	$W^+ W^- \rightarrow h^0$	166	$f_i \bar{f}_i (\rightarrow W^\pm) \rightarrow f_k \bar{f}_l$	217	$f_i \bar{f}_i \rightarrow \tilde{\chi}_2 \tilde{\chi}_2$		
91	elastic scattering	71	$Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0$		Leptoquarks:	145	$q_i \ell_j \rightarrow L_Q$	218	$f_i \bar{f}_i \rightarrow \tilde{\chi}_3 \tilde{\chi}_3$		
92	single diffraction ( <i>XB</i> )	115	$gg \rightarrow g\gamma$	72	$Z_L^0 Z_L^0 \rightarrow W_L^+ W_L^-$	162	$qg \rightarrow \ell L_Q$	219	$f_i \bar{f}_i \rightarrow \tilde{\chi}_4 \tilde{\chi}_4$		
93	single diffraction ( <i>AX</i> )	73	$Z_L^0 W_L^\pm \rightarrow Z_L^0 W_L^\pm$	163	$gg \rightarrow L_Q \bar{L}_Q$	220	$f_i \bar{f}_i \rightarrow \tilde{\chi}_1 \tilde{\chi}_2$				
94	double diffraction	76	$W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$	164	$q_i \bar{q}_i \rightarrow L_Q \bar{L}_Q$	221	$f_i \bar{f}_i \rightarrow \tilde{\chi}_1 \tilde{\chi}_3$				
95	low- $p_\perp$ production	77	$W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm$		Technicolor:	391	$ff \rightarrow G^*$	222	$f_i \bar{f}_i \rightarrow \tilde{\chi}_1 \tilde{\chi}_4$		
	Open heavy flavour: (also fourth generation)	33	$f_i \gamma \rightarrow f_i g$	149	$gg \rightarrow \eta_{tc}$	392	$gg \rightarrow G^*$	223	$f_i \bar{f}_i \rightarrow \tilde{\chi}_2 \tilde{\chi}_3$		
81	$f_i \bar{f}_i \rightarrow Q_k \bar{Q}_k$	151	$f_i \bar{f}_i \rightarrow H^0$	191	$f_i \bar{f}_i \rightarrow \rho_{tc}^0$	393	$q \bar{q} \rightarrow gG^*$	224	$f_i \bar{f}_i \rightarrow \tilde{\chi}_2 \tilde{\chi}_4$		
82	$gg \rightarrow Q_k \bar{Q}_k$	152	$gg \rightarrow H^0$	192	$f_i \bar{f}_i \rightarrow \rho_{tc}^+$	394	$qg \rightarrow qG^*$	225	$f_i \bar{f}_i \rightarrow \tilde{\chi}_3 \tilde{\chi}_4$		
83	$q_i f_j \rightarrow Q_k \bar{f}_l$	153	$\gamma\gamma \rightarrow H^0$	193	$f_i \bar{f}_i \rightarrow \omega_{tc}^0$	395	$gg \rightarrow gG^*$	226	$f_i \bar{f}_i \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp$		
84	$g\gamma \rightarrow Q_k \bar{Q}_k$	171	$f_i \bar{f}_i \rightarrow Z^0 H^0$	194	$f_i \bar{f}_i \rightarrow f_k \bar{f}_k$		Left-right symmetry:	341	$\ell_i \ell_j \rightarrow H^{\pm\pm}$		
85	$\gamma\gamma \rightarrow F_k \bar{F}_k$	172	$f_i \bar{f}_i \rightarrow W^\pm H^0$	195	$f_i \bar{f}_i \rightarrow f_k \bar{f}_l$	342	$\ell_i \ell_j \rightarrow H_R^{\pm\pm}$				
	Closed heavy flavour:	131	$f_i \gamma_T^* \rightarrow f_i g$	361	$f_i \bar{f}_i \rightarrow W_L^+ W_L^-$	343	$\ell_i^\pm \gamma \rightarrow H_L^{\pm\pm} e^\mp$				
86	$gg \rightarrow J/\psi g$	181	$gg \rightarrow Q_k \bar{Q}_k H^0$	362	$f_i \bar{f}_i \rightarrow W_L^\pm \pi_{tc}^\mp$	344	$\ell_i^\pm \gamma \rightarrow H_R^{\pm\pm} e^\mp$				
87	$gg \rightarrow \chi_{0c} g$	182	$q_i \bar{q}_i \rightarrow Q_k \bar{Q}_k H^0$	363	$f_i \bar{f}_i \rightarrow \pi_{tc}^+ \pi_{tc}^-$	345	$\ell_i^\pm \gamma \rightarrow H_L^{\pm\pm} \mu^\mp$				
88	$gg \rightarrow \chi_{1c} g$	183	$f_i \bar{f}_i \rightarrow g H^0$	364	$f_i \bar{f}_i \rightarrow \gamma \pi_{tc}^0$	346	$\ell_i^\pm \gamma \rightarrow H_R^{\pm\pm} \mu^\mp$				
89	$gg \rightarrow \chi_{2c} g$	184	$f_i g \rightarrow f_i H^0$	365	$f_i \bar{f}_i \rightarrow \gamma \pi_{tc}^0$	347	$\ell_i^\pm \gamma \rightarrow H_L^{\pm\pm} \tau^\mp$				
104	$gg \rightarrow \chi_{0c}$	185	$gg \rightarrow g H^0$	366	$f_i \bar{f}_i \rightarrow Z^0 \pi_{tc}^0$	348	$\ell_i^\pm \gamma \rightarrow H_R^{\pm\pm} \tau^\mp$				
105	$gg \rightarrow \chi_{2c}$	186	$f_i \bar{f}_i \rightarrow A^0$	367	$f_i \bar{f}_i \rightarrow Z^0 \pi_{tc}^0$	349	$f_i \bar{f}_i \rightarrow H_L^{++} H_L^{--}$				
106	$gg \rightarrow J/\psi \gamma$	187	$q_i \gamma \rightarrow q_k \pi^\pm$	368	$f_i \bar{f}_i \rightarrow W^\pm \pi_{tc}^\mp$	350	$f_i \bar{f}_i \rightarrow H_R^{++} H_R^{--}$				
107	$g\gamma \rightarrow J/\psi g$	188	$q_i \bar{q}_i \rightarrow Q_k \bar{Q}_k A^0$	370	$f_i \bar{f}_i \rightarrow W_L^\pm Z_L^0$	351	$f_i \bar{f}_i \rightarrow f_k \bar{f}_l H_L^{\pm\pm}$				
108	$\gamma\gamma \rightarrow J/\psi \gamma$	189	$f_i \bar{f}_i \rightarrow g A^0$	371	$f_i \bar{f}_i \rightarrow W_L^\pm \pi_{tc}^0$	352	$f_i \bar{f}_i \rightarrow f_k \bar{f}_l H_R^{\pm\pm}$				
	W/Z production:	176	$f_i \bar{f}_i \rightarrow Z^0 A^0$	372	$f_i \bar{f}_i \rightarrow \pi_{tc}^\pm Z_L^0$	353	$f_i \bar{f}_i \rightarrow Z_R^0$	354	$f_i \bar{f}_i \rightarrow W_R^\pm$		
1	$f_i \bar{f}_i \rightarrow \gamma^*/Z^0$	3	$f_i \bar{f}_i \rightarrow h^0$	373	$f_i \bar{f}_i \rightarrow \pi_{tc}^\pm \pi_{tc}^0$		SUSY:	201	$f_i \bar{f}_i \rightarrow \tilde{e}_L \tilde{e}_L^*$		
2	$f_i \bar{f}_j \rightarrow W^\pm$	24	$f_i \bar{f}_i \rightarrow Z^0 h^0$	374	$f_i \bar{f}_j \rightarrow \gamma \pi_{tc}^\pm$	202	$f_i \bar{f}_i \rightarrow \tilde{e}_R \tilde{e}_R^*$				
22	$f_i \bar{f}_i \rightarrow Z^0 Z^0$	26	$f_i \bar{f}_j \rightarrow W^\pm h^0$	375	$f_i \bar{f}_j \rightarrow Z^0 \pi_{tc}^\pm$	203	$f_i \bar{f}_i \rightarrow \tilde{e}_L \tilde{e}_R^*$				
23	$f_i \bar{f}_j \rightarrow Z^0 W^\pm$	32	$f_i g \rightarrow f_i h^0$	376	$f_i \bar{f}_j \rightarrow W^\pm \pi_{tc}^0$	204	$f_i \bar{f}_i \rightarrow \tilde{\mu}_L \tilde{\mu}_L^*$				
25	$f_i \bar{f}_i \rightarrow W^+ W^-$	102	$gg \rightarrow h^0$	377	$f_i \bar{f}_j \rightarrow W^\pm \pi_{tc}^0$	205	$f_i \bar{f}_i \rightarrow \tilde{\mu}_R \tilde{\mu}_R^*$				
15	$f_i \bar{f}_i \rightarrow g Z^0$	103	$\gamma\gamma \rightarrow h^0$	381	$q_i q_j \rightarrow q_i q_j$	206	$f_i \bar{f}_i \rightarrow \tilde{\mu}_L \tilde{\mu}_R^*$				
16	$f_i \bar{f}_j \rightarrow g W^\pm$	110	$f_i \bar{f}_i \rightarrow \gamma h^0$	382	$q_i \bar{q}_i \rightarrow q_k \bar{q}_k$	207	$f_i \bar{f}_i \rightarrow \tilde{\tau}_1 \tilde{\tau}_1^*$				
30	$f_i g \rightarrow f_i Z^0$	111	$f_i \bar{f}_i \rightarrow g h^0$	383	$q_i \bar{q}_i \rightarrow gg$	208	$f_i \bar{f}_i \rightarrow \tilde{\tau}_2 \tilde{\tau}_2^*$				
31	$f_i g \rightarrow f_k W^\pm$	112	$f_i g \rightarrow f_i h^0$	384	$f_i g \rightarrow f_i g$	209	$f_i \bar{f}_i \rightarrow \tilde{\tau}_1 \tilde{\tau}_2^*$				
19	$f_i \bar{f}_i \rightarrow \gamma Z^0$	113	$gg \rightarrow g h^0$	385	$gg \rightarrow q_k \bar{q}_k$						
20	$f_i \bar{f}_j \rightarrow \gamma W^\pm$	121	$gg \rightarrow Q_k \bar{Q}_k h^0$	386	$gg \rightarrow gg$						
35	$f_i \gamma \rightarrow f_i Z^0$	122	$q_i \bar{q}_i \rightarrow Q_k \bar{Q}_k h^0$	387	$f_i \bar{f}_i \rightarrow Q_k \bar{Q}_k$						
		123	$f_i \bar{f}_i \rightarrow f_i f_j h^0$	388	$gg \rightarrow Q_k \bar{Q}_k$						
		124	$f_i \bar{f}_j \rightarrow f_k \bar{f}_l h^0$								

# Process simulation

Many specialized processes already available in Pythia8/Herwig++

but, processes usually only implemented at the lowest non-trivial order ...

Need external programs that ...

1. include higher order loop corrections or, alternatively, do kinematic dependent rescaling
3. allow matching of higher order ME generators [otherwise need to trust parton shower description ...]
5. provide correct spin correlations often absent in PS ...[e.g. top produced unpolarized, while  $t \rightarrow bW \rightarrow blv$  decay correct]
7. simulate newly available physics scenarios ...[appear quickly; need for many specialised generators]

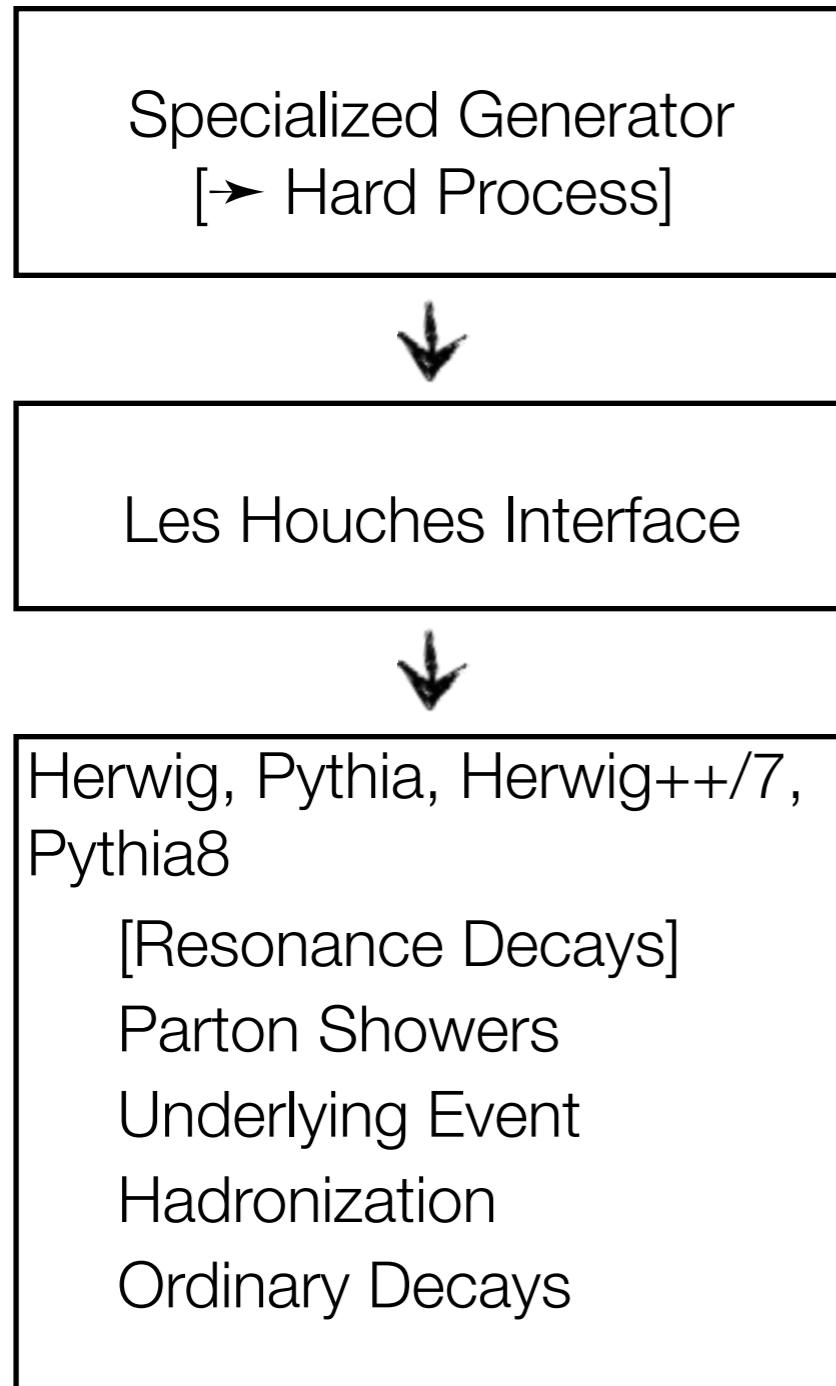
## Les Houches Accord ...

Specifies how parton-level information about the hard process and sequential decays can be encoded and passed on to a general-purpose generator.

Les Houches: regular annual meeting between theoreticians and experimentalists on MC generator developments.



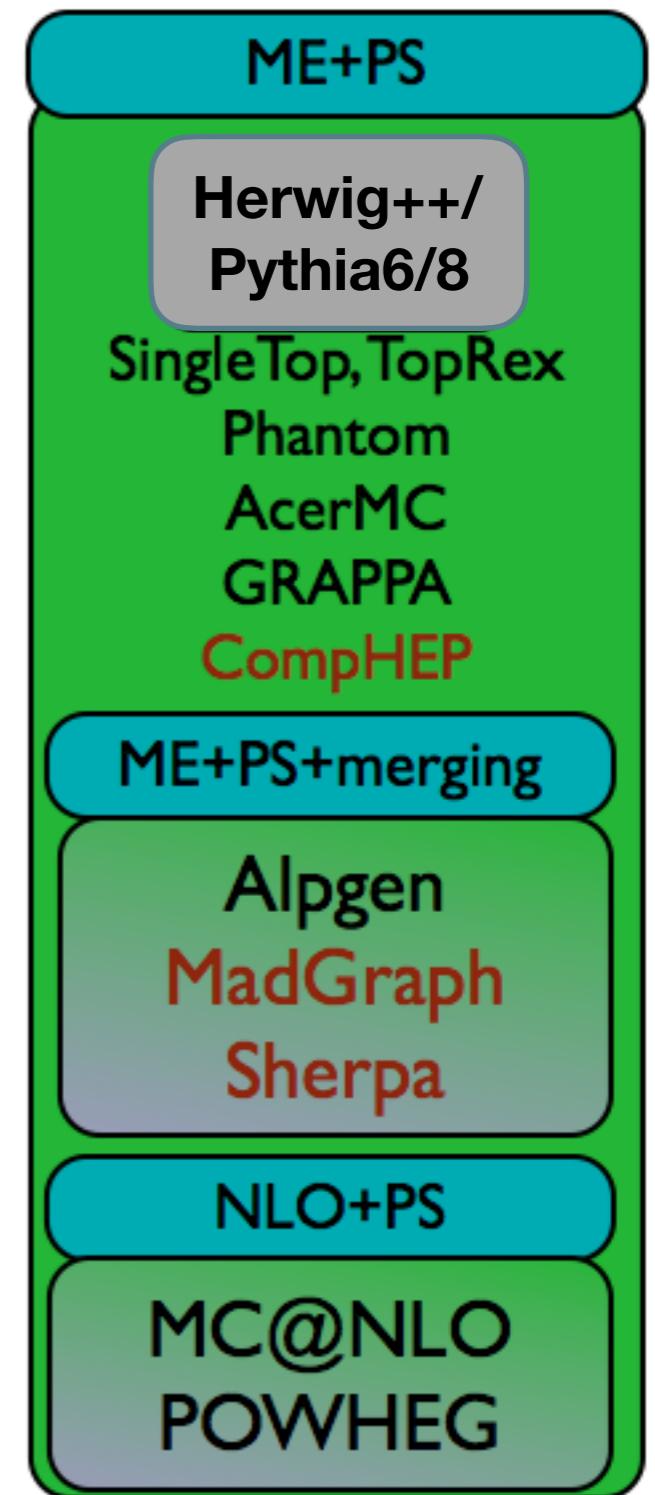
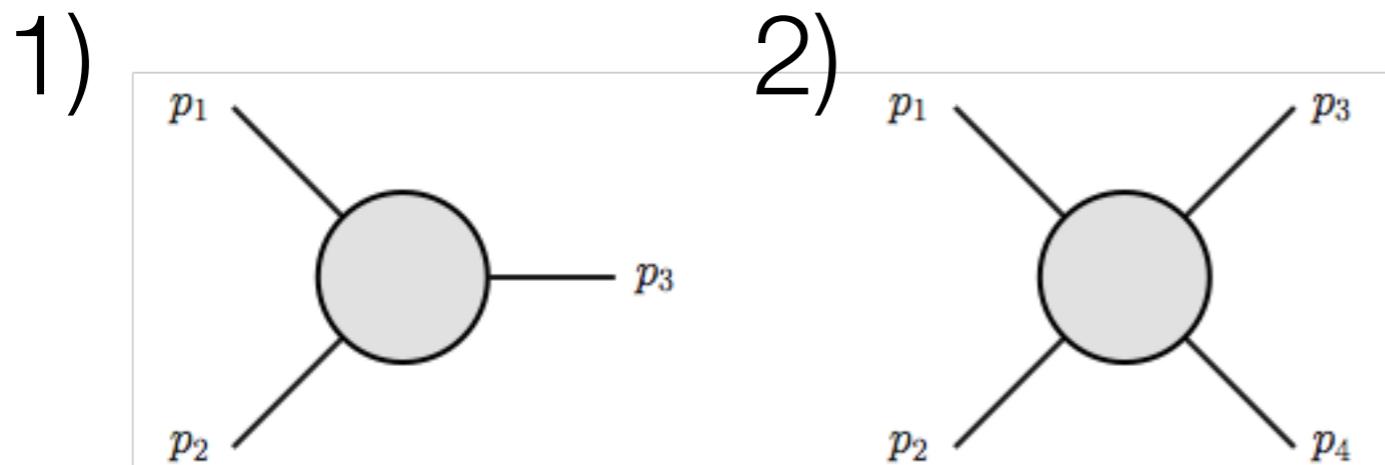
# Specialised Generators [some examples]



AcerMC :	ttbb, .single top
ALPGEN	: W/Z + $\leq 6j$ , nW + mZ + kH + $\leq 3j$ , ...
AMEGIC++	: generic LO
CompHEP	: generic LO
GRACE [+Bases/Spring]	: generic LO [+ some NLO loops]
GR@PPA	: bbbb
MadCUP	: W/Z+ $\leq 3j$ , ttbb
HELAS & MadGraph	: generic LO
MCFM	: NLO W/Z+ $\leq 2j$ , WZ, WH, H+ $\leq 1j$
O'Mega & WHIZARD	: generic LO
VECBOS	: W/Z+ $\leq 4j$
HRES	: Higgs boson production @NNLO
DYNNLO	: W/Z production @NNLO

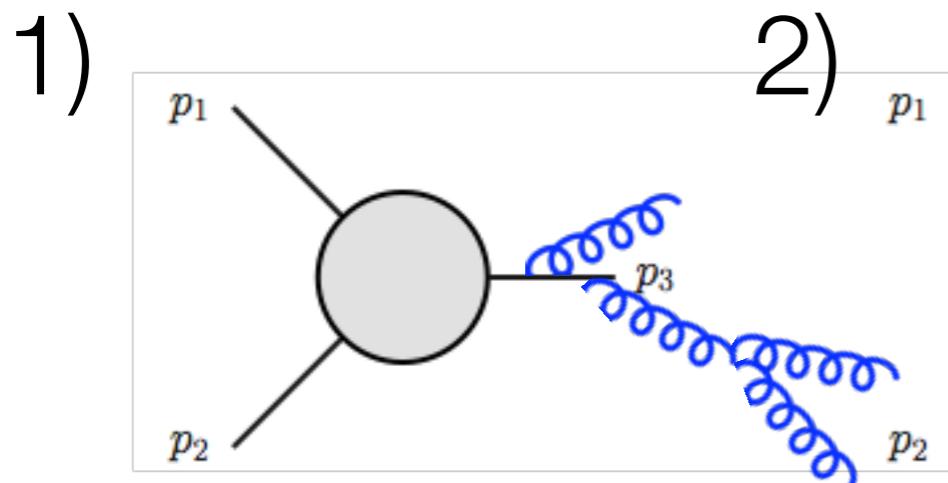
# Type I : Leading order matrix element & leading log parton shower

LO ME for hard processes  
[ $2 \rightarrow 1$  or  $2 \rightarrow 2$ ]

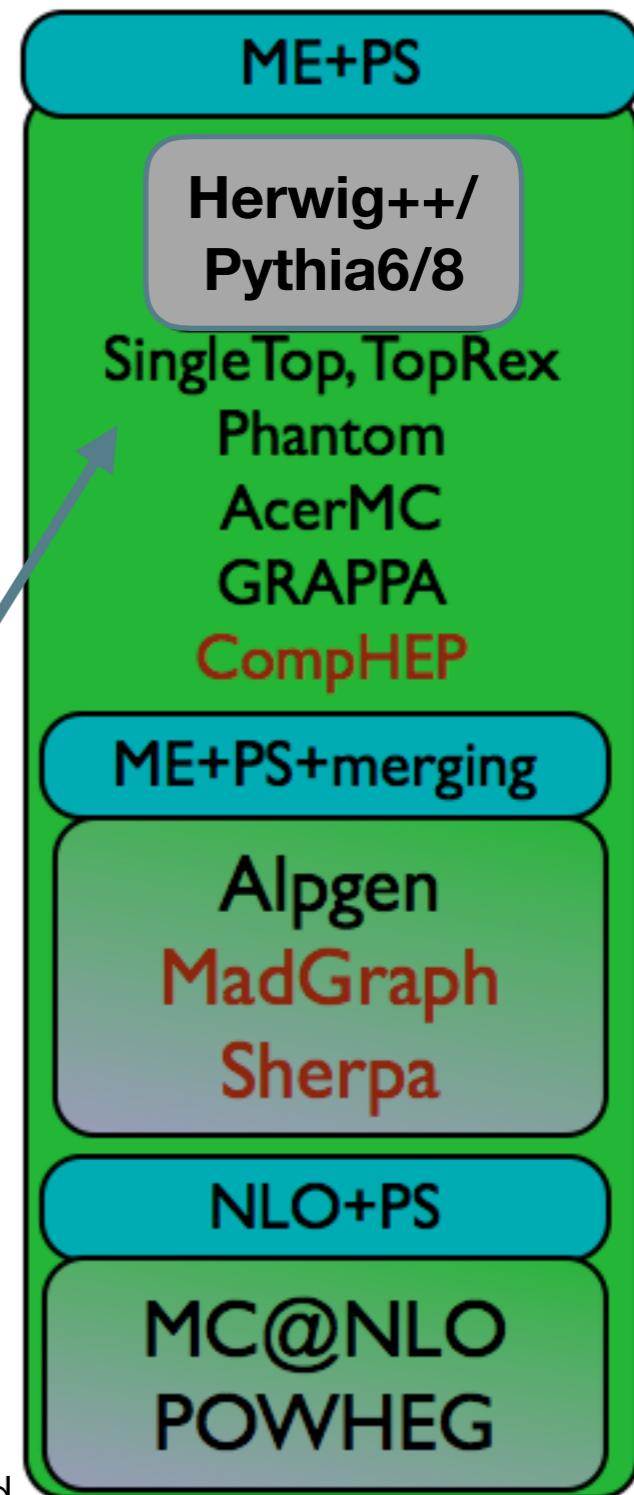


# Type I : Leading order matrix element & leading log parton shower

LO ME for hard processes  
[ $2 \rightarrow 1$  or  $2 \rightarrow 2$ ]



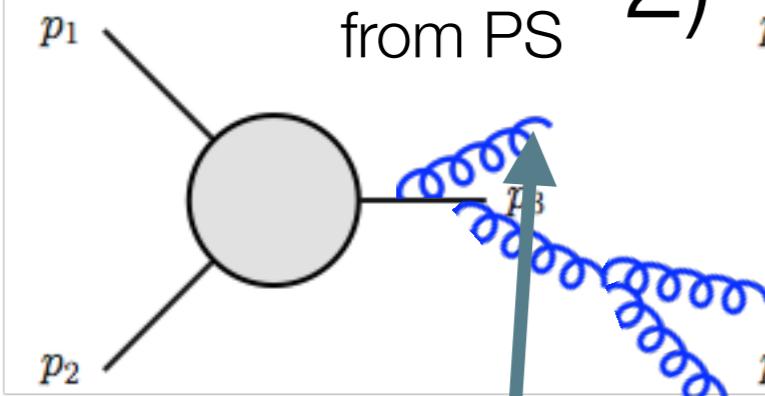
- Parton Shower: attaches gluons at each leg.
- only in soft-collinear approximation
- typically underestimate large angle/hard emission
- 1) or 2) at ME (different generations, different accuracy: cannot be combined)



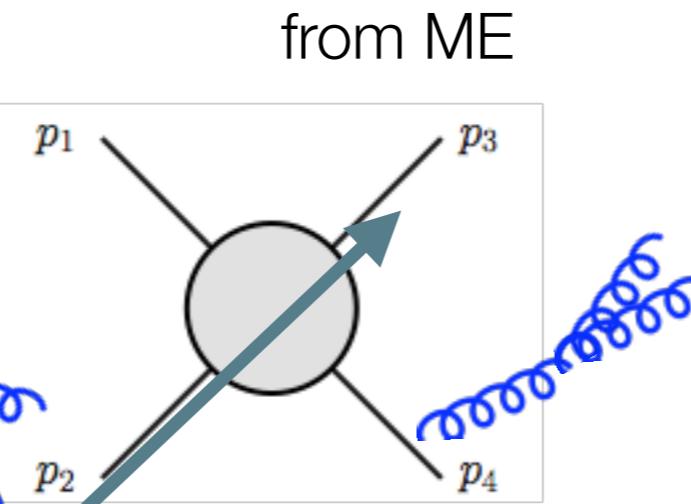
# Type 2 : Leading order matrix element & leading log parton shower + merging

LO ME for hard processes  
[ $2 \rightarrow 1$  or  $2 \rightarrow 2$ ]

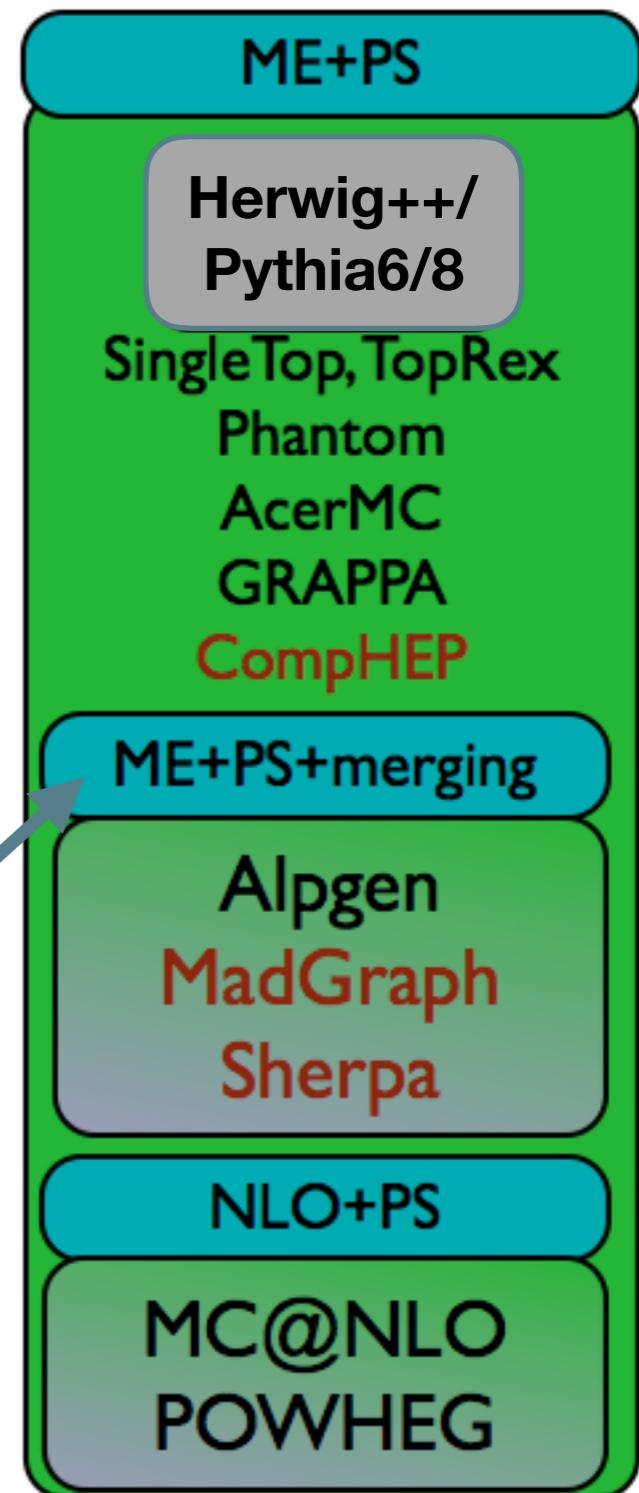
1)



2)



- Type 1 can be improved using 1) + 2)
- use ME calculation for hard/large angle jets
- but needs to remove double-counting: merging
  - CKKW: Catani, Krauss, Kuhn, Weber (Sherpa)
  - MLM
- very good description of high jet multiplicity kinematics



# Merging @LO

---

## MLM matching (simplified)

1) define matching cuts:

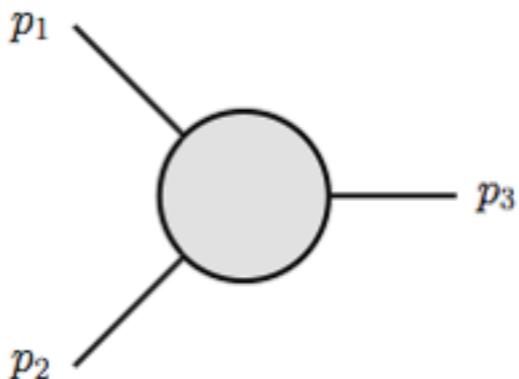
for example  $p_{TJ} > 20 \text{ GeV}$ ,  $\Delta R = 0.4$

# Merging @LO

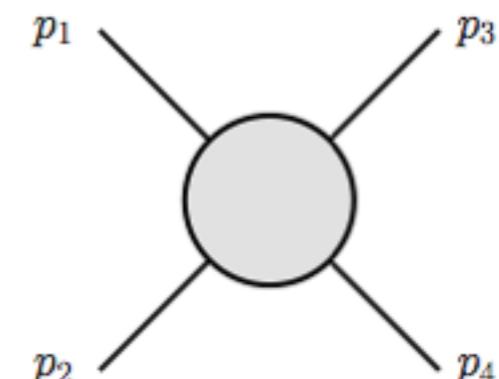
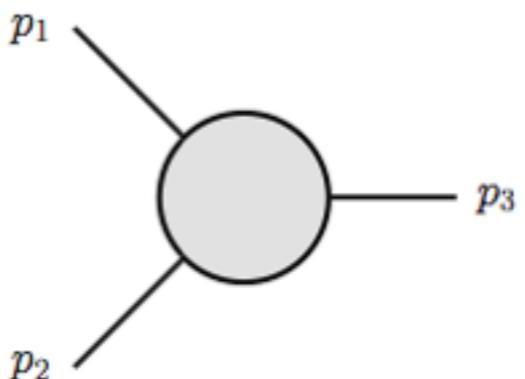
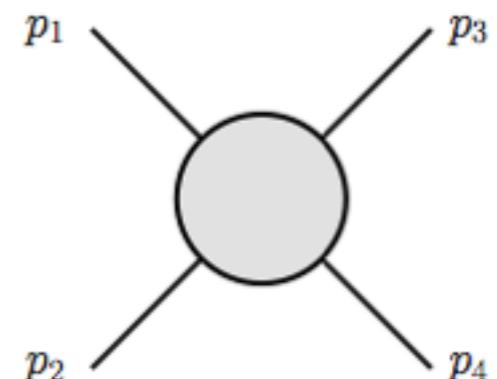
## MLM matching (simplified)

- 1) define matching cuts:  
for example  $p_{TJ} > 20 \text{ GeV}$ ,  $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets

1 parton



2 partons

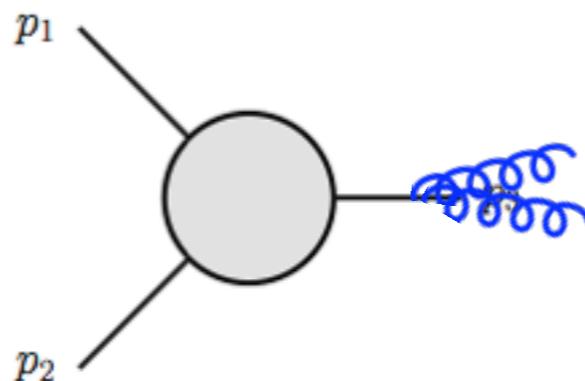


# Merging @LO

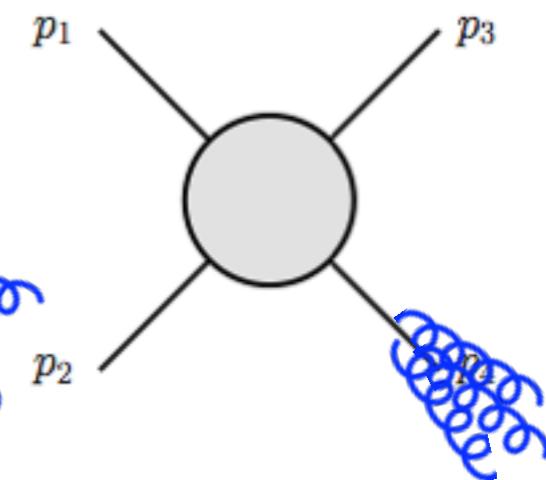
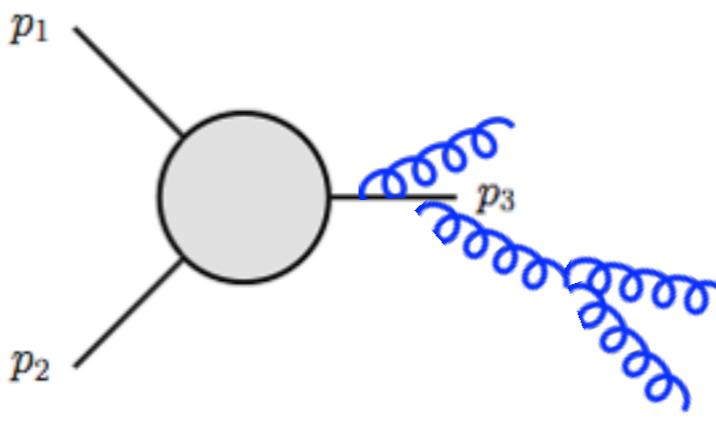
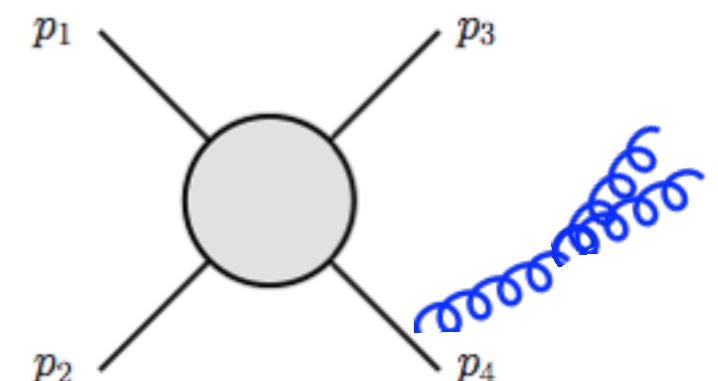
## MLM matching (simplified)

- 1) define matching cuts:  
for example  $p_{TJ} > 20 \text{ GeV}$ ,  $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events

1 parton



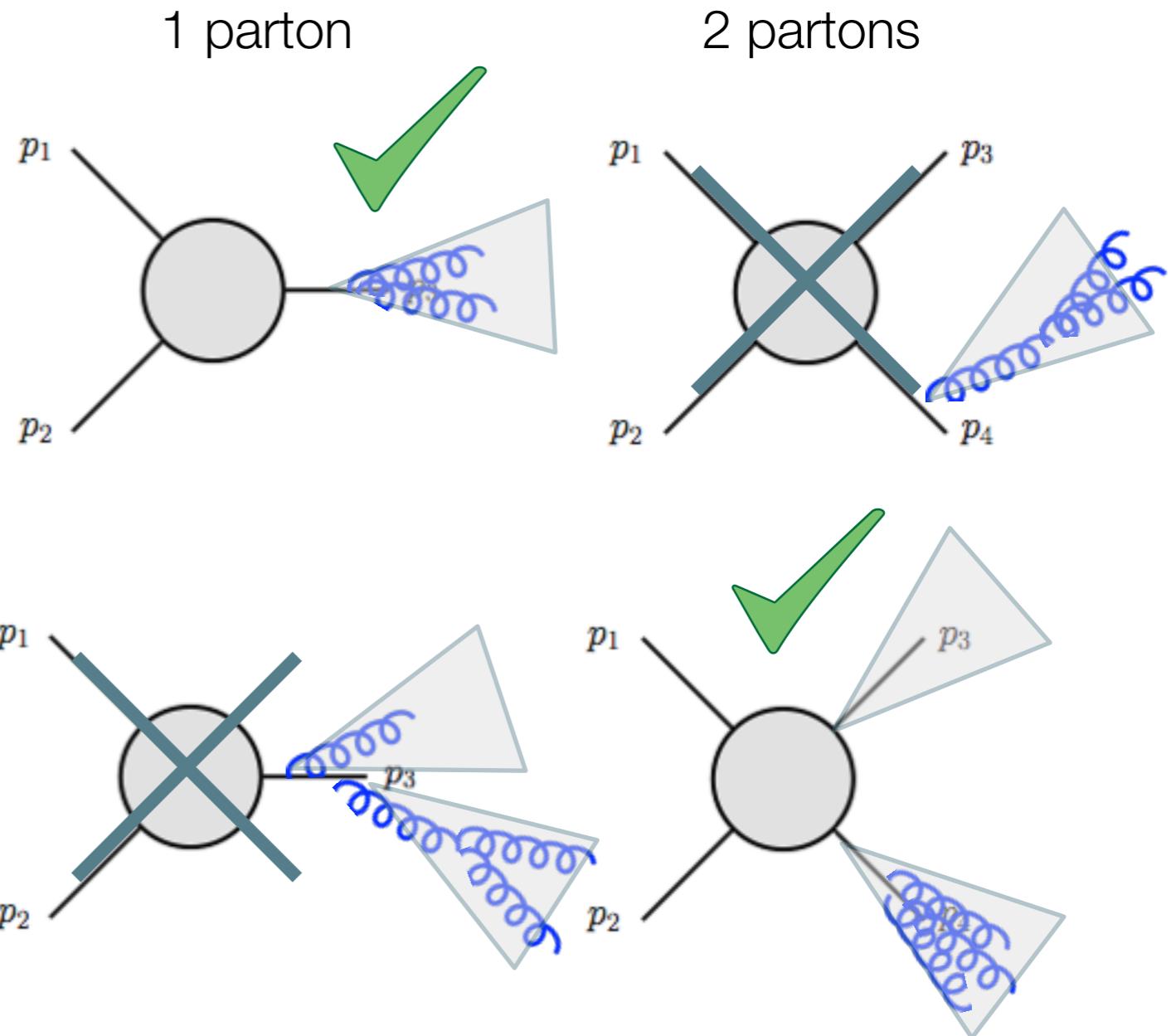
2 partons



# Merging @LO

## MLM matching (simplified)

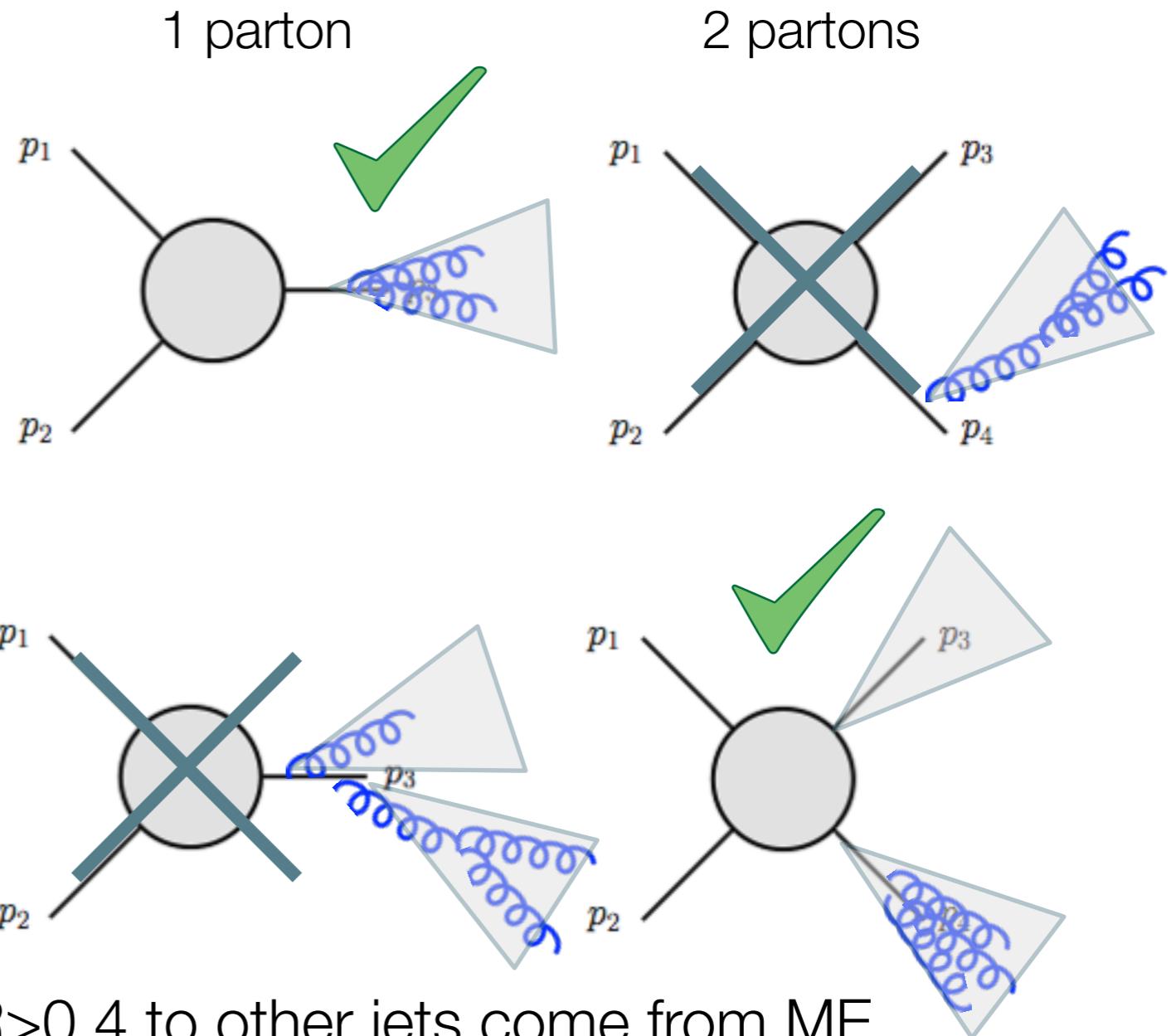
- 1) define matching cuts:  
for example  $p_{TJ} > 20 \text{ GeV}$ ,  $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events
- 4) select only events where jets above the  $p_T$  threshold match with final partons



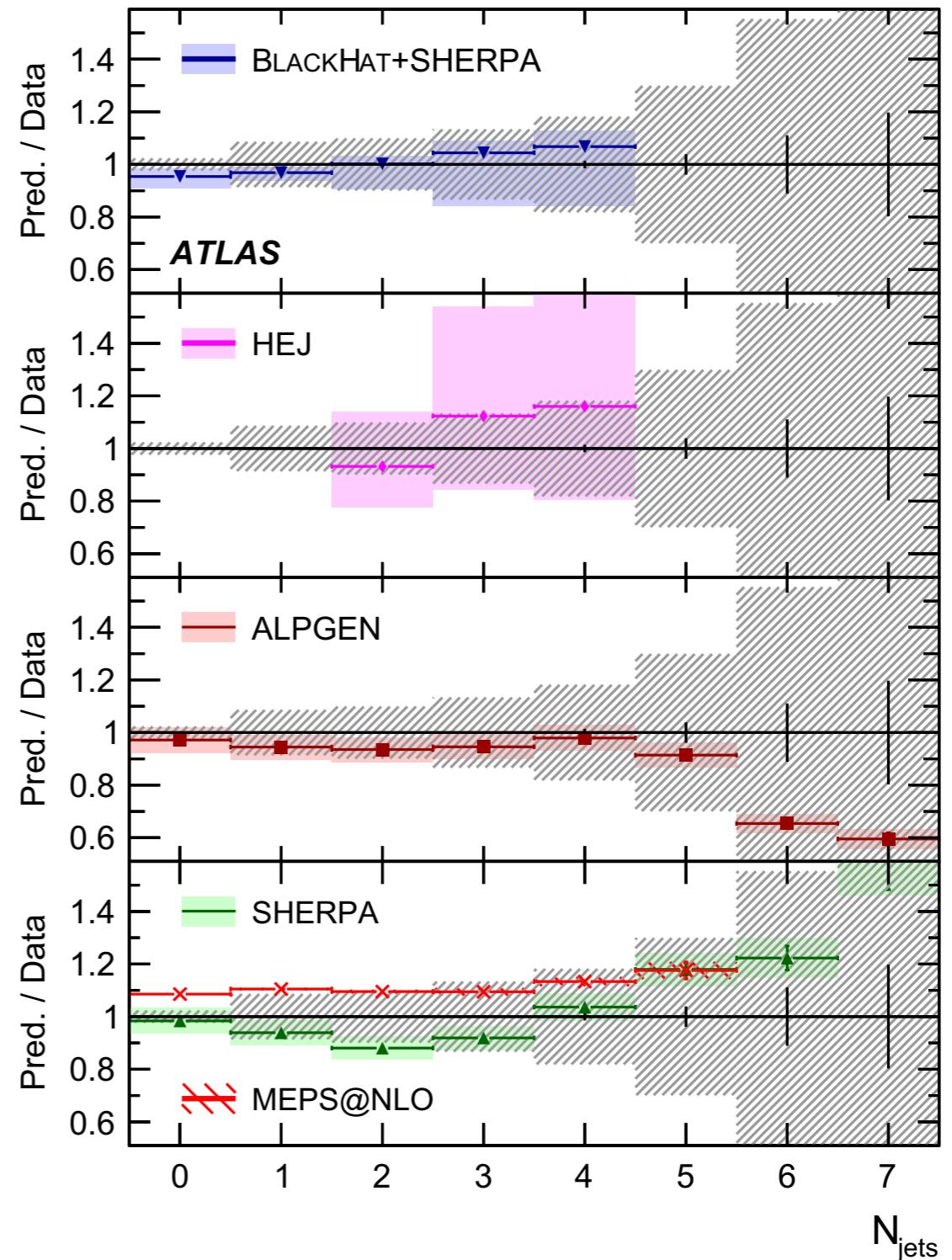
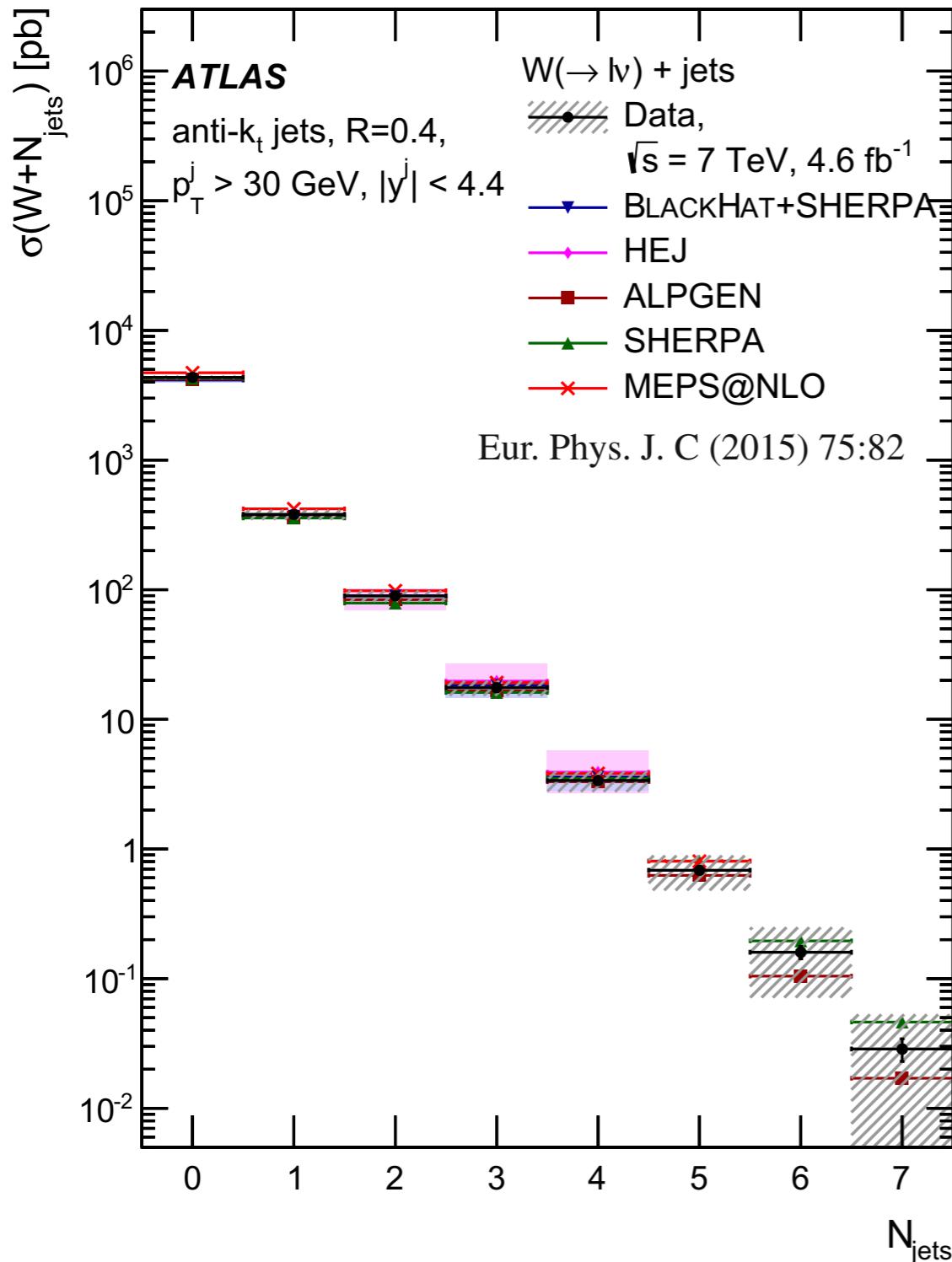
# Merging @LO

## MLM matching (simplified)

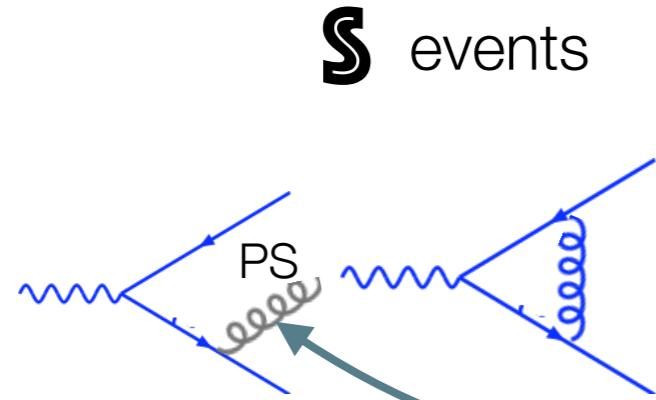
- 1) define matching cuts:  
for example  $p_{TJ} > 20 \text{ GeV}$ ,  $\Delta R = 0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events
- 4) select only events where jets above the  $p_T$  threshold match with final partons



# W+jets distributions



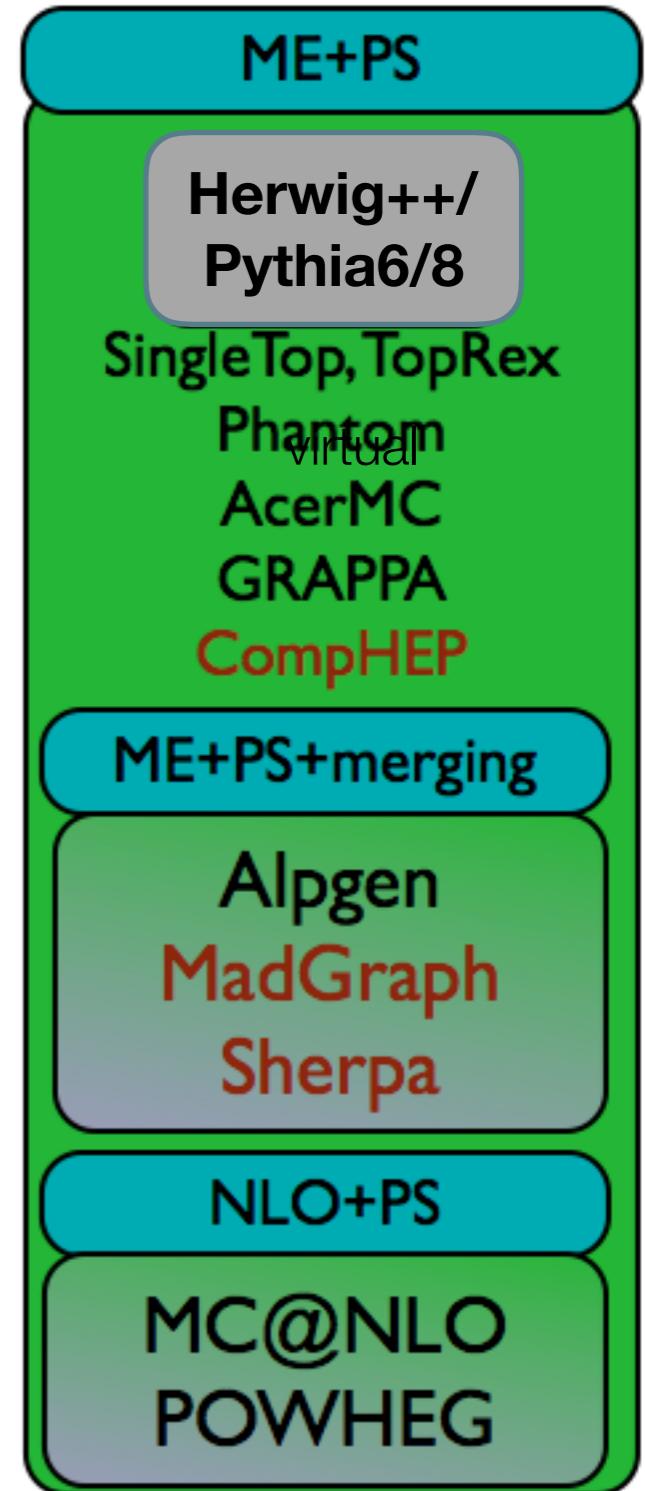
# Type III : Next-to-leading order ME & leading-log parton shower



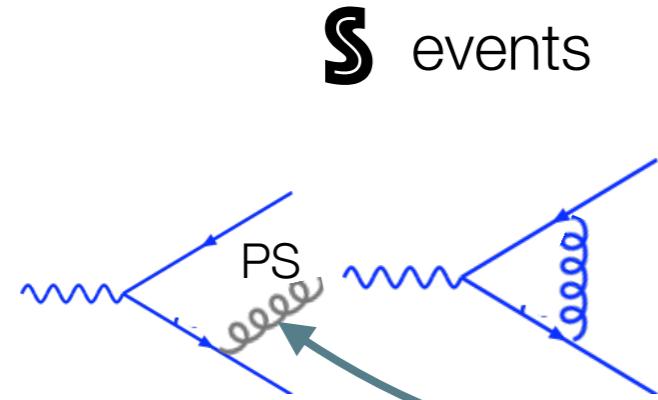
**H** events

need to remove double-counting

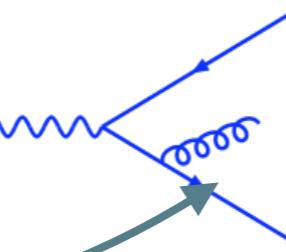
- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions



# Type III : Next-to-leading order ME & leading-log parton shower



**H** events



need to remove double-counting

- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions

Two matching methods:

1. Powheg

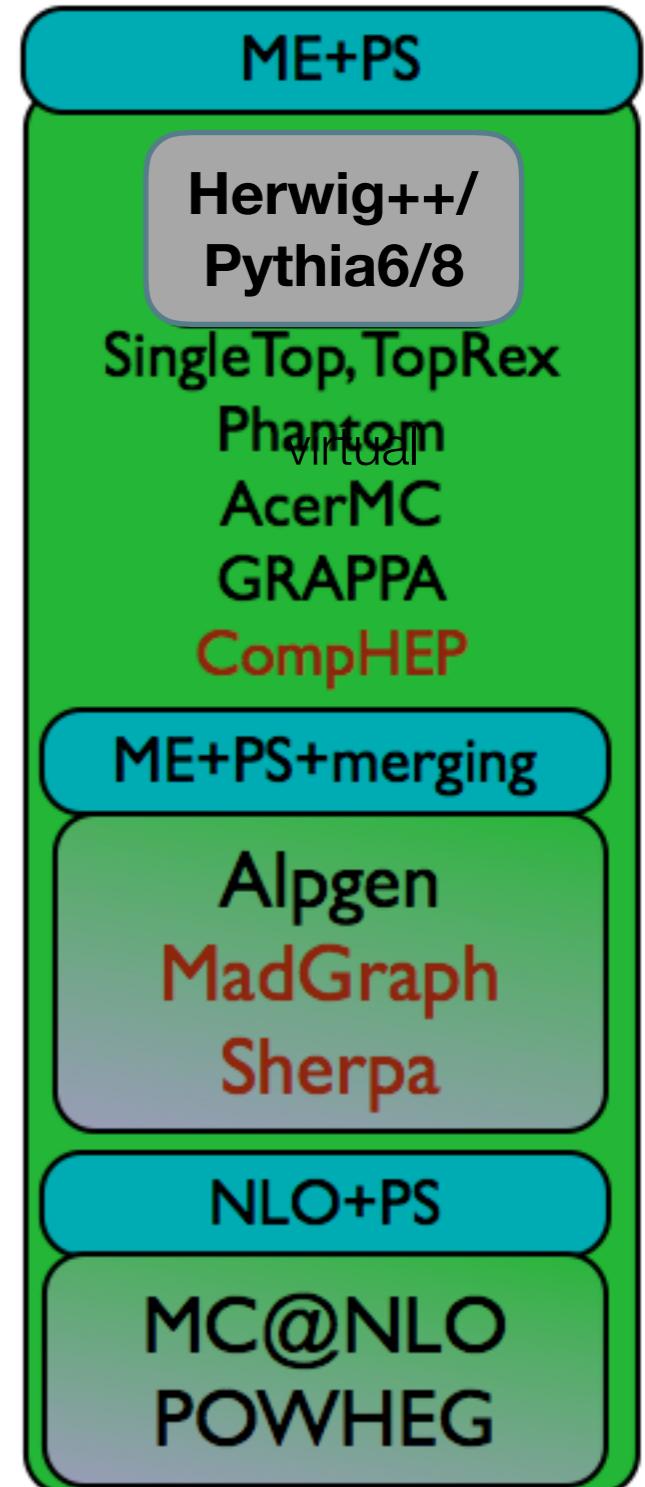
Truncated showers:

- 1) first emission produced by the ME;
- 2) don't allow the PS to produce patrons harder than the first emission;
- 3) not exact at NLO (contains unbalanced higher order terms)

2. MC@NLO:

$$|ME|^2 = |ME + PS - PS(\text{up to } \alpha_s^2)|^2$$

- + Result is exact at NLO...
- produce some negative weights, need retuning for each PS

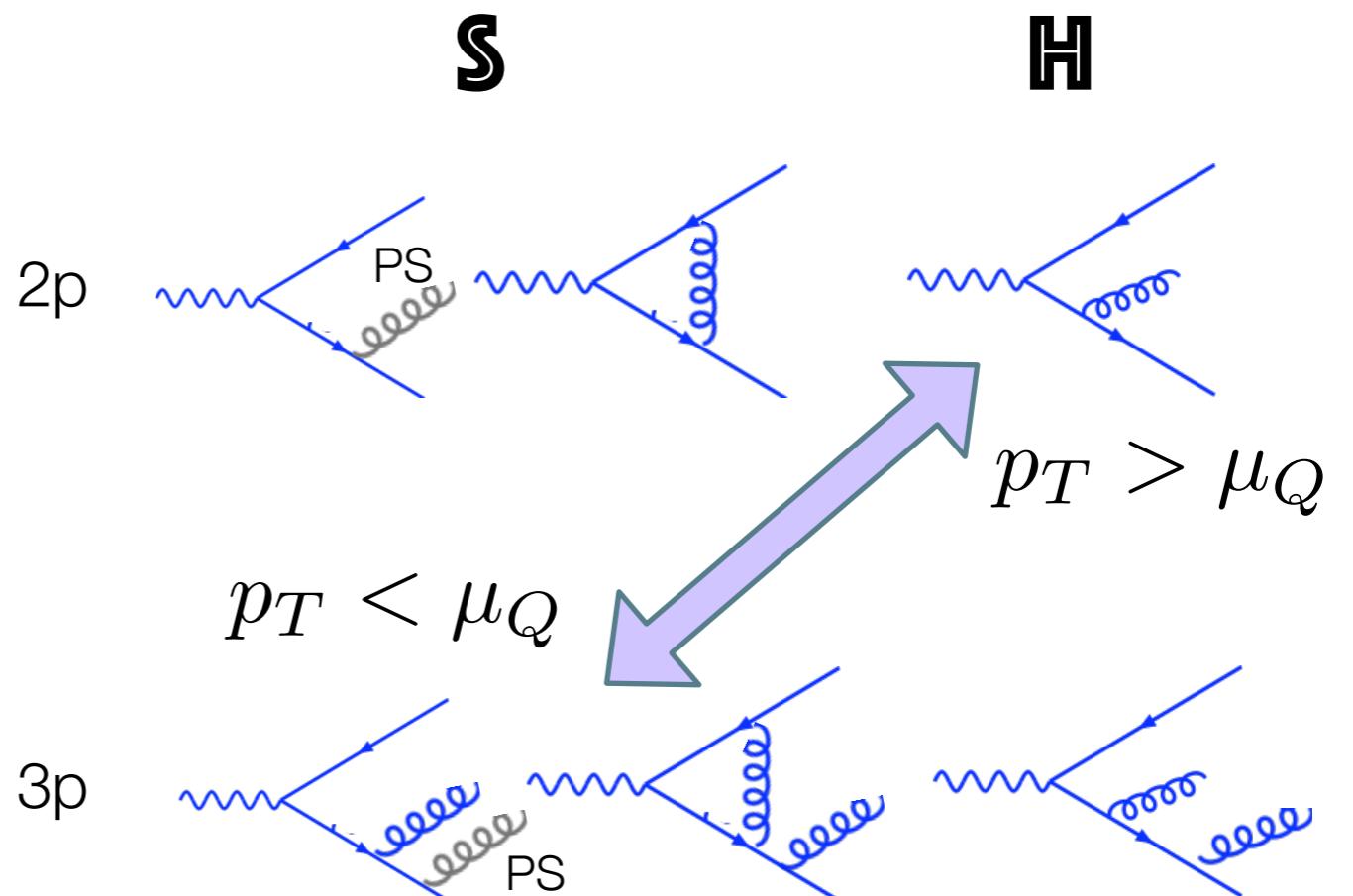


# Merging @NLO (quite new, used now at 13 TeV)

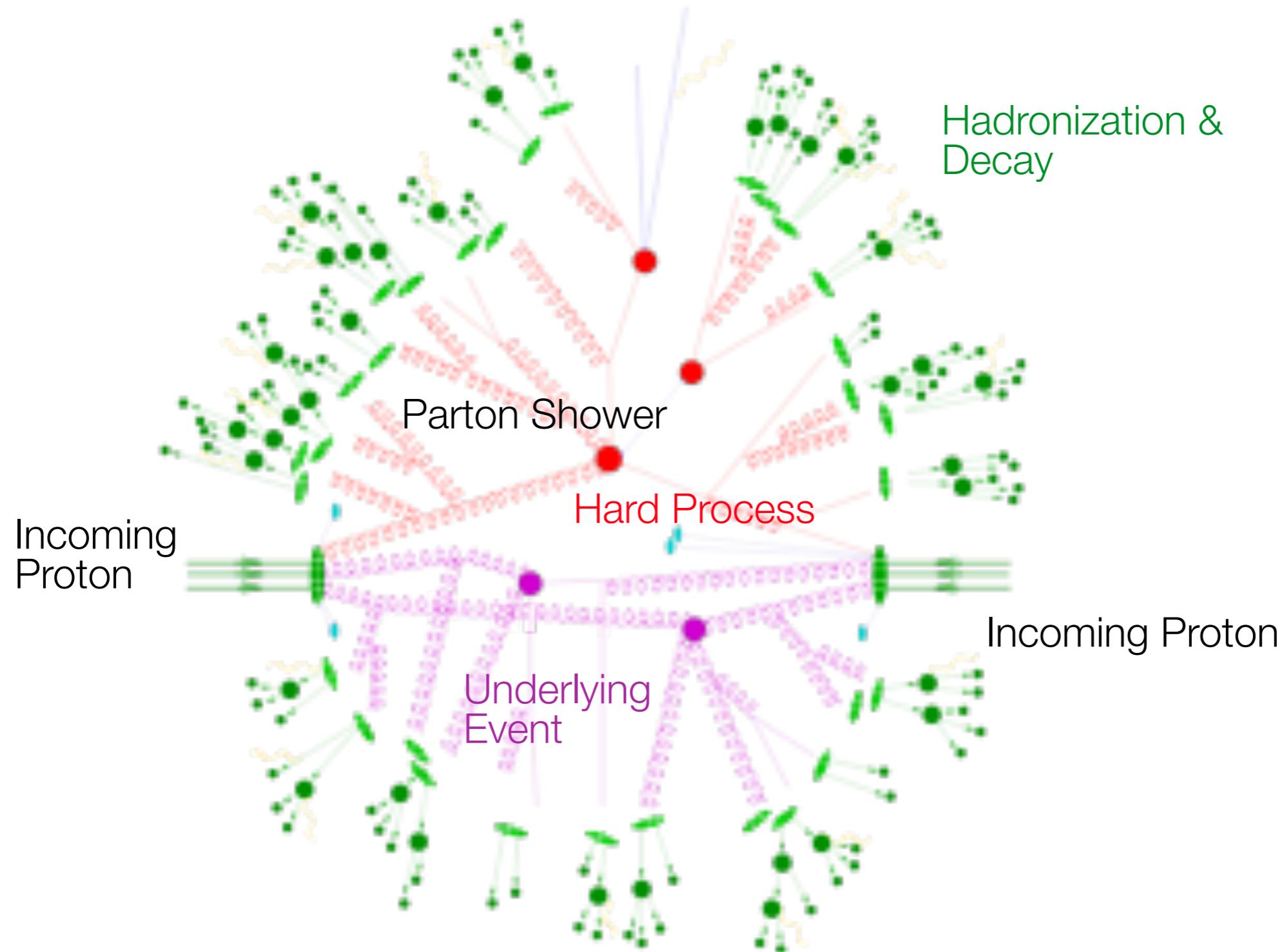
JHEP12(2012)061

## FxFx (Frederix-Frixione) merging

- 1) define a matching scale  $\mu_Q$ ;
- 2) don't allow **S** events with  $p_T > \mu_Q$   
(those will be provided by **H** events of  $n-1$  partons NLO real emission);  
the restriction is imposed both at ME  
and on the shower starting scale  $\mu < \mu_Q$
- 3) treat the obtained events as LO ones  
and apply an LO-style merging (this  
allow to produce smoother  
distributions)



# Let's recap



# From partons to color neutral hadrons

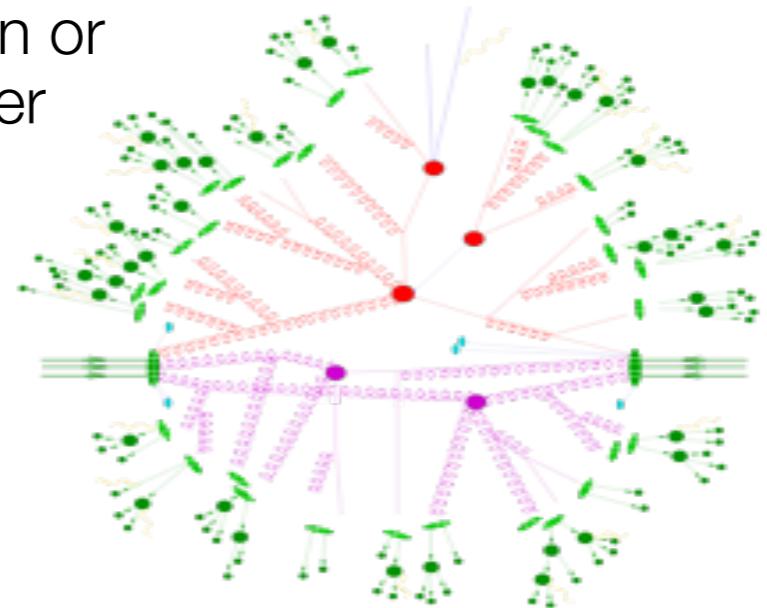
## Fragmentation:

Parton splitting into other partons

[QCD: re-summation of leading-logs]

["Parton shower"]

Fragmentation or  
Parton Shower



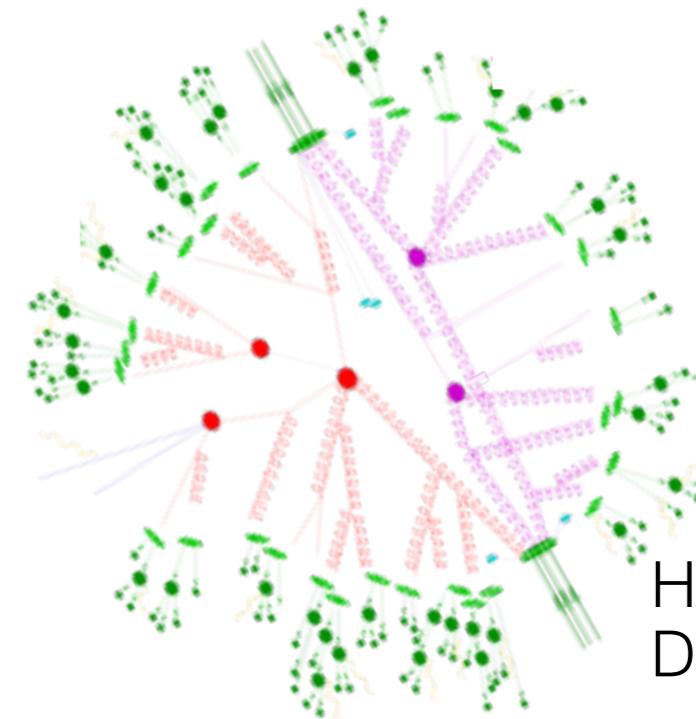
## Hadronization:

Parton shower forms hadrons

[non-perturbative, only models]

## Decay of unstable hadrons

[perturbative QCD, electroweak theory]



Hadronization &  
Decays

# Non-perturbative transition from partons to hadrons ...

---

[Modelling relies on **phenomenological models** available]

Models based on MC simulations  
very successful:

Generation of **complete final states** ...

[Needed by experimentalists in detector simulation]

Caveat: **tunable ad-hoc parameters**

Most popular MC models:

Pythia/8 : **Lund string model**

Herwig/++ : **Cluster model**

# Independent fragmentation of each parton

Simplest approach:  
[Field, Feynman, Nucl. Phys. B136 (1978) 1]

Start with original quark

Generate quark-antiquark pairs  
from vacuum

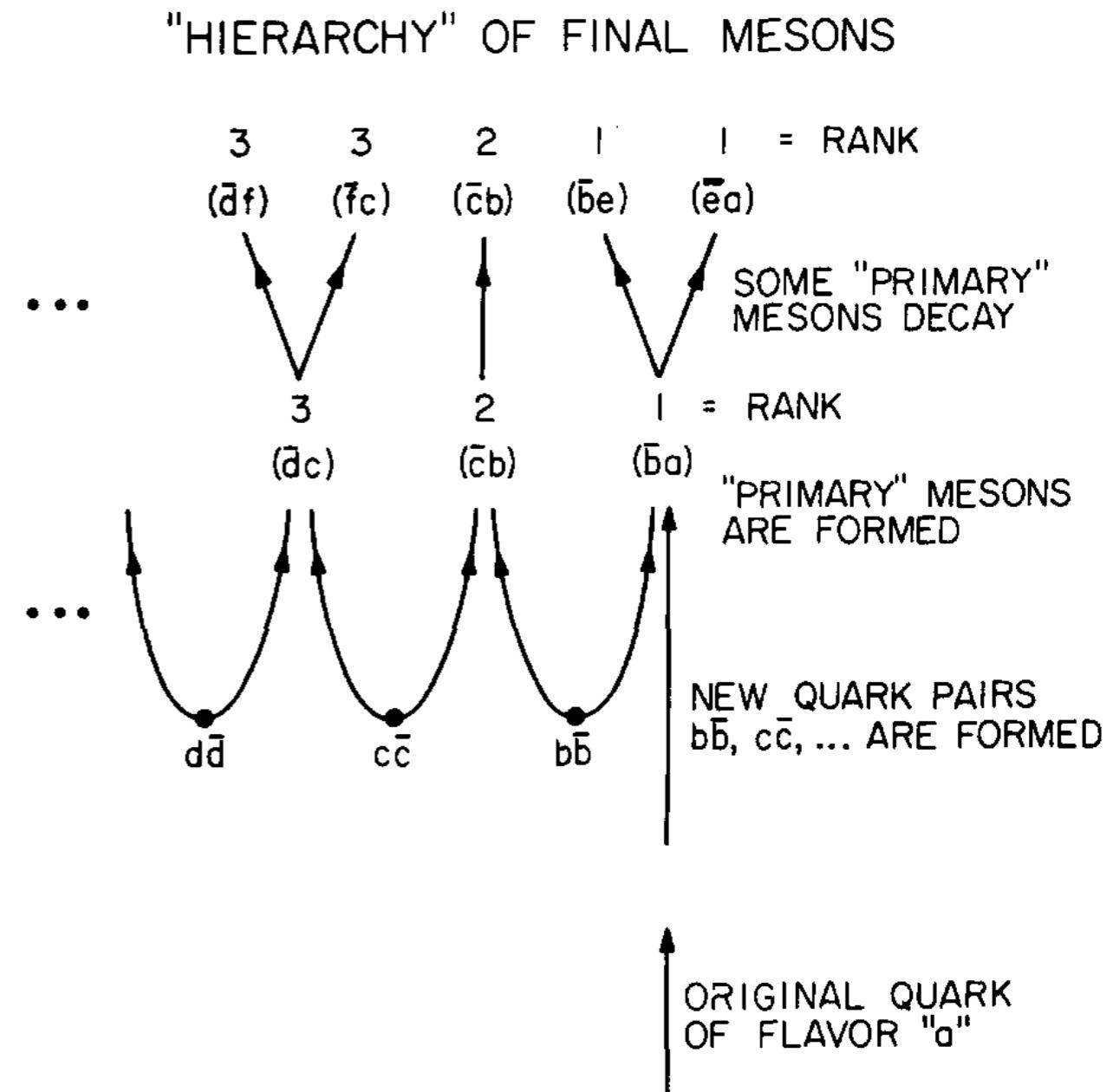
→ form “primary meson”  
with energy fraction  $z$

Continue with leftover quark  
with energy fraction  $1-z$

Stop at low energies (cut-off)

Include flavour non-perturbative  
fragmentation functions  $D(z)$

$D(z)$ : probability to find a meson/hadron  
with energy fraction  $z$  in jet ...



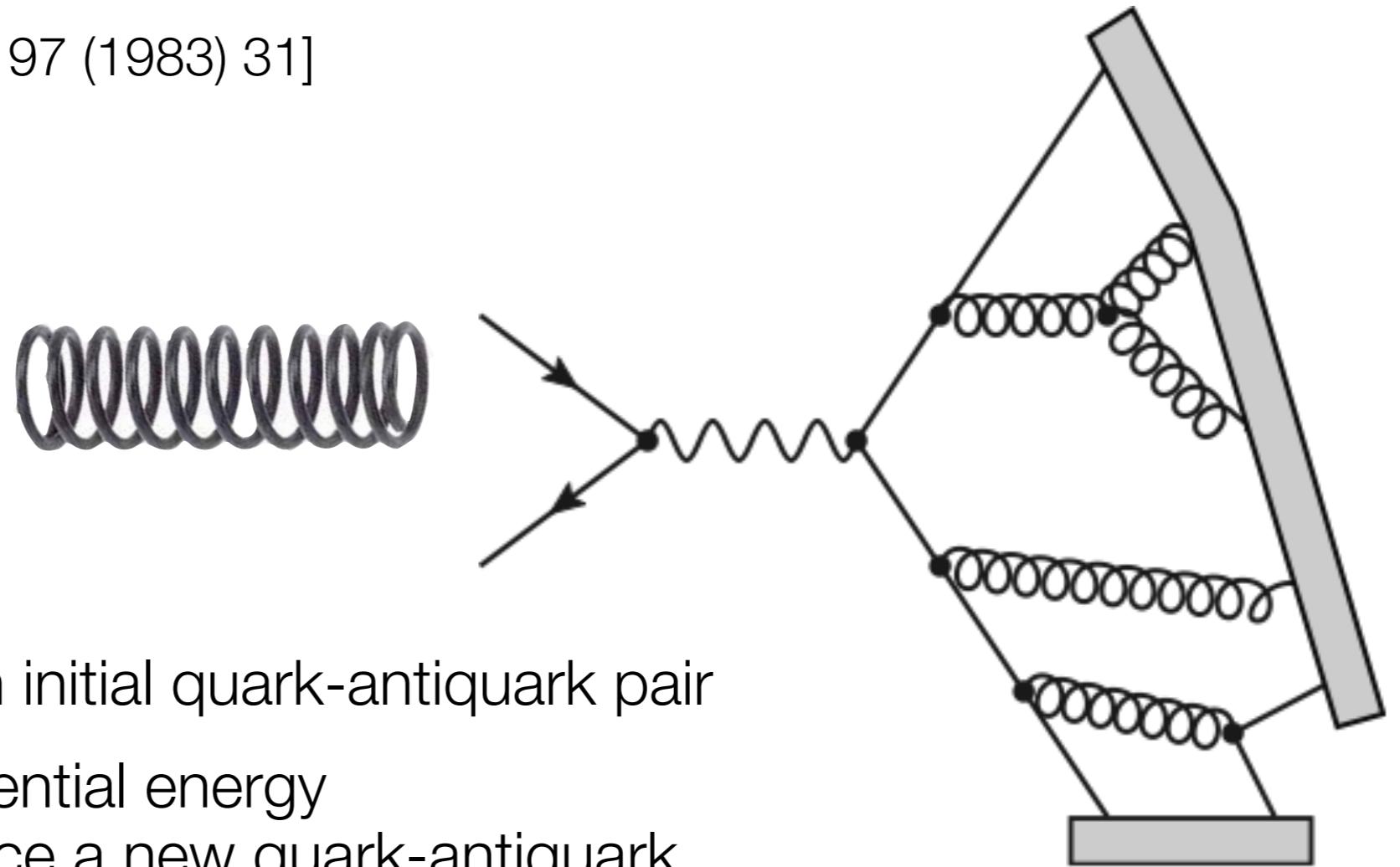
# Lund String Model

[Andersson et al., Phys. Rep. 97 (1983) 31]

QCD potential:

$$V(r) = -\frac{4 \alpha_s(1/r^2)}{3} + kr$$

neglected



String formation between initial quark-antiquark pair

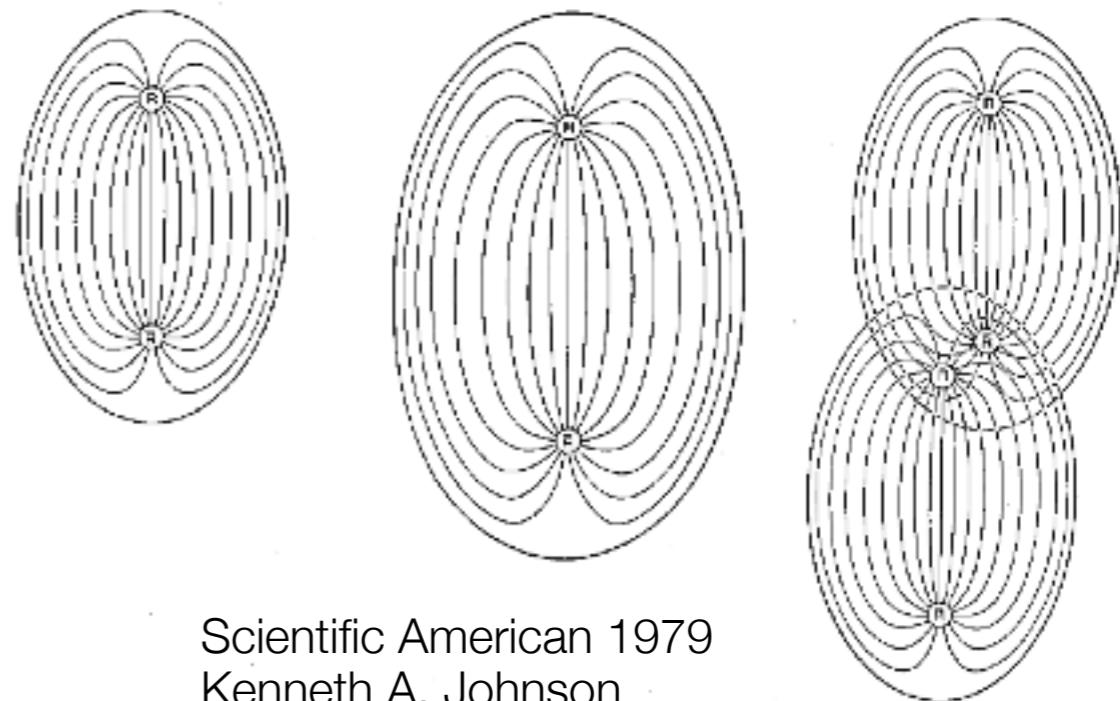
- String breaks up if potential energy large enough to produce a new quark-antiquark pair
- Gluons = 'kinks' in string
- At low energy: hadron formation
- Very widely used ... [default in Pythia 6/8]

# Lund String Model

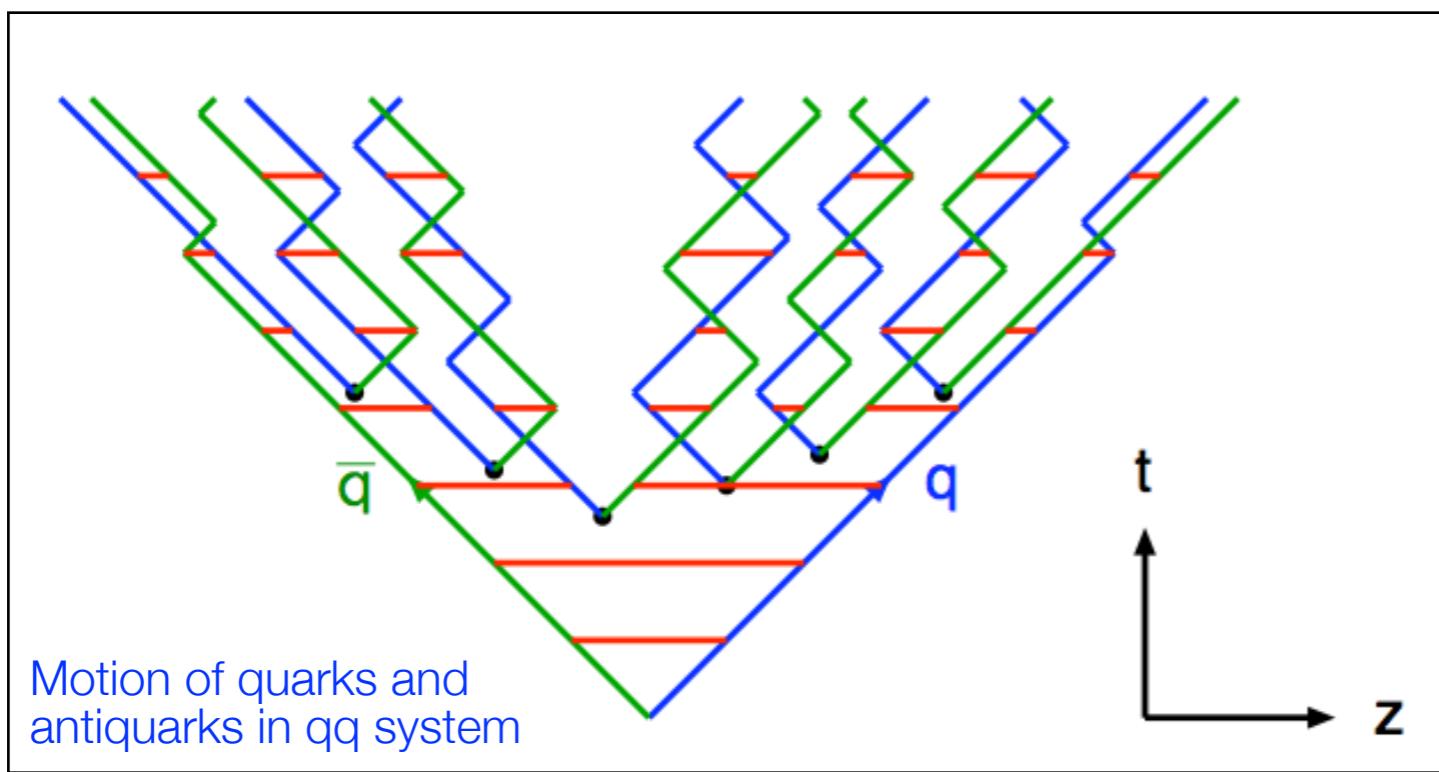
Repeated string breaks for large system  
with pure  $V(r) = \kappa \cdot r$ , i.e. neglect Coulomb part

$$\left| \frac{dE}{dz} \right| = \left| \frac{dp_z}{dz} \right| = \left| \frac{dE}{dt} \right| = \left| \frac{dp_z}{dt} \right| = \kappa$$

Energy-momentum quantities can be  
read from space-time quantities ...



Scientific American 1979  
Kenneth A. Johnson



Simple but powerful picture  
of hadron production

[with extensions to massive quarks, baryons, ...]

$$\begin{aligned} \mathcal{P} &\propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) \\ &\propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right) \end{aligned}$$

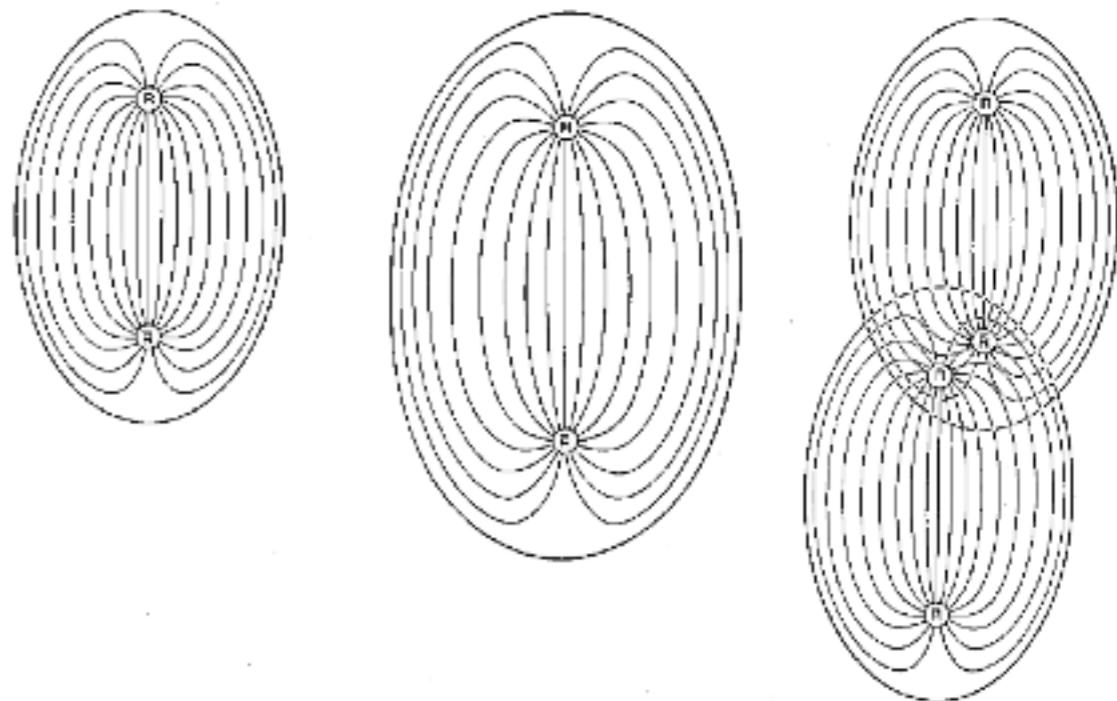
Yields: Common Gaussian  $p_\perp$  spectrum  
Heavy quark suppression

# Lund String Model

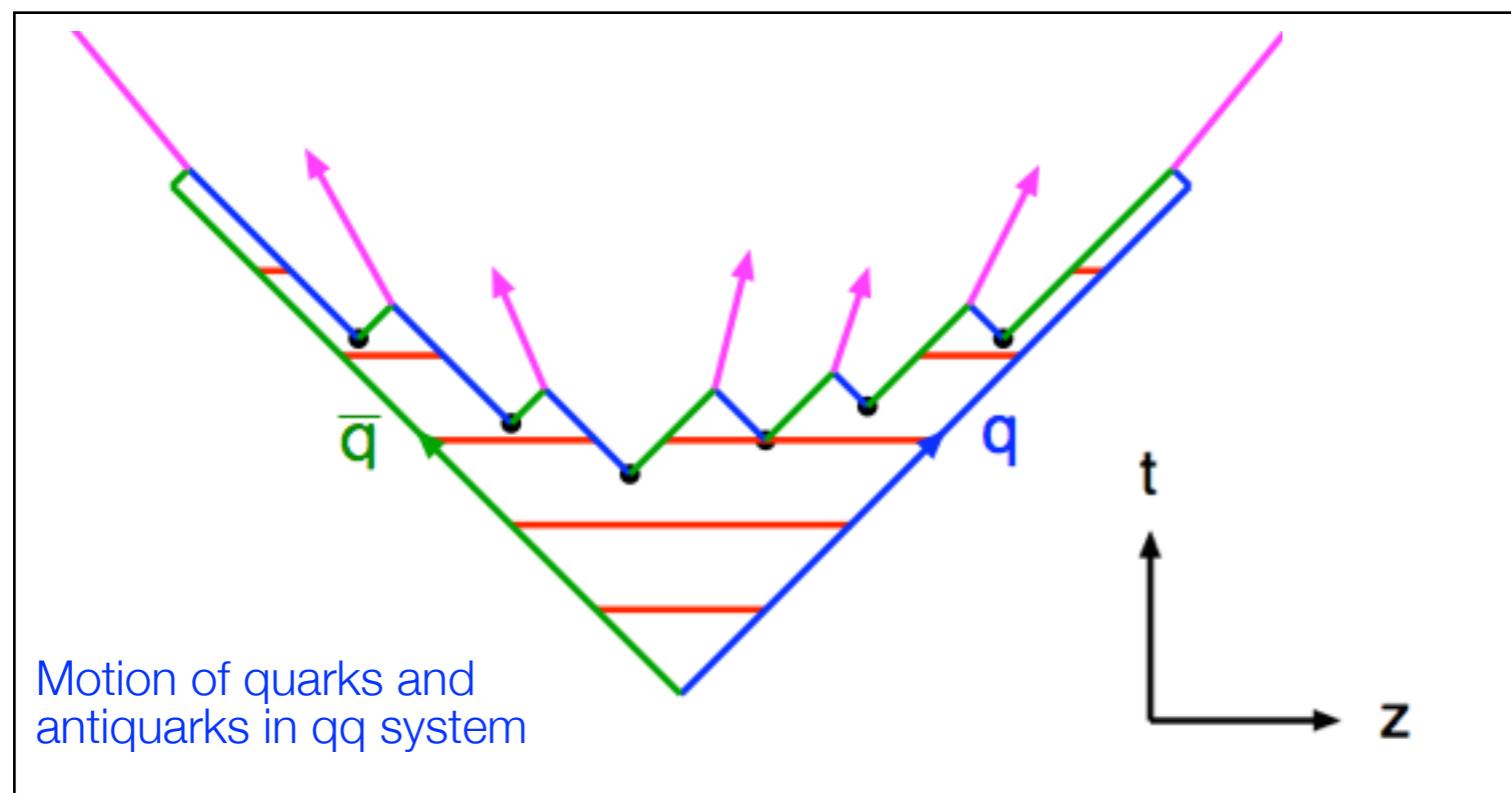
Repeated string breaks for large system  
with pure  $V(r) = \kappa \cdot r$ , i.e. neglect Coulomb part

$$\left| \frac{dE}{dz} \right| = \left| \frac{dp_z}{dz} \right| = \left| \frac{dE}{dt} \right| = \left| \frac{dp_z}{dt} \right| = \kappa$$

Energy-momentum quantities can be  
read from space-time quantities ...



Kenneth A. Johnson



Simple but powerful picture  
of hadron production

[with extensions to massive quarks, baryons, ...]

$$\begin{aligned}\mathcal{P} &\propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) \\ &\propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)\end{aligned}$$

Yields: Common Gaussian  $p_\perp$  spectrum  
Heavy quark suppression

# Cluster Model

[Webber, Nucl. Phys. B238 (1984) 492]

Color flow confined during hadronisation process

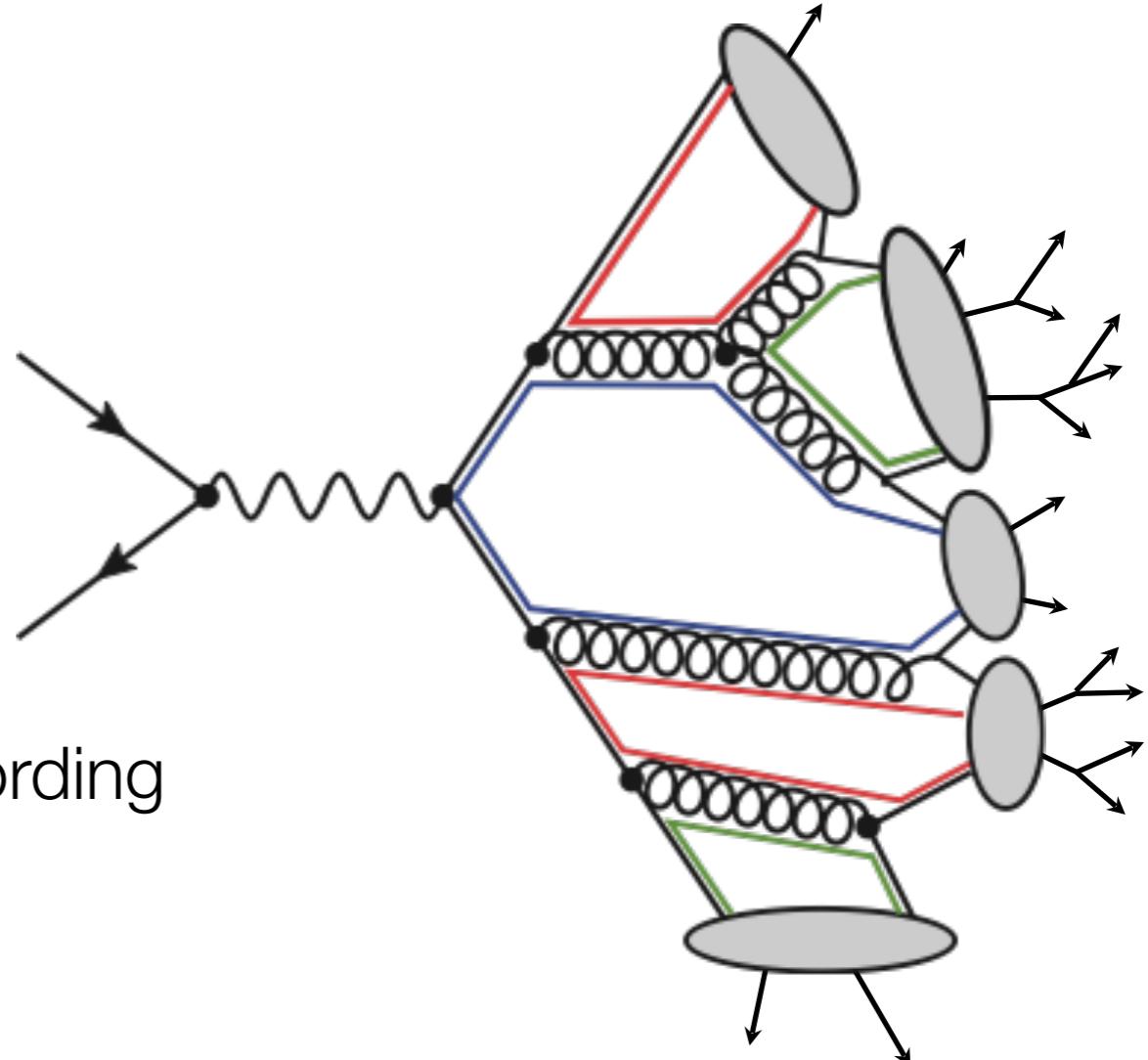
→ Formation of color-neutral parton clusters

Gluons (color-anticolor) split to quark-antiquark pairs

Clusters decay into 2 hadrons according to phase-space, i.e. isotropically

→ no free tuning parameters  
parton clusters

Very widely used ...  
[default in Herwig/Herwig++]



# Hadronisation models summary

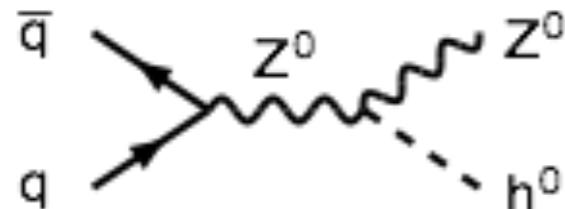
Model	Pythia6/8 (string)	Herwig/Herwig++ / Sherpa(cluster)
Energy-mom. picture	powerful predictive	simple unpredictive
Parameters	few	many
Flavour composition	messy unpredictive	simple in-between
Parameters	many	few

# Structure of basic generator process [by order of consideration]

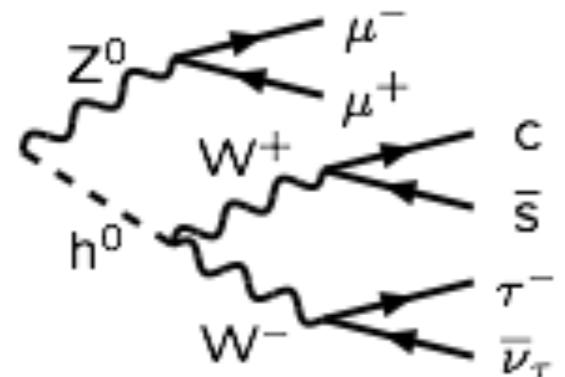
From the 'simple' to the 'complex' or from 'calculable' at large scales to 'modelled; at small

## Matrix elements (ME)

1. Hard subprocess:  
 $|M|^2$ , Breit Wigners, PDFs

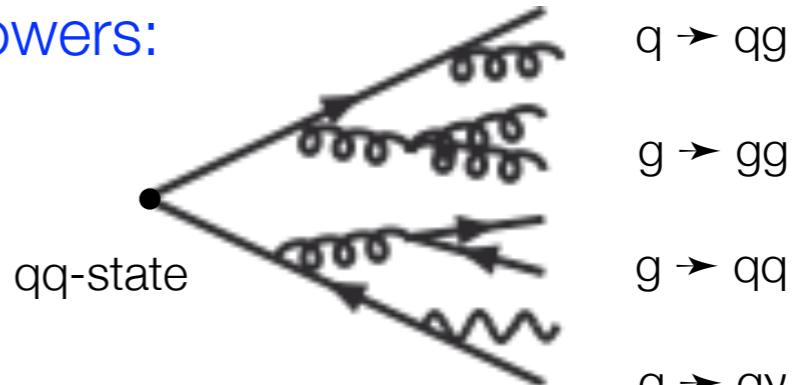


2. Resonance decays:  
Includes particle correlations

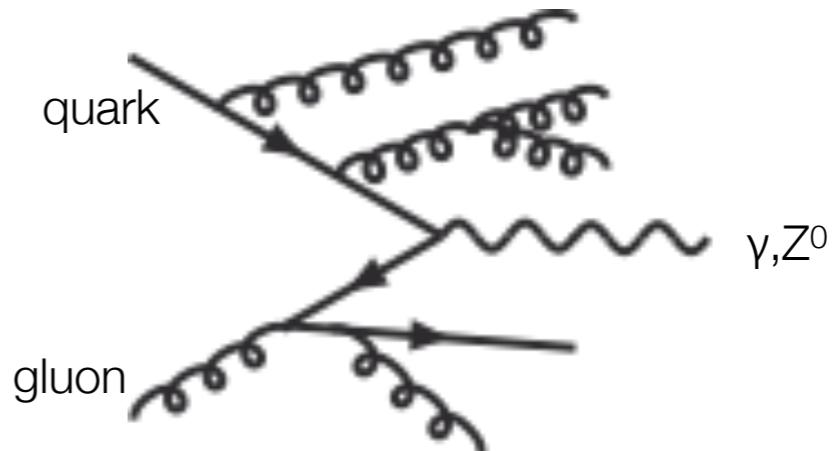


## Parton Shower (PS)

3. Final-state parton showers:



4. Initial-state parton showers:

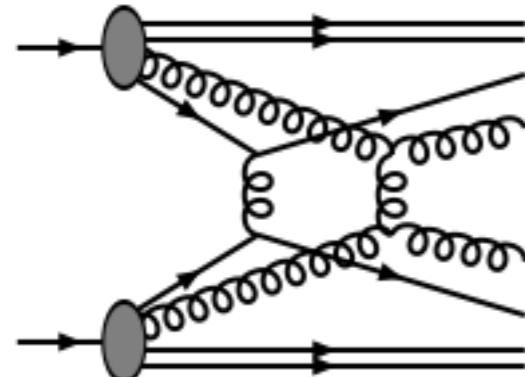


# Conclusions: Structure of basic generator process

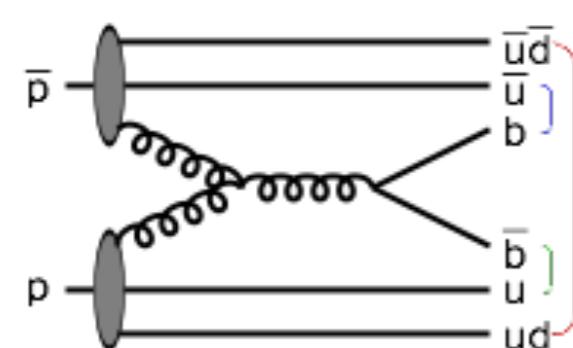
From the 'simple' to the 'complex' or from 'calculable' at large scales to 'modelled; at small

## Underlying Event (UE)

### 5. Multi-parton interaction:

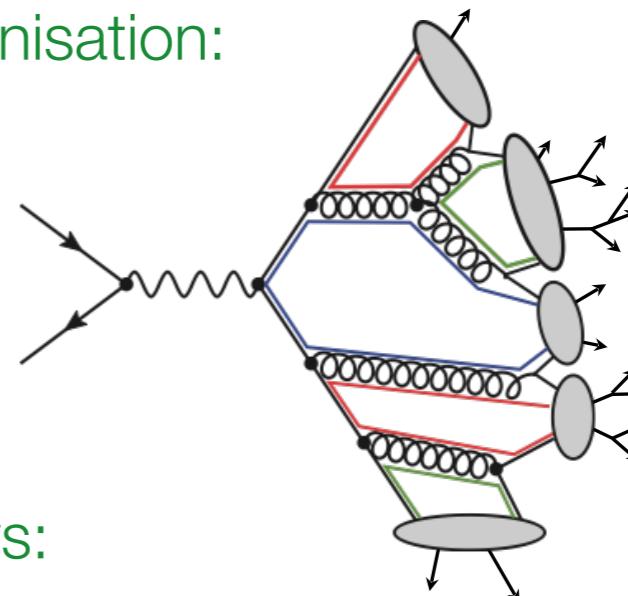


### 6. Beam remnants:

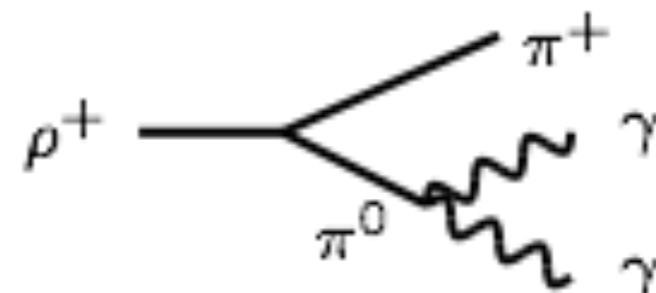


## Stable Particle State

### 7. Hadronisation:



### 8. Decays:



The DGLAP evolution equation is said to resum large collinear logarithms. So where are these logarithms, and where is the resummation?

Let's perform the integration of the DGLAP equation and expand the result:

$$\begin{aligned} f(x, t) &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \int_x^1 \frac{dz}{z} P(z) g\left(\frac{x}{z}, t'\right) \\ &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \left\{ f_0\left(\frac{x}{z}\right) + \right. \\ &\quad \left. + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') \left[ f_0\left(\frac{x}{zz'}\right) + \dots \right] \right\} \\ &= f_0(x) + \frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right) + \\ &\quad + \frac{1}{2!} \left[ \frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \right]^2 \int_x^1 \frac{dz}{z} P(z) \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) + \dots \end{aligned}$$

As suggested by the last step, it is indeed a resummation of all terms proportional to  $\left[\frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right)\right]^n$ .

