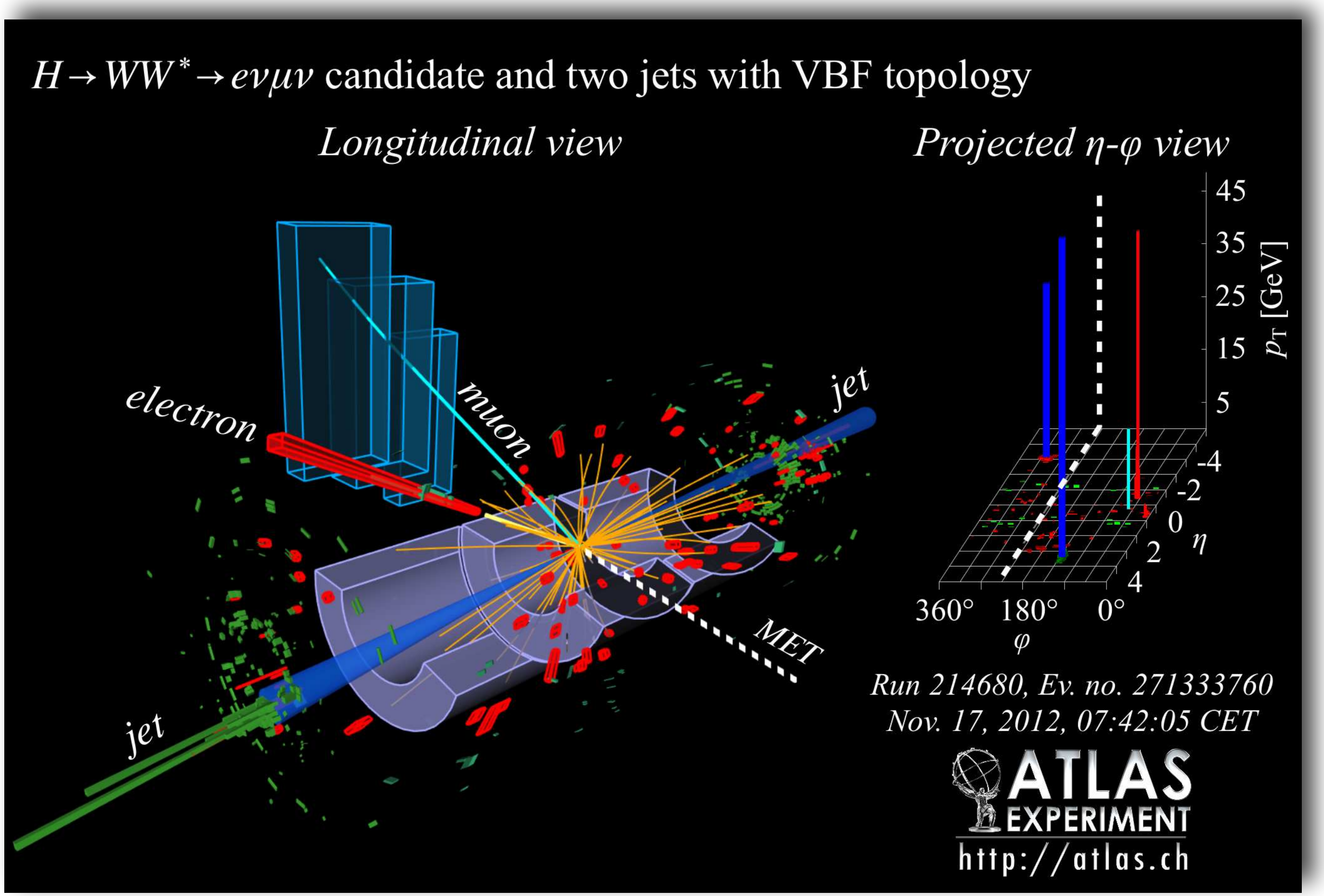
Monte Carlo Generators at colliders

High energy physics simulation

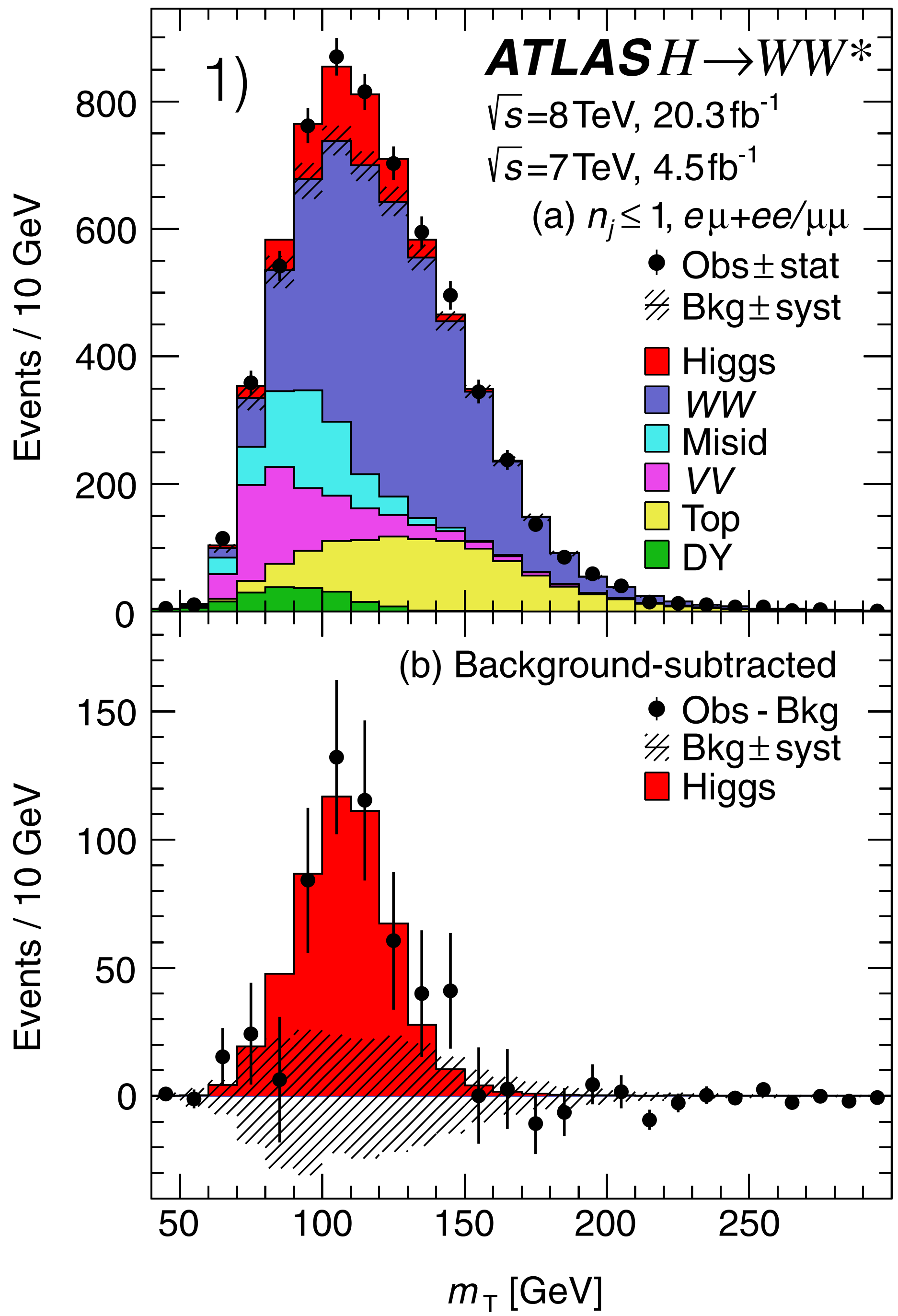


 [Thanks to C.C. Coulon, U.Husemann, G.Herten]

Acknowledgements

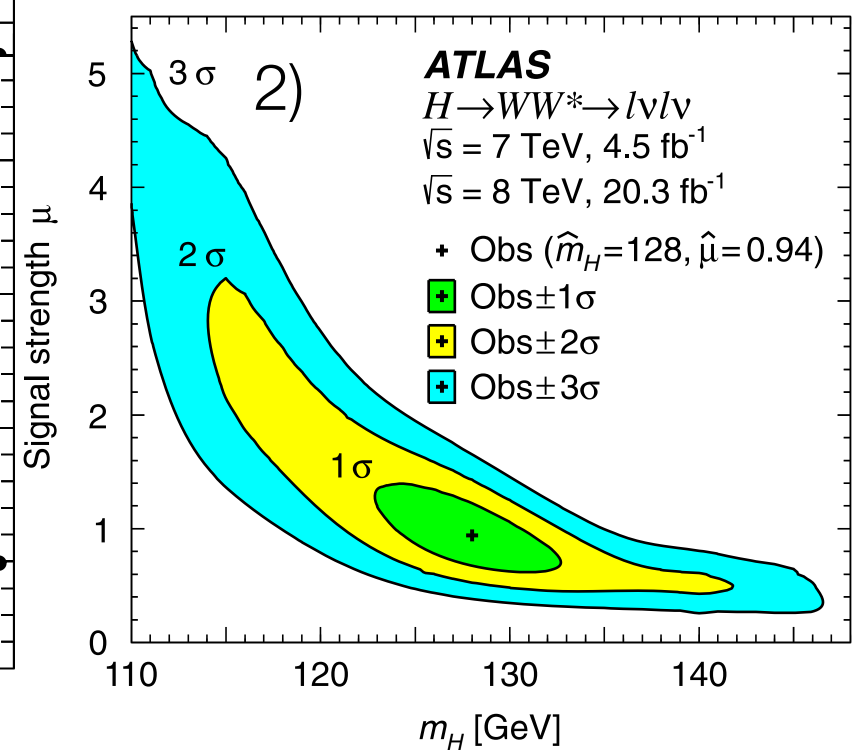
These slides have been prepared using heavily the material prepared by Christian Schultz Coulon and clarifying few points plus updating to recent developments in proton-proton physics.

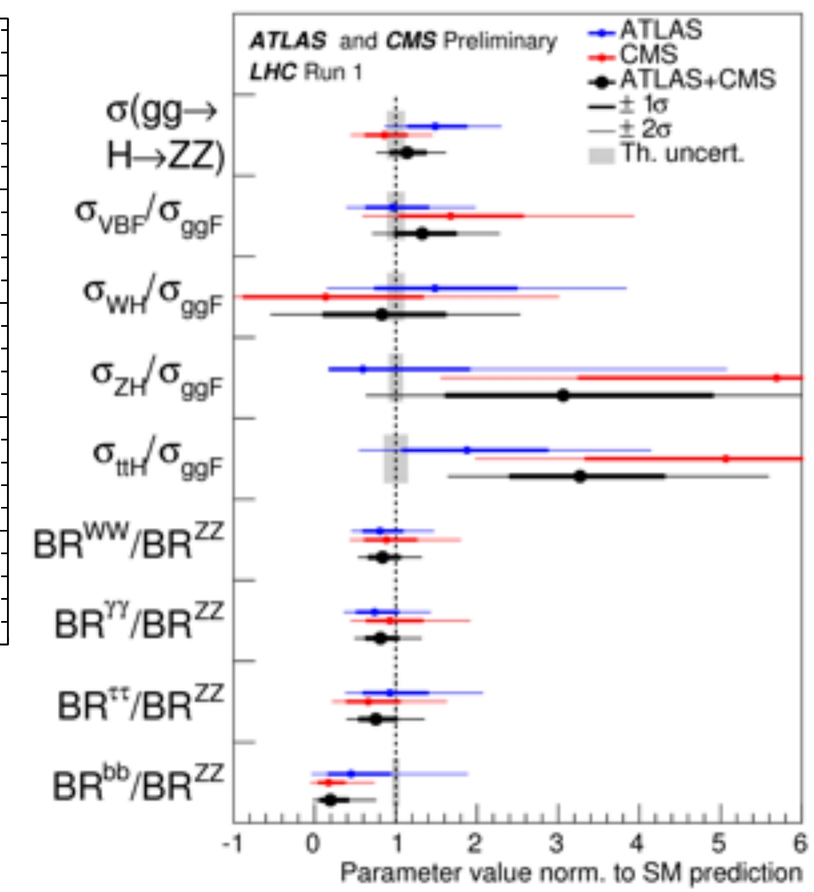
Why MC simulation?



1) to extract an interesting signal we need to subtract the expectation from known processes;

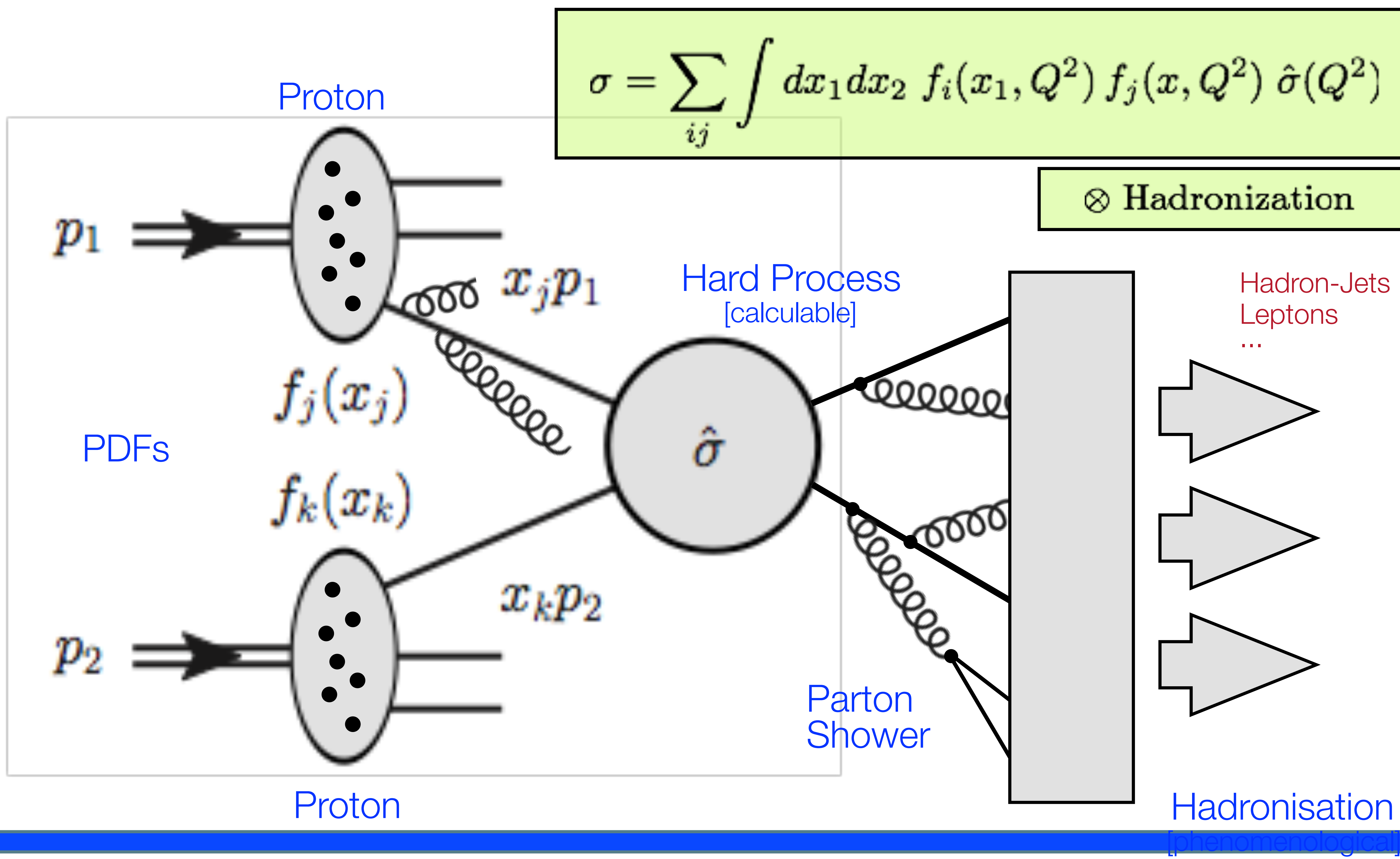
2) signal needs also to be modelled in order to compute detection efficiency and estimate production cross sections and couplings





[from G.Herten]

The simulation chain



MC simulations in particle physics

How Monte Carlo simulation works

- Numerical process generation based on random numbers
- Method very powerful in particle physics

Event generation programs:

Pythia6, Pythia8, Herwig, Herwig++, Sherpa ...

Hard partonic subprocess + fragmentation and hadronisation ...

Detector simulation:

Geant4
Fluka low energy hadron interactions...

interaction & response of all produced particles ...

Event Generator
simulate physics process
(quantum mechanics: probabilities!)

↓

Detector Simulation
simulate interaction with detector material

↓

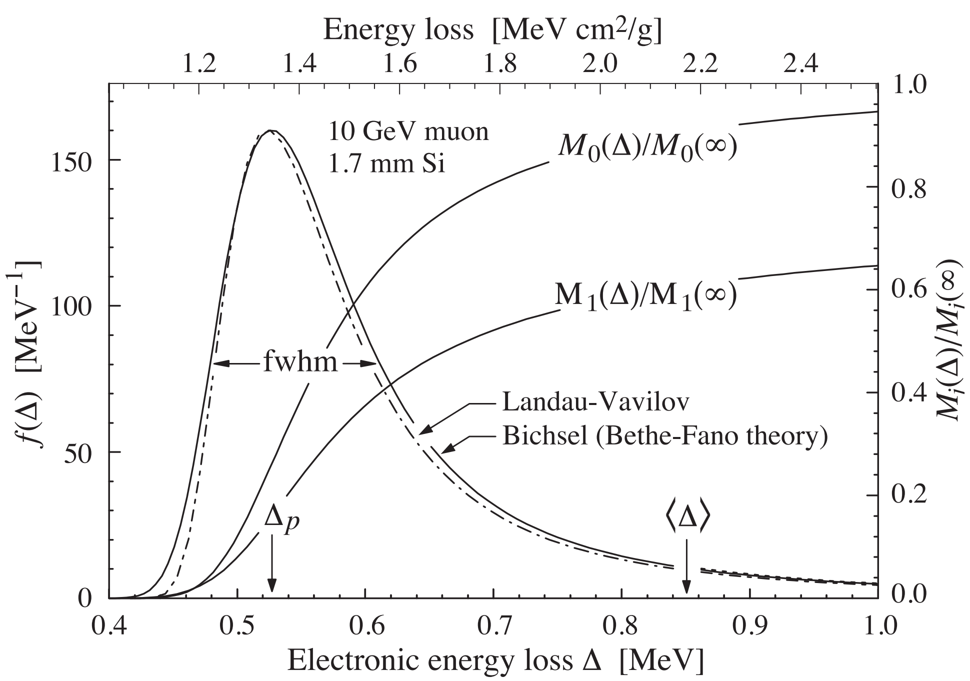
Digitisation
translate interactions with detector into realistic signals

↓

Reconstruction/Analysis
as for real data

Baseline of the simulation process

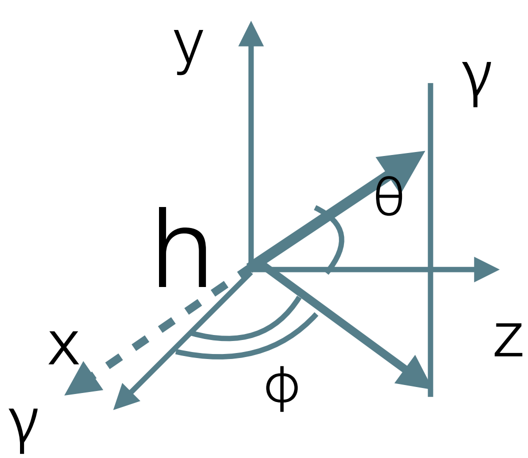
Typically, we need to generate a continuous variable following some distribution
i.e. energy loss of a particle in a given material segment;
angle of a photon in the h reference frame for the h →γγ decay



$$dP = f(x, ..)dx$$

↳ distribution formula

↳ probability to get an x_0 value between x and x+dx



$$dP = f(\theta, \phi)d\theta d\phi = sen\theta d\theta d\phi$$

flat distribution in ϕ
non flat in θ

Distribution function transformation properties

1) software libraries provide basic functions to produce flat distributed random numbers in the interval [0,1] (ex. root TRandom3 class), they are typically fast and accurate uniform random numbers generators;
2) starting from uniform distributed random numbers, it is possible to generate numbers following any distribution using different techniques

$$dP_x = f(x)dx \qquad y = g(x)$$

$$x \in [x_a, x_b]$$

How "y" distributes in [$g(x_a)$, $g(x_b)$]?

Because y is a monotonic function of x the probability to have y between g(x) and g(x+dx) is equal to the probability to have x between x and x+dx

$$dP_y = h(y)dy = h(y)g'(x)dx$$

$$h(y)g'(x) = f(x) \Rightarrow h(y) = \frac{f(x)}{g'(x)} = \frac{f(g^{-1}(y))}{g'(g^{-1}(y))}$$

Ex.: range map

$$[0,1] \rightarrow [a,b] \qquad y = (b-a)x + a$$

$$f(x) = 1 \quad g'(x) = b - a \quad h(y) = \frac{1}{b-a}$$

uniform

y is uniformly distributed in [a,b]

Distribution function transformation properties

Ex. 2: integration method:

$$g(x) = \frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' \qquad g'(x) = \frac{f(x)}{\int_a^b f(x')dx'}$$

$$h(y) = \frac{f(x)}{g'(x)} = \frac{f[g^{-1}(y)]}{f[g^{-1}(y)]} \cdot \int_a^b f(x')dx' = \int_a^b f(x')dx'$$

y is uniformly distributed:

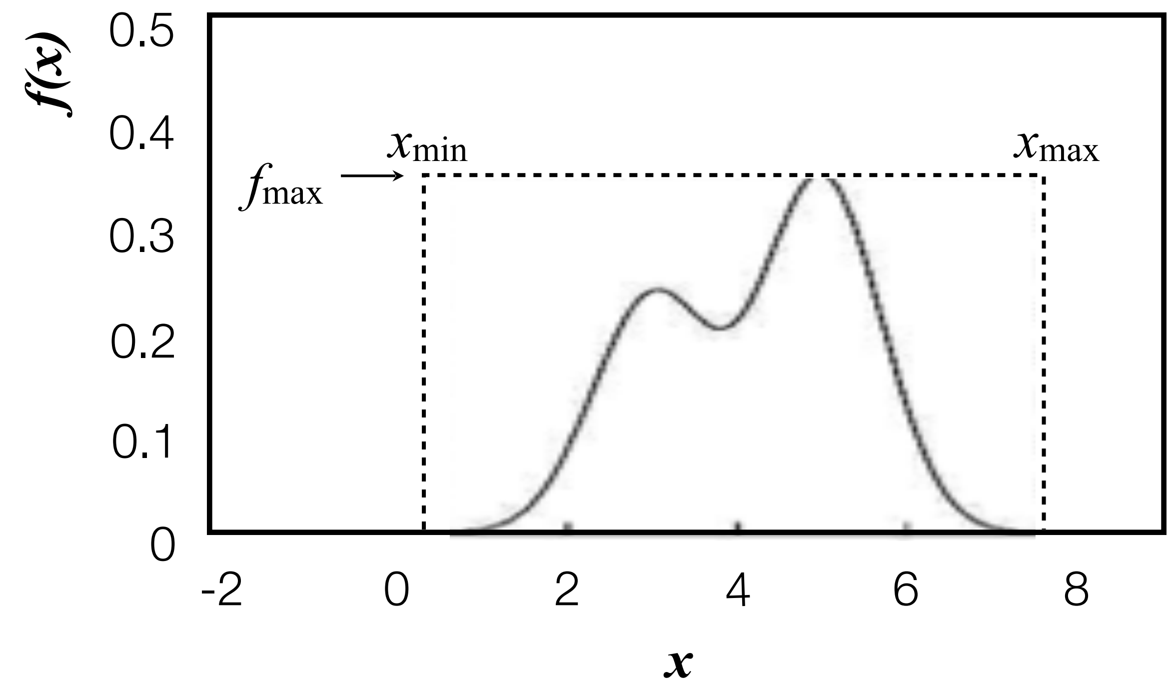
1) generate y flat in [f_{min}, f_{max}];
2) compute x = g^{-1}(y), x will be distributed in $g^{-1}(f_{min})$, $g^{-1}(f_{max})$

Finding $g^{-1}(y)$ is equivalent to solve the equation:

$$\frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' = y$$

Hit or miss method.

1) generate x flat in x_{min}, x_{max}
2) generate y flat in 0, f_{max}
3) if y < f(x) accept the event, otherwise ignore it



for a given x in x, x+dx the fraction of accepted events is proportional to f(x)dx -> dPx = f(x)dx

1) advantages:

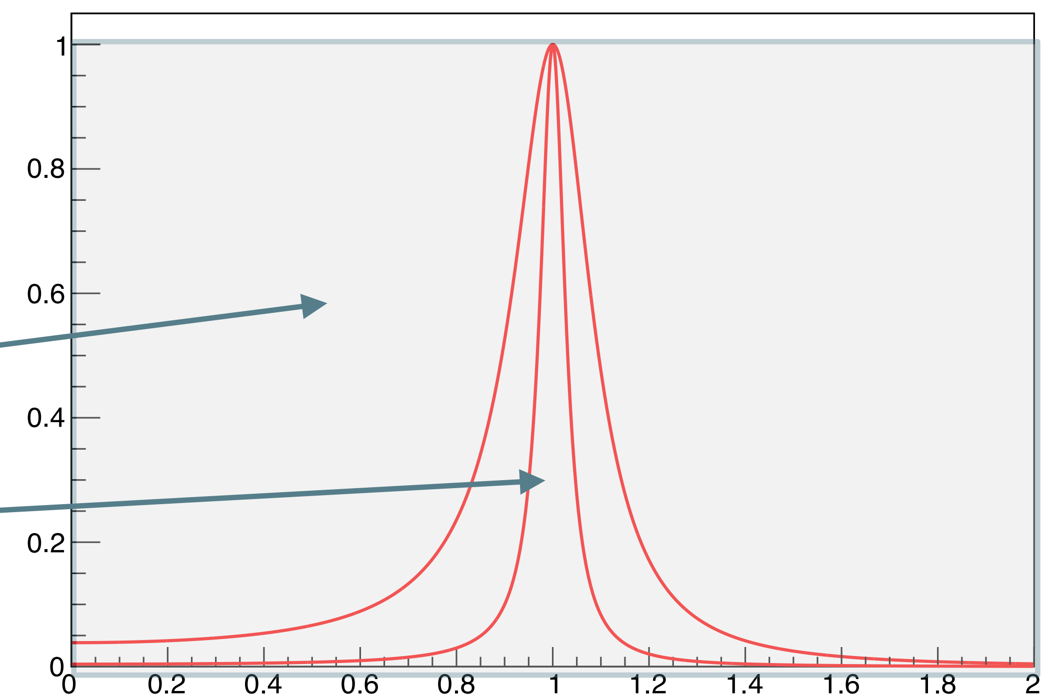
- can be used for all functions, even non continuous …
- can be extended to N-dimension (generate $x_1, x_2, \ldots, x_n$), y accept if y < $f(x_1, x_2, .., x_n)$

2) disadvantages

- can be extremely slow

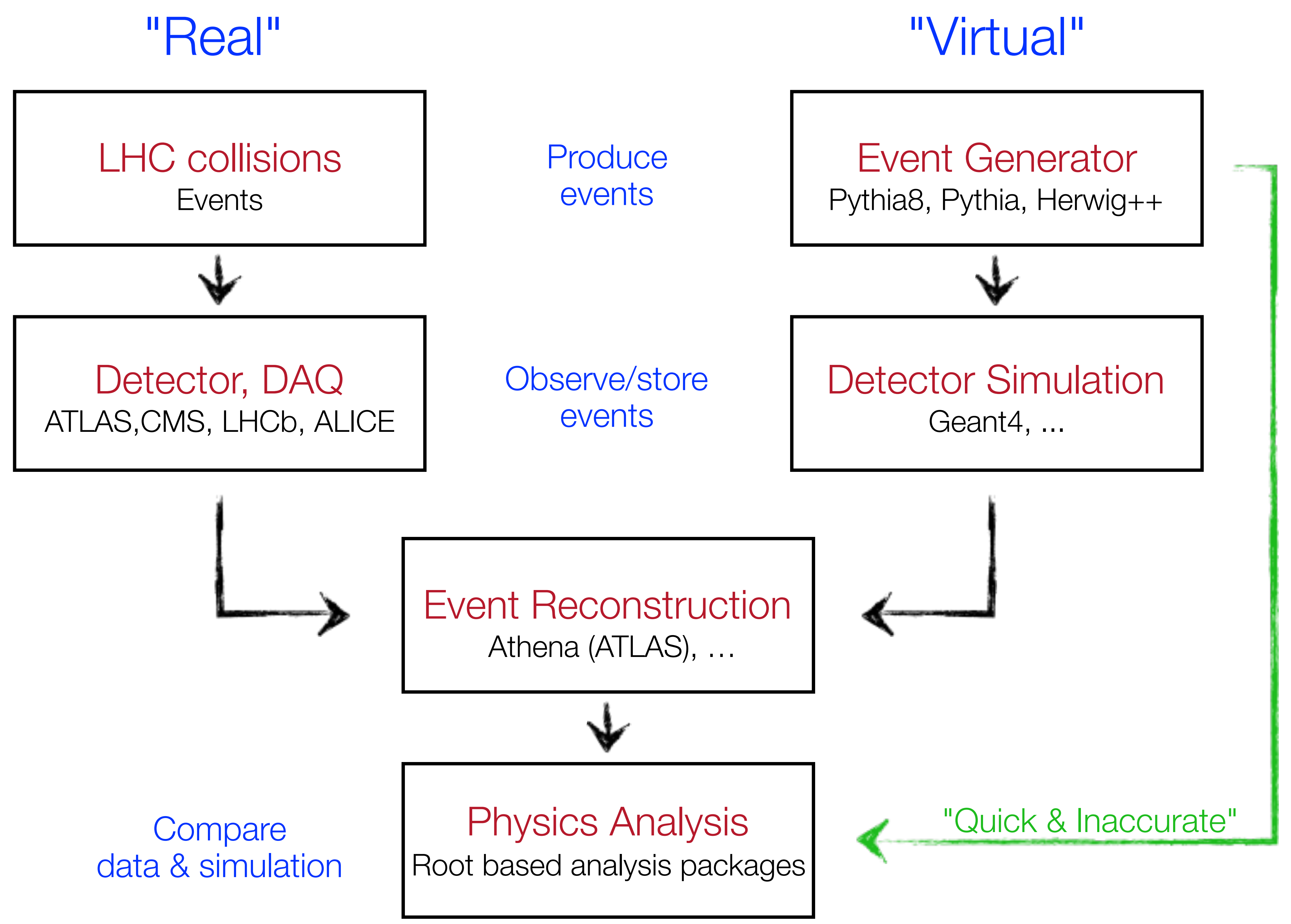
points generated uniformly in the square

points accepted only below the curve



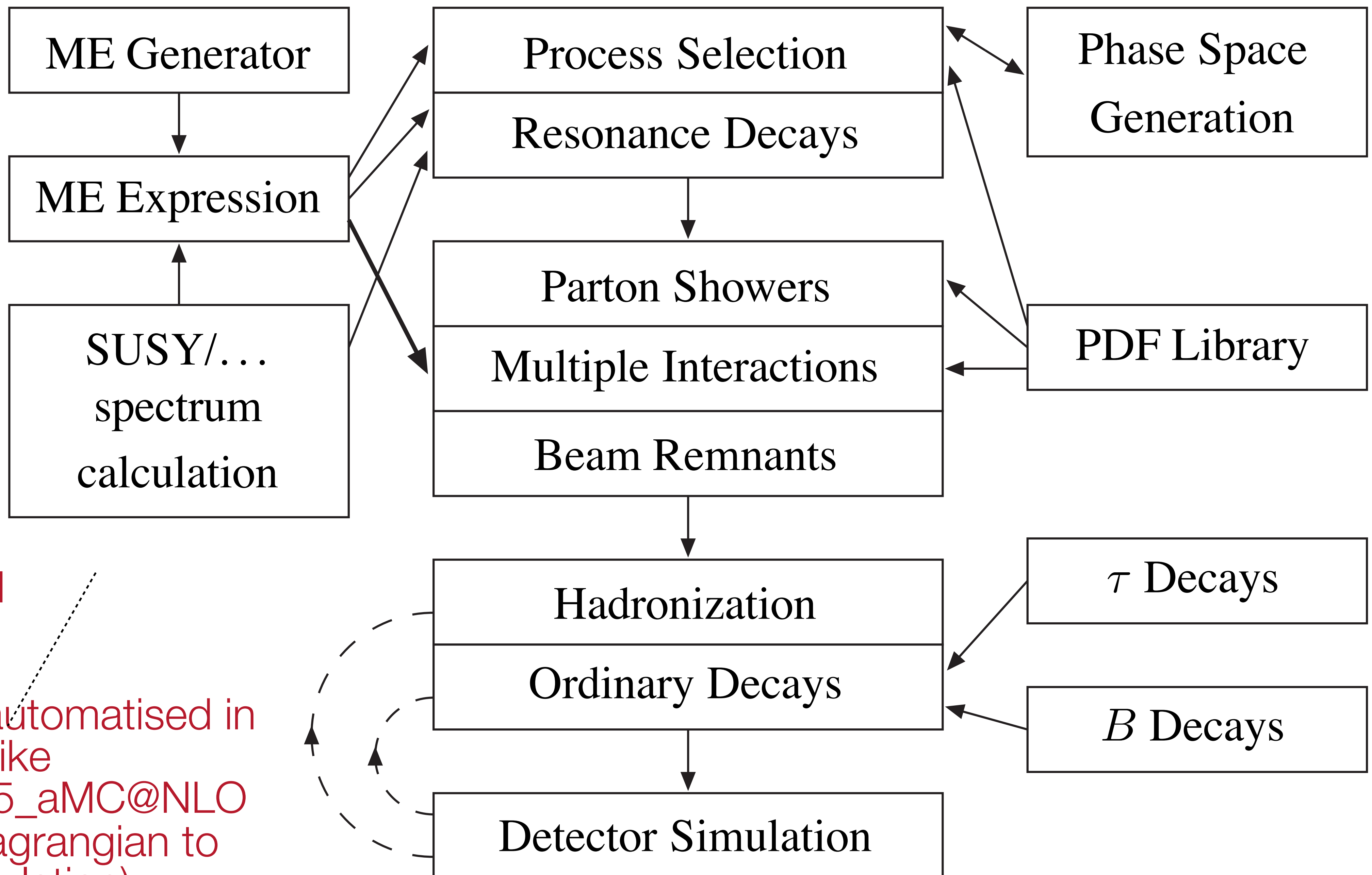
MC generators implement "smart" generation techniques to increase efficiencies

Comparison between real and simulated events



[after T.Sjöstrand]

Simulation elements

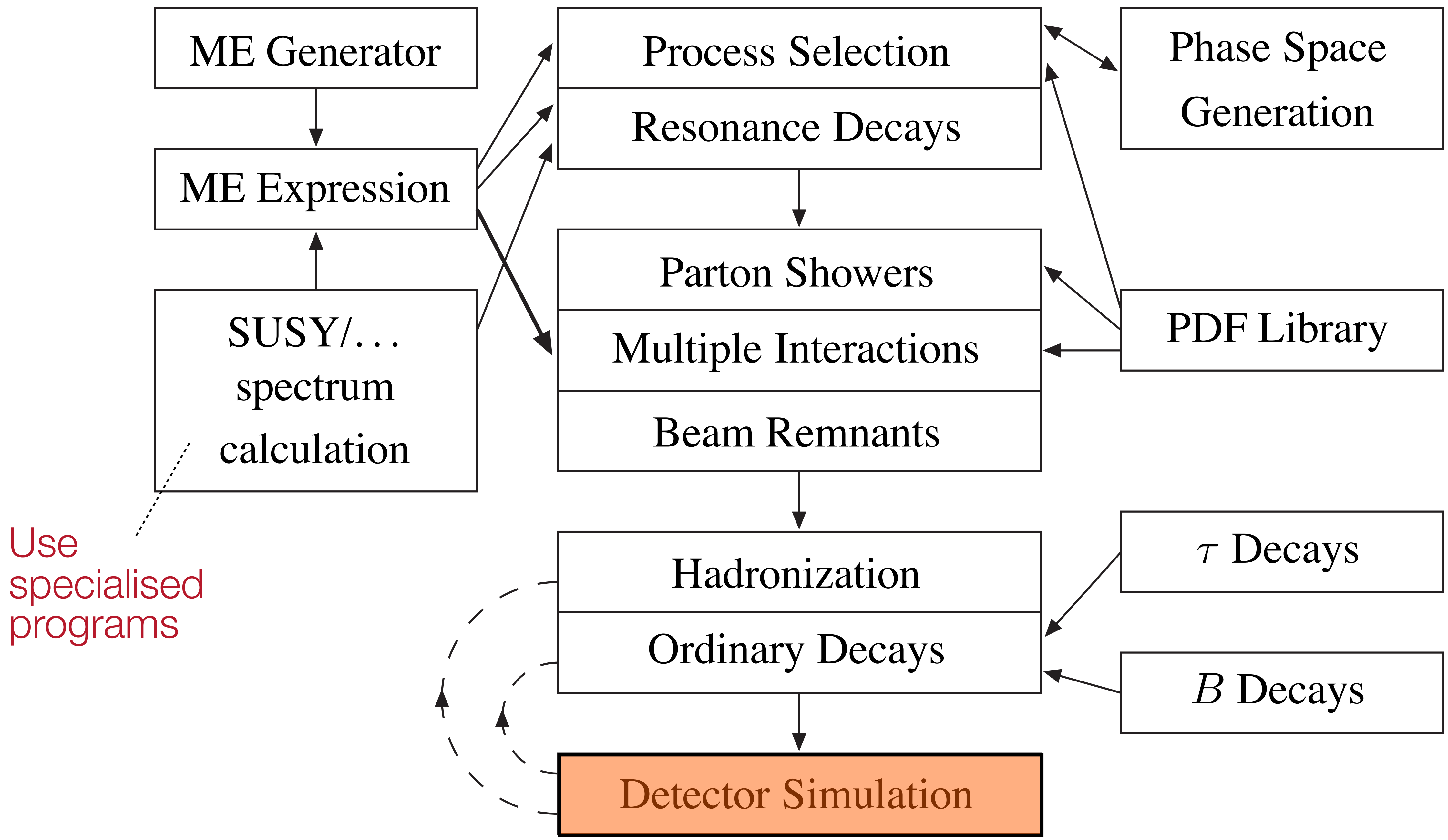


Use specialised programs

Now fully automatised in programs like Madgraph5_aMC@NLO (from the lagrangian to the full simulation)

[Sjöstrand, arXiv:hep-ph/0611247v1]

Simulation elements



[Sjöstrand, arXiv:hep-ph/0611247v1]

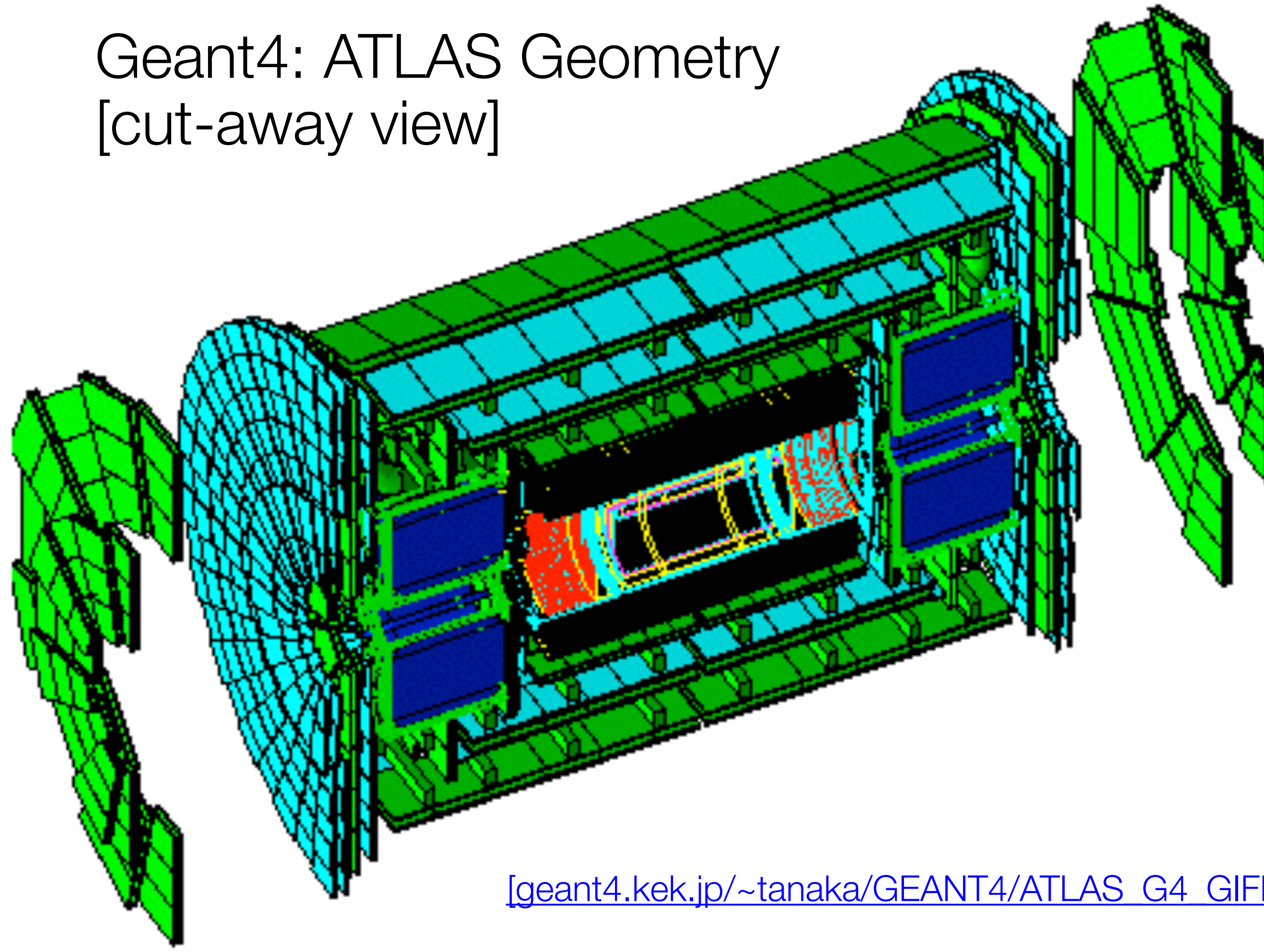
GEANT Geometry And Tracking

Detailed description of detector geometry
[sensitive & insensitive volumes]

Tracking of all particles through detector material ...

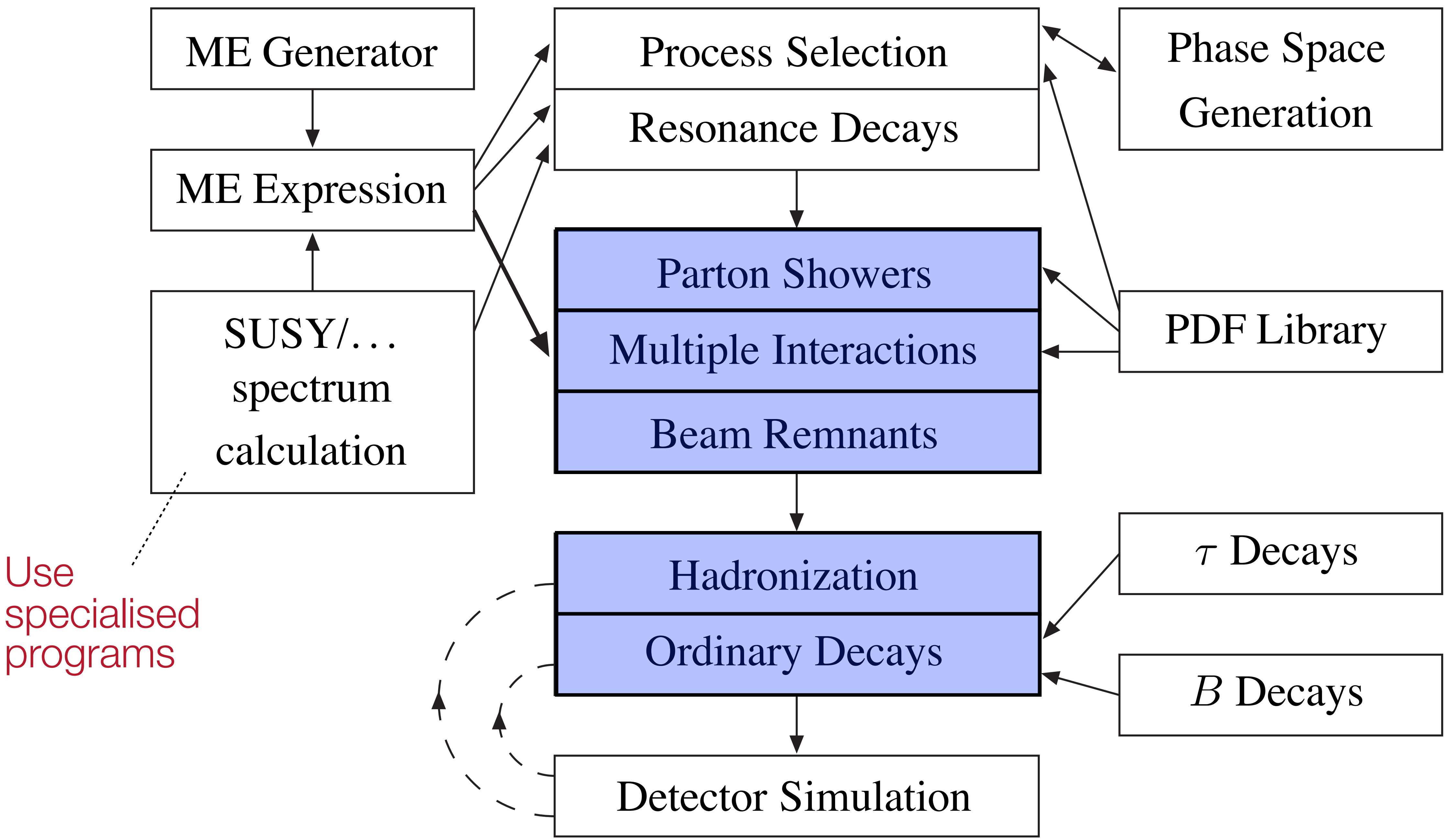
➛ Detector response

Geant4: ATLAS Geometry
[cut-away view]



[geant4.kek.jp/~tanaka/GEANT4/ATLAS_G4_GIFFIG/]

Developed at CERN since 1974 (FORTRAN)
[Today: Geant4; programmed in C^{++}]



[Sjöstrand, arXiv:hep-ph/0611247v1]

Strong interactions:

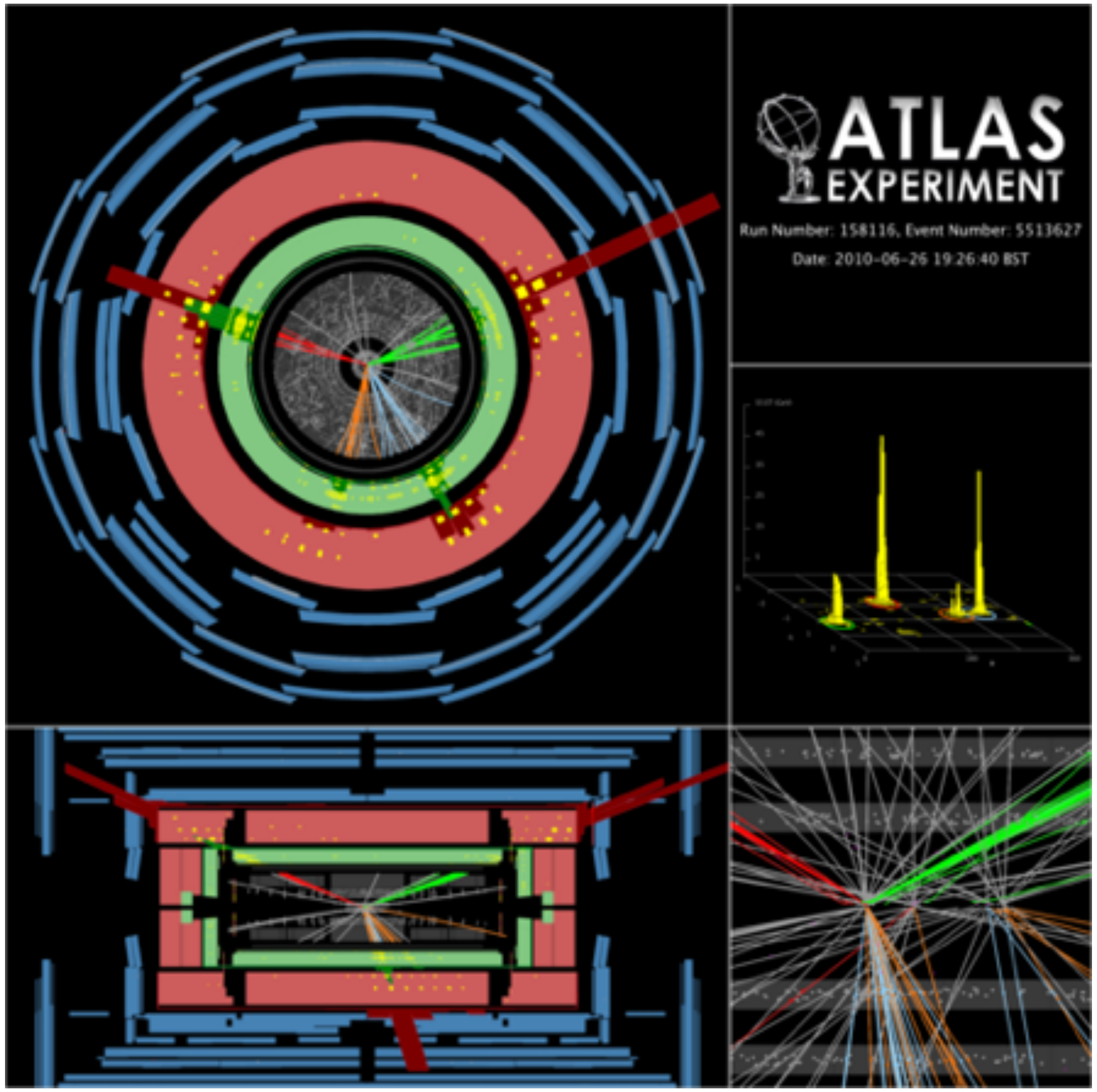
No free Quarks

Expect jets

i.e. bundles of particles at high energies
[hadron p_T range limited w.r.t. initial parton]

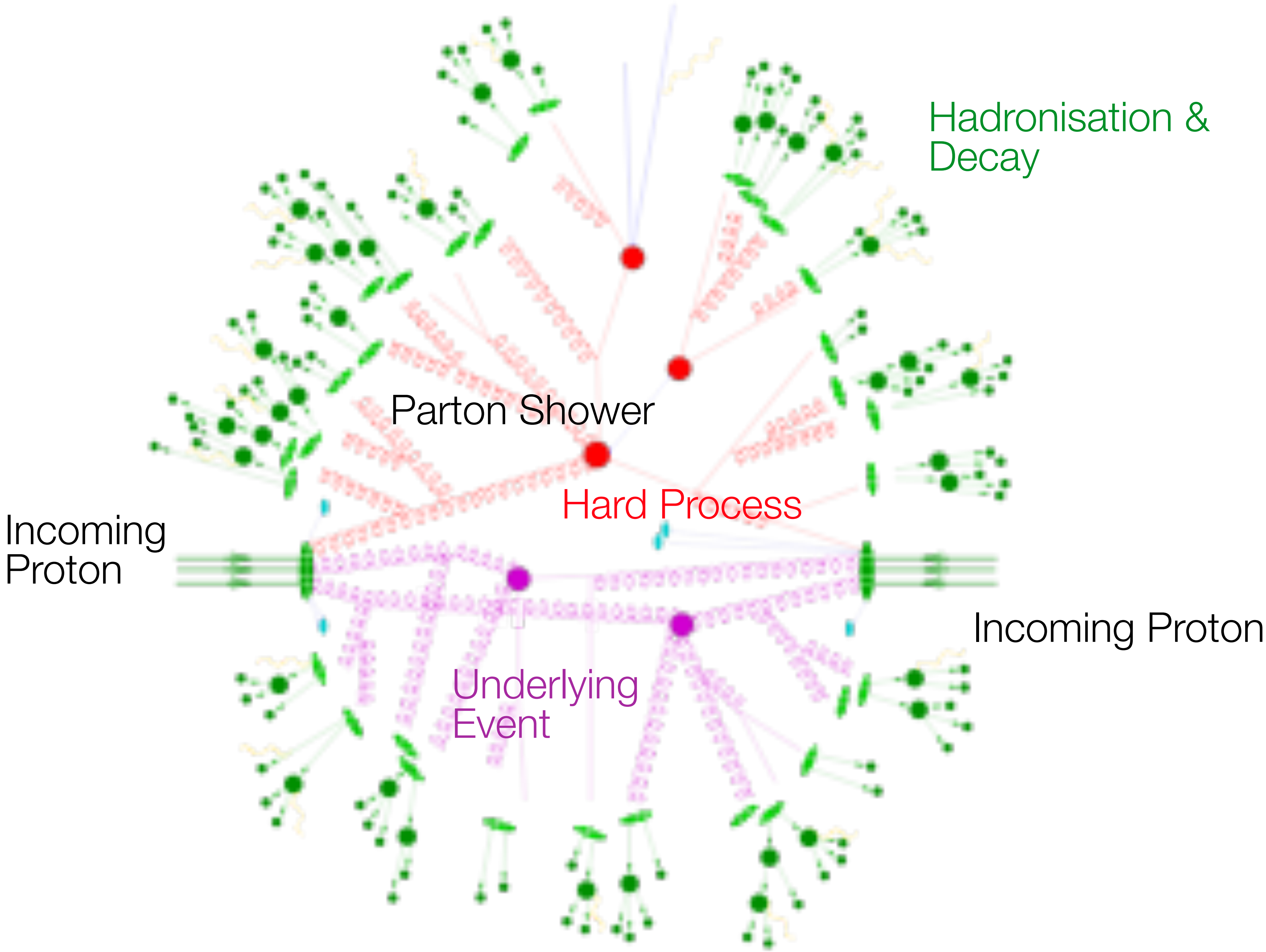
First observation of jets
in e^+e^- collisions @ $E_{CMS} > 6$ GeV
[SPEAR, SLAC, 1975]

Later also observed in
hadron-hadron collisions
[e.g. @ CERN ISR]



An event with 4 jets @ LHC

Goal: Infer parton properties from jet properties
[need to calculate and/or model fragmentation & hadronisation process]



[T. Gleisberg et al., JHEP02 (2004) 056]

Pure matrix element (ME) simulation:

MC integration of cross section & PDFs, no hadronisation (recall: cross section = $|\text{matrix element}|^2 \otimes$ phase space)

Useful for theoretical studies, no exclusive events generated

[Example: MCFM (http://mcfm.fnal.gov); many LHC processes up to NLO, HNNLO (http://theory.fi.infn.it/grazzini/codes.html) Higgs production at NNLO]

Event generators:

Combination of ME and parton showers ...

Typical: generator for leading order ME
combined with leading log (LL) parton shower MC (see later)

Exclusive events ➛ useful for experimentalists ...

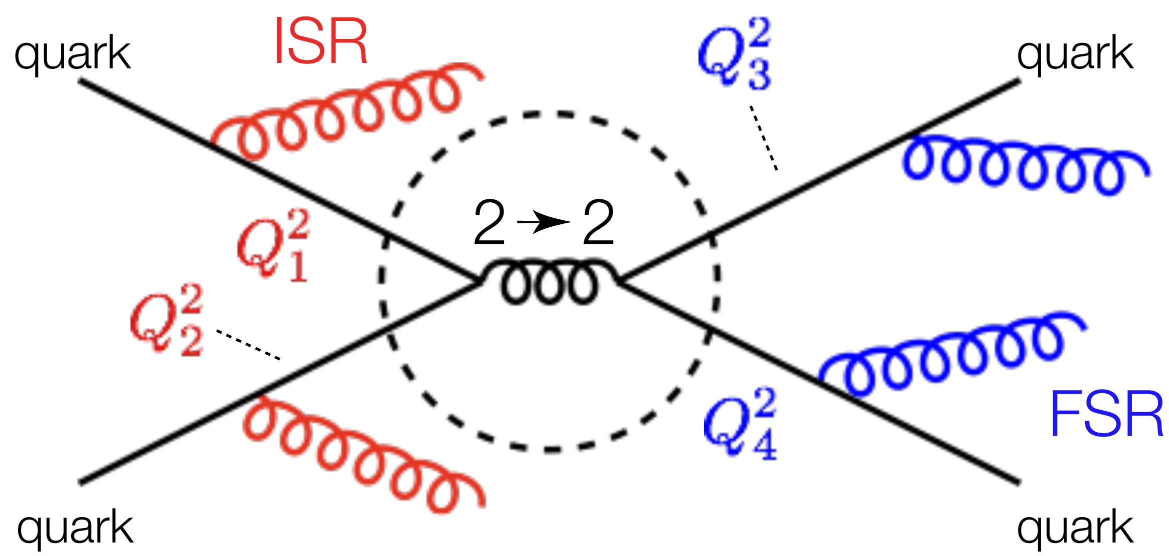
Parton showers

[Sjöstrand, arXiv:hep-ph/0611247v1]

A realistic simulation needs many particles in the final state, it is quite difficult (sometimes impossible) to compute a pp (2) → many particles process

(2 ➛ n) = ...

... = (2 ➛ 2) ⊕ ISR ⊕ FSR



FSR: Final state radiation
$Q^2 \sim m^2 > 0$ decreasing
[time-like shower]

ISR: Initial state radiation
$Q^2 \sim -m^2 > 0$ increasing
[space-like shower]

Hard process [2 ➛ 2]:

$$\sigma = \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}\hat{t}\; f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

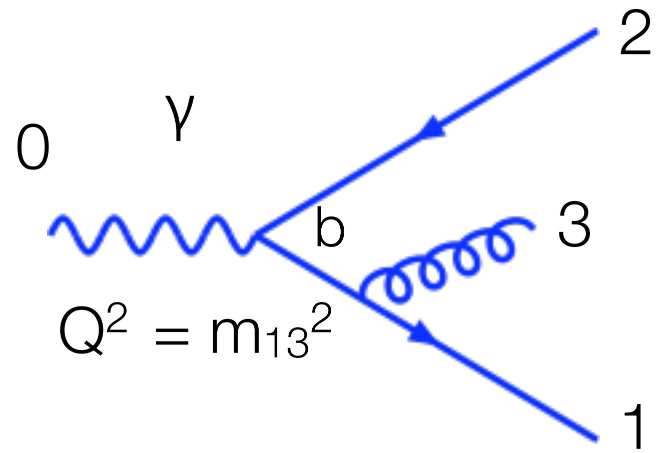
Calculable

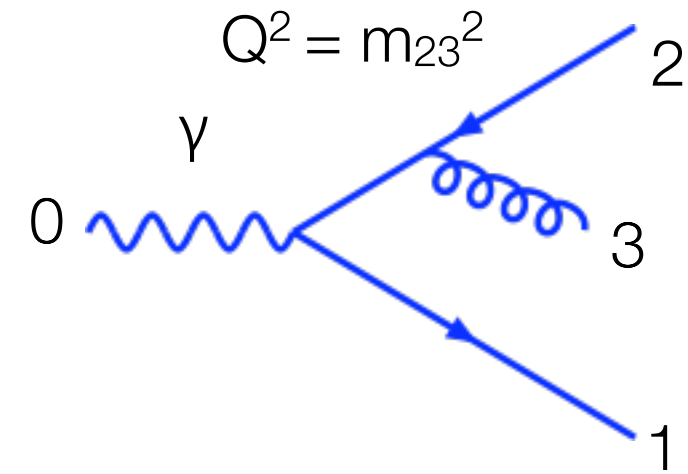
Shower evolution:

Viewed as probabilistic process, which occurs with unit total probability; cross section not directly affected; only indirectly via changed event shape.

Parton showers

$e^+e^- \to qqg$





$$x_i = \frac{2E_i}{E_{cm}} \qquad x_1 + x_2 + x_3 = 2$$

Cross Section: $$\frac{d\sigma_{qqg}}{dx_1 dx_2} = \frac{4}{3}\frac{\alpha_s}{2\pi} \cdot \sigma_0 \cdot \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Cross section has large contributions for $x_1, x_2 \to 1$

[$m_q = 0$; see e.g. Halzen/Martin]

from p_T balance $$1 - x_2 = \frac{m_{13}^2}{E_{cm}^2} = \frac{Q^2}{E_{cm}^2} \quad m_{13}^2 \sim 2E_1E_2(1-cos\theta)\, x_2 \to 1 \Rightarrow m_{13}^2 \to 0 \quad \Longrightarrow \quad \theta \to 0 \text{ collinear limit}$$

$$dx_2 = -\frac{dQ^2}{E_{cm}^2}$$

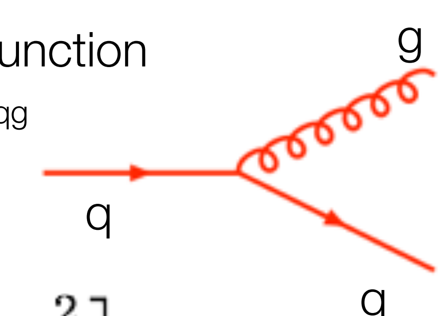
$$E_q = E_1 = zE_b \quad E_g = E_3 = (1-z)E_b$$

Rewrite for $x_2 \to 1$:
[qg collinear limit]

$$x_1 \approx z \qquad dx_1 \approx dz$$

$$x_3 \approx 1 - z$$

Splitting Function
$P_{q \to qg}$



$$d\mathcal{P} = \frac{d\sigma_{qqg}}{\sigma_0} = \frac{4}{3}\frac{\alpha_s}{2\pi} \cdot \frac{dx_2}{(1-x_2)} \cdot \frac{x_1^2 + x_2^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \cdot \frac{dQ^2}{Q^2} \cdot \frac{4}{3}\left[\frac{1+z^2}{1-z}\right] dz$$

$z \to 1 \Rightarrow E_g \to 0$ soft divergence

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}P_{a\to bc}(z)dz$$

Splitting probability determined by splitting functions $P_{q\to qg}$

Analogous splitting functions used in PDF evolution

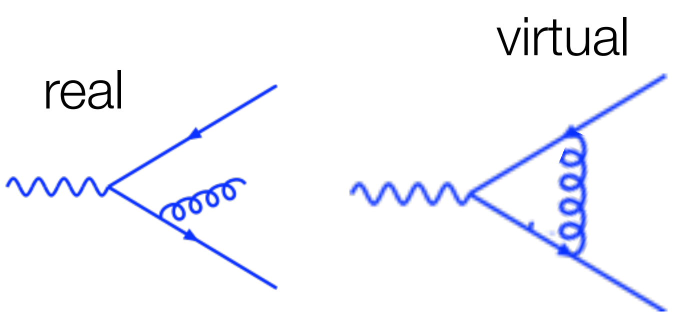
$$P_{\mathrm{q}\to\mathrm{qg}} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{\mathrm{g}\to\mathrm{gg}} = 3\frac{(1-z(1-z))^2}{z(1-z)}$$

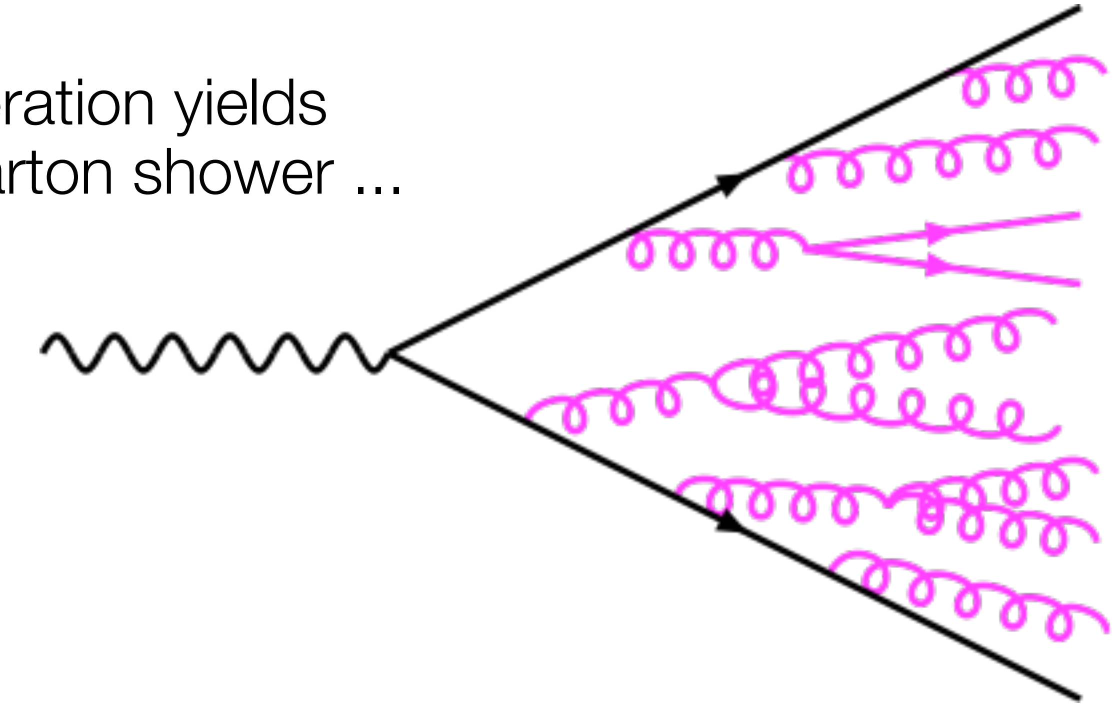
$$P_{\mathrm{g}\to\mathrm{q\bar{q}}} = \frac{n_f}{2}(z^2+(1-z)^2)$$

z : fractional momentum of radiated parton
n_f : number of quark flavours

In NLO calculations soft and collinear divergencies cancelled by virtual contributions: they persist in LO calculations.



Iteration yields parton shower ...



Need soft/collinear cut-offs to avoid non-perturbative regions ... [divergencies!]

Details model-dependent

e.g. $Q > m_0 = \min(m_{ij}) \approx 1$ GeV,
$z_{min}(E,Q) < z < z_{max}(E,Q)$ or
$p_\perp > p_{\perp min} \approx 0.5$ GeV

Parton shower evolution 1

[Sjöstrand, arXiv:hep-ph/0611247v1]

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

Time evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \le T) = \mathcal{P}_{\text{nothing}}(0 < t \le T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \le T)$$

$$\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \le T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \le T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(- \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1}) \right) \\
&= \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t} \mathrm{d}t \right)
\end{aligned}$$

$e^{-x} \approx 1 - x$

[Taylor]

$$\rightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\, \exp\left(- \int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t} \mathrm{d}t \right)$$

Parton shower evolution 2

[Sjöstrand, arXiv:hep-ph/0611247v1]

Instead of evolving to later and later times
need to evolve to smaller and smaller Q^2 ...
[Heisenberg: Q ~ 1/t]

Sudakov
Form Factor

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}P_{a\to bc}(z)\,dz\;\exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_{\max}}\frac{dQ'^2}{Q'^2}\int\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\,dz'\right)$$

Probability to radiated
with virtuality Q^2

No radiation for higher
virtualities i.e. for Q^2 ... Q^2_{max}

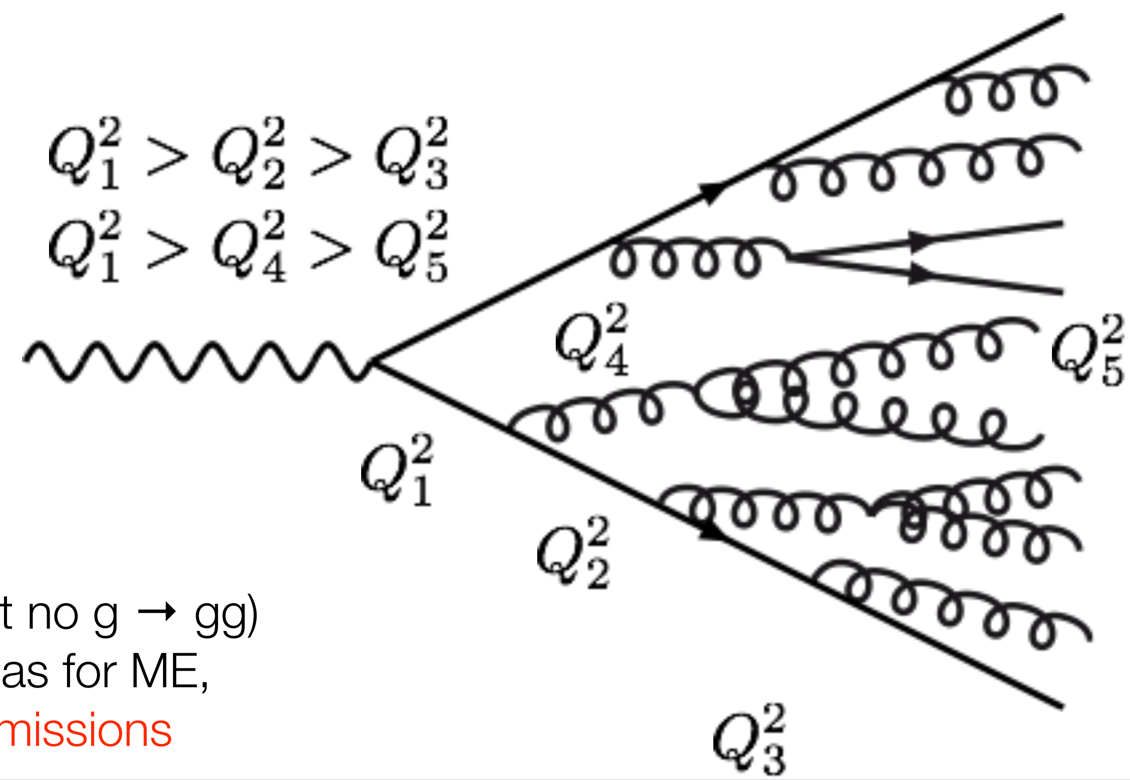
Note that $\sum_{b,c} \iint dP_{a\to bc} \equiv 1$...
[Convenient for Monte Carlo]

Sudakov form factor ...

... provides "time" ordering of shower ...
[lower Q^2 ⇔ longer times]

... regulates singularity for first emission ...

But in the limit of repeated soft emissions q → qg (but no g → gg)
one obtains the same inclusive Q emission spectrum as for ME,
i.e. divergent ME spectrum ⇔ infinite number of PS emissions



Sudakov picture of parton showers

Basic algorithm: Markov chain

[each step requires only knowledge only of previous step]

(i) Start with virtuality Q_1 and momentum fraction x_1

(ii) Generate target virtuality Q_2 with random number R_T uniform distributed in [0,1]

Probability to not have $Q_x > Q_2$

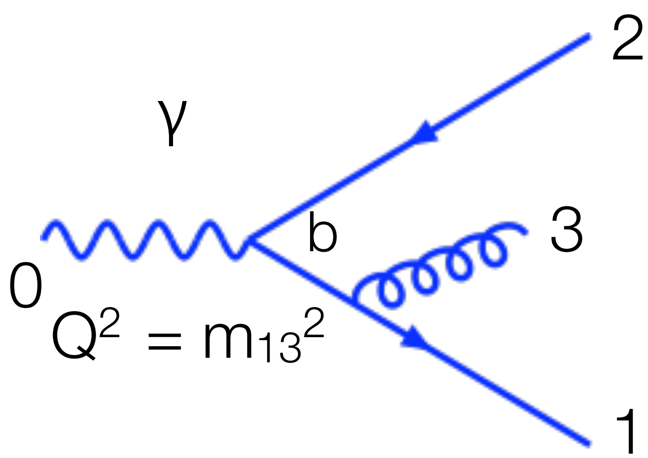
using:

$$\Delta(Q_i^2) = \exp\left(-\sum_{b,c}\int_{Q_i^2}^{Q_{max}^2}\frac{dQ'^2}{Q'^2}\int\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\,dz'\right)$$

solve the equation for Q_2

$$R_t = \frac{\Delta(Q_2^2)}{\Delta(Q_1^2)}$$

[probability to evolve from t_1 to t_2 without radiation]



(iii) Q_2 known (x_2 known), need to compute $x_1 \sim z$

$$P_{q\to qg} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$R_z = \frac{\int_0^z P(z')dz'}{\int_0^1 P(z')dz'}$$

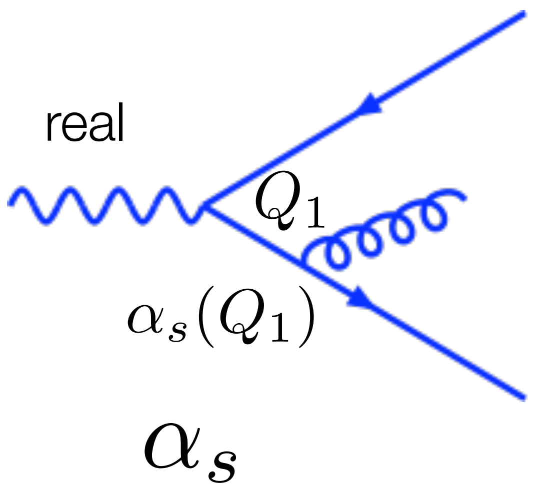
flat distributed

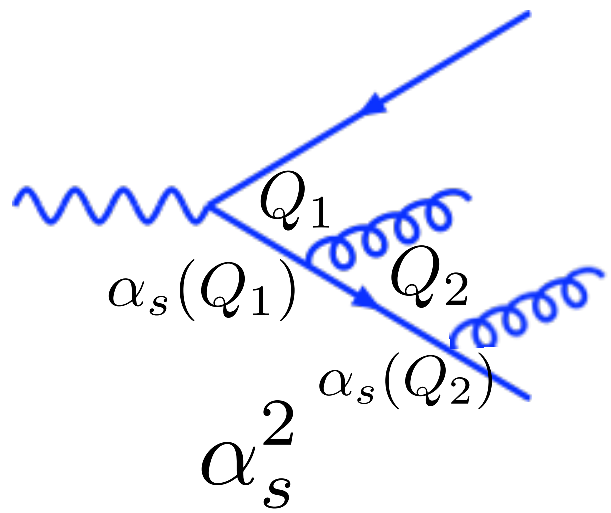
$$R_z \in [0,1]$$

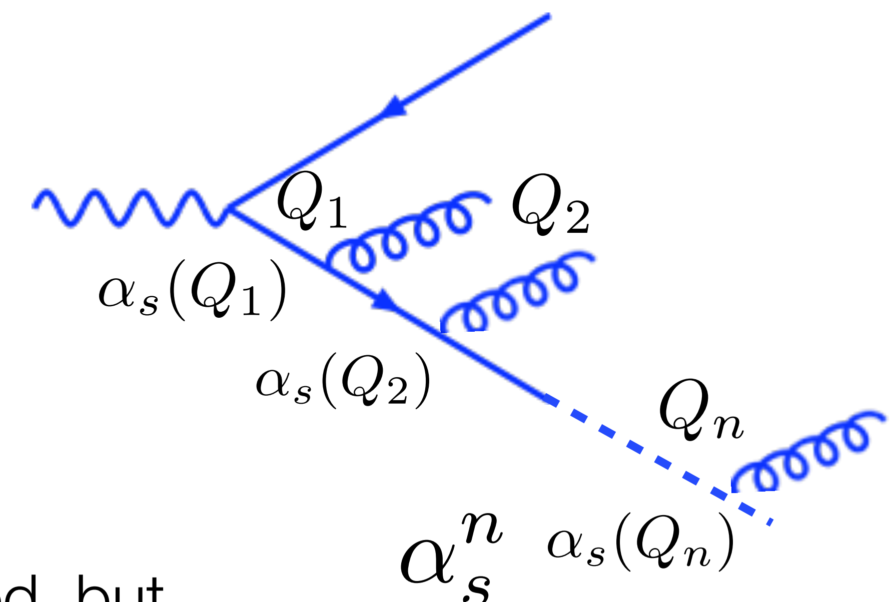
(iv) Generate random azimuthal angle Φ flat distributed

Process ends when partons are below threshold (p_T,Q)

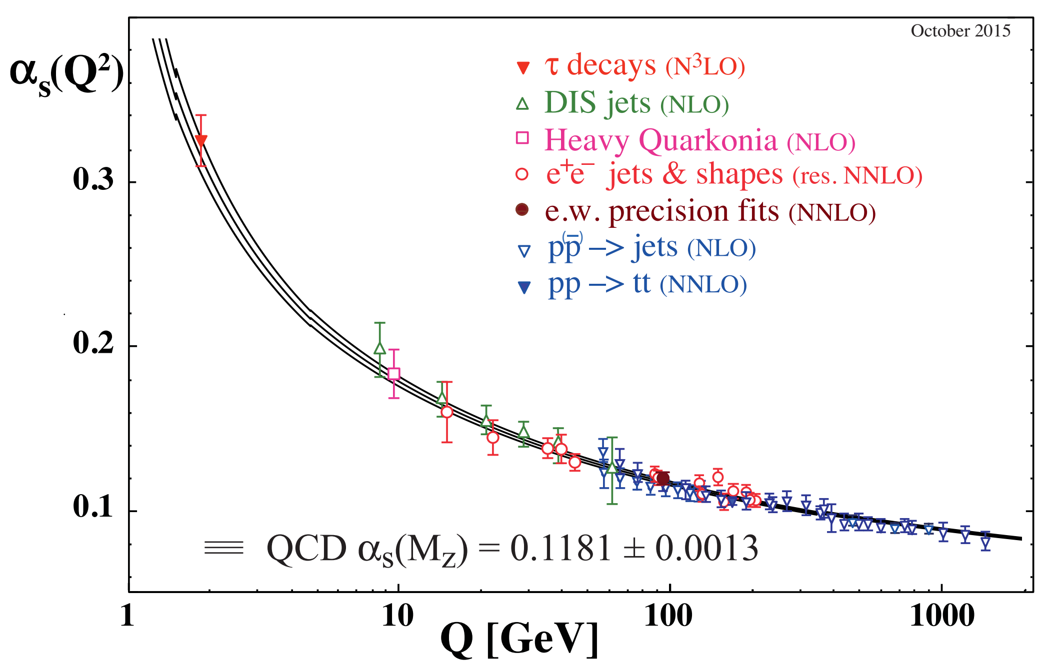
Parton shower and logarithmic resummation







If α_s is small higher contributions are power suppressed, but…



α_s increases at small Q^2

$$\alpha_s(Q_n) \sim \alpha_s(Q_1) ln(Q_1/Q_n)$$

$$\alpha_s(Q_1) + \alpha_s(Q_1)\alpha_s(Q_2) + \ldots + \alpha_s(Q_1)\cdot \ldots \cdot \alpha_s(Q_n)$$

$$\sim [\alpha_s(Q_1) ln(Q_1)]^2 \sim [\alpha_s(Q_1) ln(Q_1)]^n$$

if $\alpha_s(Q_1) ln(Q_1)$ is large, the expansion is broken, PS allow to sum up all the large contribution [Leading Log resummation]

Parton shower ordering

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}P_{a\to bc}(z)\,dz\ \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_{\max}}\frac{dQ'^2}{Q'^2}\int\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\,dz'\right)$$

In the splitting function appears only dQ^2/Q^2, therefore if $P = f(z)Q^2$ $dP/P = dQ^2/Q^2$

Three main approaches to showering in use:

$p_\perp^2 \approx z(1-z)m^2$ p_T ordered showers $\quad E^2\theta^2 \approx m^2/(z(1-z))$ angular ordered showers

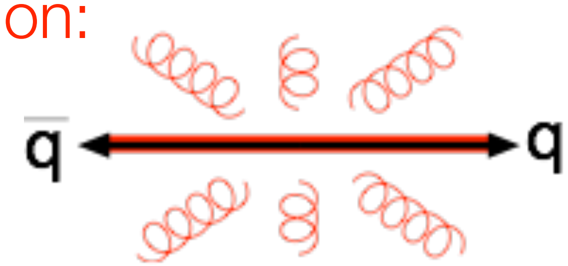
Two are based on the standard shower language of a ➛ bc successive branchings:

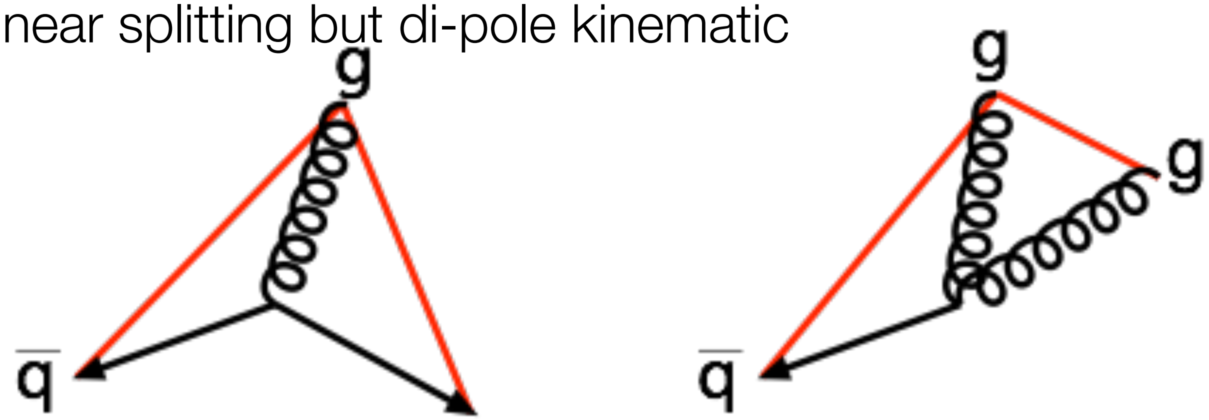
HERWIG, HERWIG++ : $Q^2 \approx E^2(1-\cos\theta) \approx E^2\theta^2/2$

PYTHIA, 8 (basic) : $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

PYTHIA6, 8 (p_T oredered) : mixture: collinear splitting but di-pole kinematic

One is based on a picture of dipole emission:



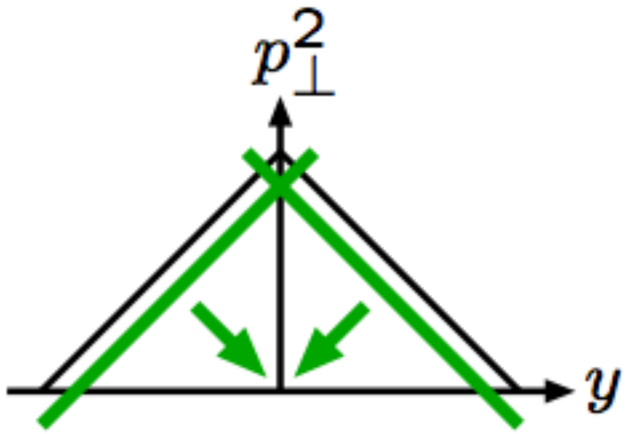


Ariadne : $Q^2 = p^2_\perp$; FSR mainly, ISR is primitive ...

consider the full recoil and not only the branching

[from G.Herten]

PYTHIA: $Q^2 = m^2$



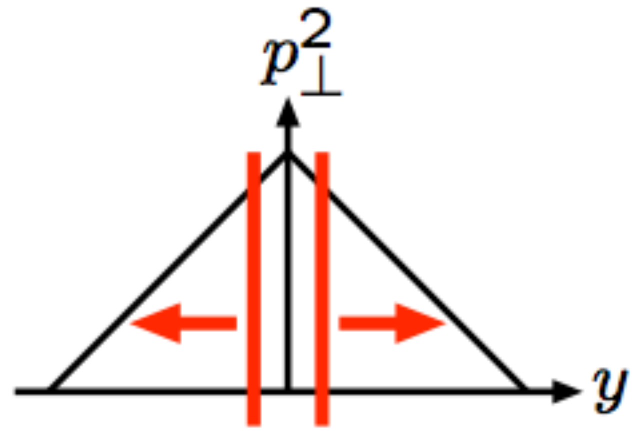
Large mass first

["hardness" ordered]

Covers phase space
ME merging simple
g ➛ qq simple
not Lorentz invariant
no stop/restart

ISR: m^2 ➛ $-m^2$

HERWIG/++: $Q^2 \sim E^2\theta^2$



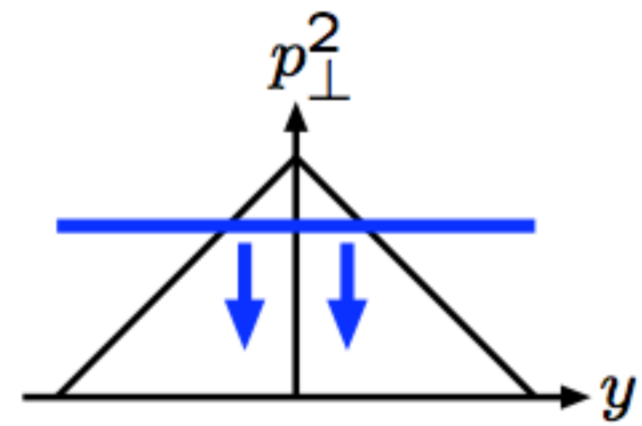
Large angle first

[not "hardness" ordered]

Gaps in coverage
ME merging messy
g ➛ qq simple
not Lorentz invariant
no stop/restart

ISR: θ ➛ θ

ARIADNE/Pythia8: $Q^2 = p^2_\perp$



Large $p_\perp$ first

["hardness" ordered]

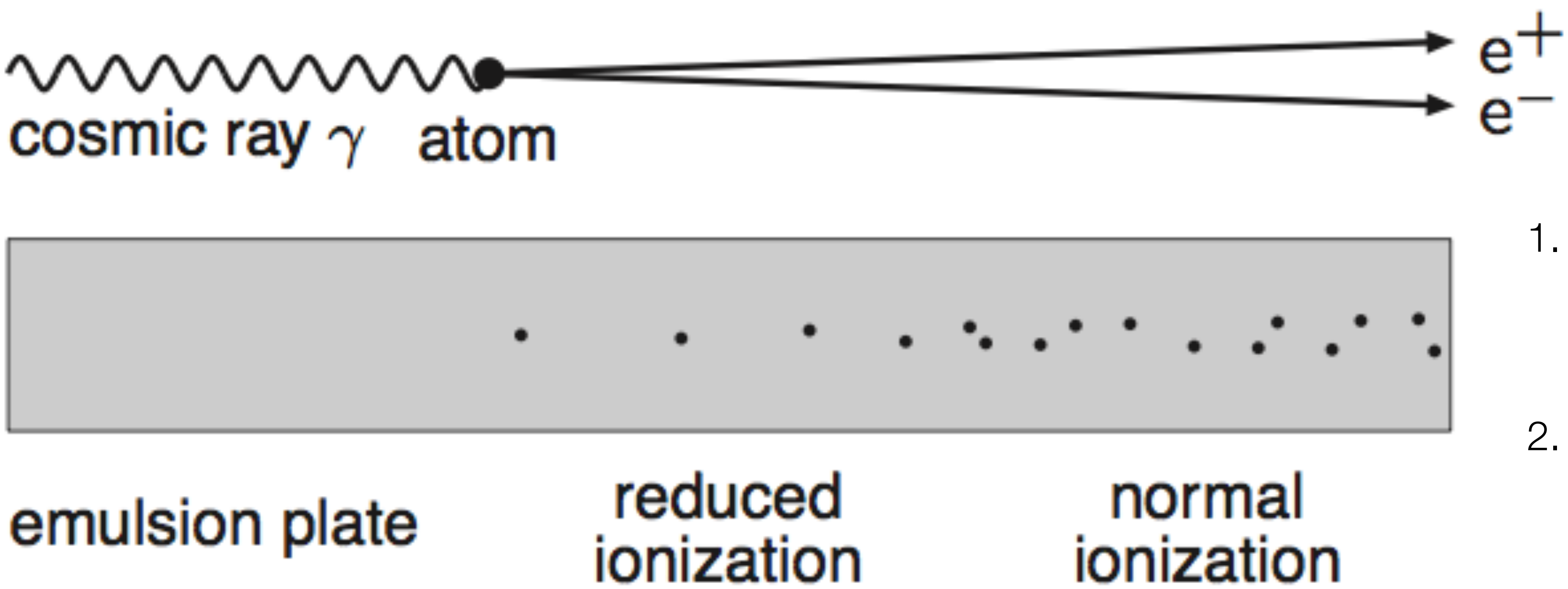
Covers phase space
ME merging simple
g ➛ qq messy
Lorentz invariant
can stop/restart

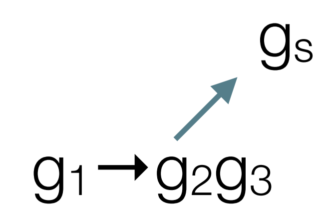
ISR: complicated

[from G.Herten]

Color coherence

QED: Chudakov effect (mid-fifties)



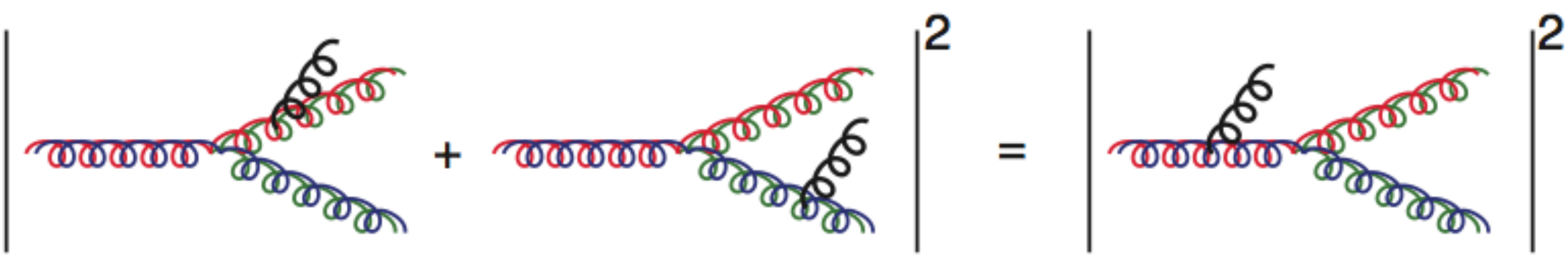


1. soft gluons see the pair of split gluons as a whole, color screening reduce their emission

2. angular ordered and p_T ordered PS reproduce the correct color coherence

3. Pythia Q^2 needs aposteriori corrections

QCD: colour coherence for **soft** gluon emission



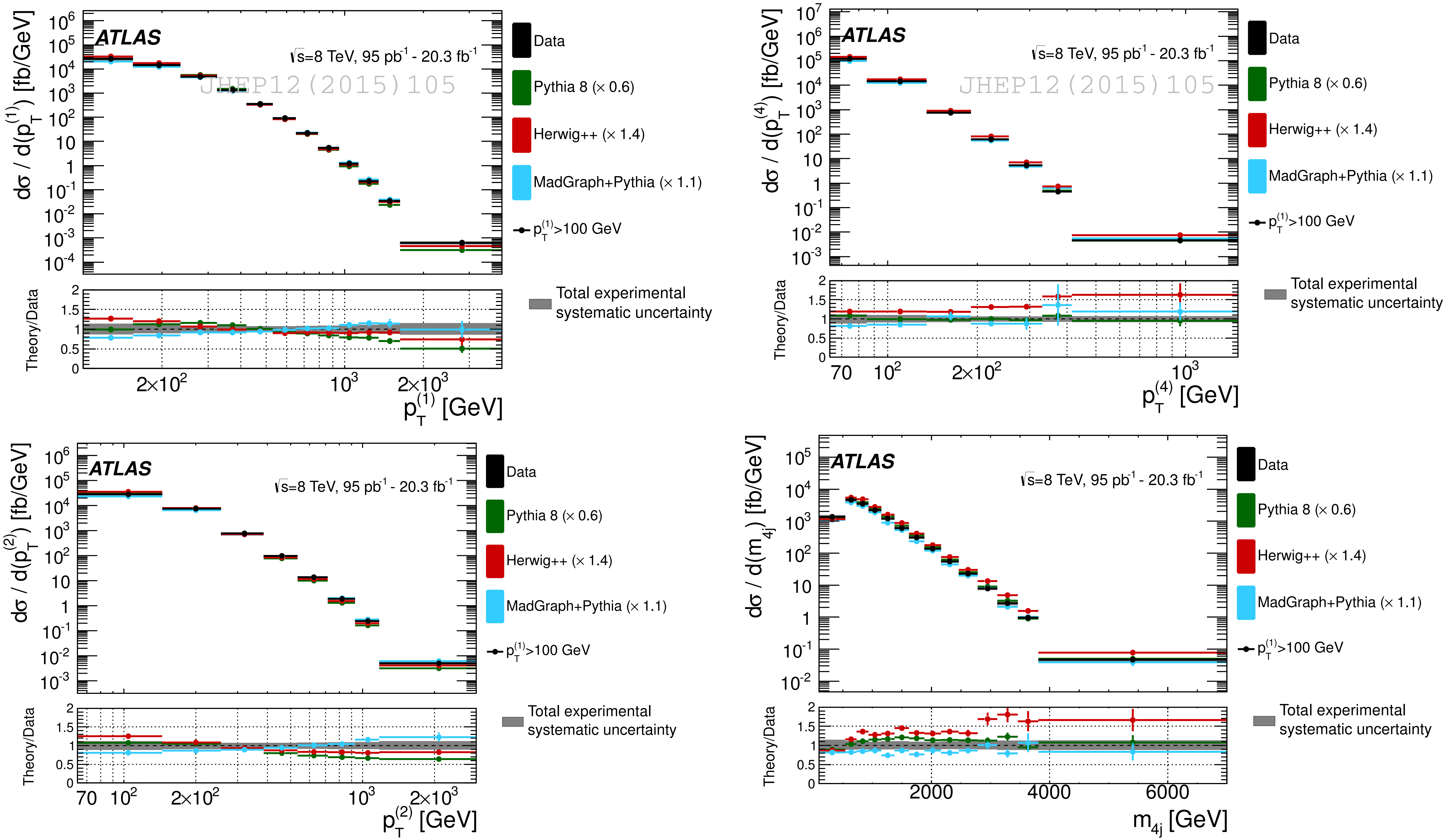
solved by • requiring emission angles to be decreasing

or • requiring transverse momenta to be decreasing

[from G.Herten]

Compariosn to LHC data

4jets cross section: $p_T^{(1)} > p_T^{(2)} > p_T^{(3)} > p_T^{(4)}$



Example of processes implemented in Pythia6

No.	Subprocess
	Hard QCD processes:
11	$f_i f_j \to f_i f_j$
12	$f_i \bar{f}_i \to f_k \bar{f}_k$
13	$f_i \bar{f}_i \to gg$
28	$f_i g \to f_i g$
53	$gg \to f_k \bar{f}_k$
68	$gg \to gg$
	Soft QCD processes:
91	elastic scattering
92	single diffraction (XB)
93	single diffraction (AX)
94	double diffraction
95	low-$p_\perp$ production
	Open heavy flavour: (also fourth generation)
81	$f_i \bar{f}_i \to Q_k \bar{Q}_k$
82	$gg \to Q_k \bar{Q}_k$
83	$q_i f_j \to Q_k f_l$
84	$g\gamma \to Q_k \bar{Q}_k$
85	$\gamma\gamma \to F_k \bar{F}_k$
	Closed heavy flavour:
86	$gg \to J/\psi g$
87	$gg \to \chi_{0c} g$
88	$gg \to \chi_{1c} g$
89	$gg \to \chi_{2c} g$
104	$gg \to \chi_{0c}$
105	$gg \to \chi_{2c}$
106	$gg \to J/\psi\gamma$
107	$g\gamma \to J/\psi g$
108	$\gamma\gamma \to J/\psi\gamma$
	W/Z production:
1	$f_i \bar{f}_i \to \gamma^*/Z^0$
2	$f_i \bar{f}_j \to W^\pm$
22	$f_i \bar{f}_i \to Z^0 Z^0$
23	$f_i \bar{f}_j \to Z^0 W^\pm$
25	$f_i \bar{f}_i \to W^+ W^-$
15	$f_i \bar{f}_i \to g Z^0$
16	$f_i \bar{f}_j \to g W^\pm$
30	$f_i g \to f_i Z^0$
31	$f_i g \to f_k W^\pm$
19	$f_i \bar{f}_i \to \gamma Z^0$
20	$f_i \bar{f}_j \to \gamma W^\pm$
35	$f_i \gamma \to f_i Z^0$

No.	Subprocess
36	$f_i \gamma \to f_k W^\pm$
69	$\gamma\gamma \to W^+ W^-$
70	$\gamma W^\pm \to Z^0 W^\pm$
	Prompt photons:
14	$f_i \bar{f}_i \to g\gamma$
18	$f_i \bar{f}_i \to \gamma\gamma$
29	$f_i g \to f_i \gamma$
114	$gg \to \gamma\gamma$
115	$gg \to g\gamma$
	Deeply Inel. Scatt.:
10	$f_i f_j \to f_k f_l$
99	$\gamma^* q \to q$
	Photon-induced:
33	$f_i \gamma \to f_i g$
34	$f_i \gamma \to f_i \gamma$
54	$g\gamma \to f_k \bar{f}_k$
58	$\gamma\gamma \to f_k \bar{f}_k$
131	$f_i \gamma^*_T \to f_i g$
132	$f_i \gamma^*_L \to f_i g$
133	$f_i \gamma^*_T \to f_i \gamma$
134	$f_i \gamma^*_L \to f_i \gamma$
135	$g\gamma^*_T \to f_i \bar{f}_i$
136	$g\gamma^*_L \to f_i \bar{f}_i$
137	$\gamma^*_T \gamma^*_T \to f_i \bar{f}_i$
138	$\gamma^*_T \gamma^*_L \to f_i \bar{f}_i$
139	$\gamma^*_L \gamma^*_T \to f_i \bar{f}_i$
140	$\gamma^*_L \gamma^*_L \to f_i \bar{f}_i$
80	$q_i \gamma \to q_k \pi^\pm$
	Light SM Higgs:
3	$f_i \bar{f}_i \to h^0$
24	$f_i \bar{f}_i \to Z^0 h^0$
26	$f_i \bar{f}_j \to W^\pm h^0$
32	$f_i g \to f_i h^0$
102	$gg \to h^0$
103	$\gamma\gamma \to h^0$
110	$f_i \bar{f}_i \to \gamma h^0$
111	$f_i \bar{f}_i \to g h^0$
112	$f_i g \to f_i h^0$
113	$gg \to g h^0$
121	$gg \to Q_k \bar{Q}_k h^0$
122	$q_i \bar{q}_i \to Q_k \bar{Q}_k h^0$
123	$f_i f_j \to f_i f_j h^0$
124	$f_i f_j \to f_k f_l h^0$

No.	Subprocess
	New gauge bosons:
141	$f_i \bar{f}_i \to \gamma/Z^0/Z'^0$
142	$f_i \bar{f}_j \to W'^+$
144	$f_i \bar{f}_j \to R$
	Heavy SM Higgs:
5	$Z^0 Z^0 \to h^0$
8	$W^+ W^- \to h^0$
71	$Z^0_L Z^0_L \to Z^0_L Z^0_L$
72	$Z^0_L Z^0_L \to W^+_L W^-_L$
73	$Z^0_L W^\pm_L \to Z^0_L W^\pm_L$
76	$W^+_L W^-_L \to Z^0_L Z^0_L$
77	$W^\pm_L W^\pm_L \to W^\pm_L W^\pm_L$
	BSM Neutral Higgs:
151	$f_i \bar{f}_i \to H^0$
152	$gg \to H^0$
153	$\gamma\gamma \to H^0$
171	$f_i \bar{f}_i \to Z^0 H^0$
172	$f_i \bar{f}_j \to W^\pm H^0$
173	$f_i f_j \to f_i f_j H^0$
174	$f_i f_j \to f_k f_l H^0$
181	$gg \to Q_k \bar{Q}_k H^0$
182	$q_i \bar{q}_i \to Q_k \bar{Q}_k H^0$
183	$f_i \bar{f}_i \to g H^0$
184	$f_i g \to f_i H^0$
185	$gg \to g H^0$
156	$f_i \bar{f}_i \to A^0$
157	$gg \to A^0$
158	$\gamma\gamma \to A^0$
176	$f_i \bar{f}_i \to Z^0 A^0$
177	$f_i \bar{f}_j \to W^\pm A^0$
178	$f_i f_j \to f_i f_j A^0$
179	$f_i f_j \to f_k f_l A^0$
186	$gg \to Q_k \bar{Q}_k A^0$
187	$q_i \bar{q}_i \to Q_k \bar{Q}_k A^0$
188	$f_i \bar{f}_i \to g A^0$
189	$f_i g \to f_i A^0$
190	$gg \to g A^0$
	Charged Higgs:
143	$f_i \bar{f}_j \to H^+$
161	$f_i g \to f_k H^+$
401	$gg \to \bar{t} b H^+$
402	$q\bar{q} \to \bar{t} b H^+$

No.	Subprocess
	Higgs pairs:
297	$f_i \bar{f}_j \to H^\pm h^0$
298	$f_i \bar{f}_j \to H^\pm H^0$
299	$f_i \bar{f}_i \to A^0 h^0$
300	$f_i \bar{f}_i \to A^0 H^0$
301	$f_i \bar{f}_i \to H^+ H^-$
	Leptoquarks:
145	$q_i \ell_j \to L_Q$
162	$qg \to \ell L_Q$
163	$gg \to L_Q \bar{L}_Q$
164	$q_i \bar{q}_i \to L_Q \bar{L}_Q$
	Technicolor:
149	$gg \to \eta_{tc}$
191	$f_i \bar{f}_i \to \rho^0_{tc}$
192	$f_i \bar{f}_j \to \rho^+_{tc}$
193	$f_i \bar{f}_i \to \omega^0_{tc}$
194	$f_i \bar{f}_i \to f_k \bar{f}_k$
195	$f_i \bar{f}_j \to f_k \bar{f}_l$
361	$f_i \bar{f}_i \to W^+_L W^-_L$
362	$f_i \bar{f}_i \to W^\pm_L \pi^\mp_{tc}$
363	$f_i \bar{f}_i \to \pi^+_{tc} \pi^-_{tc}$
364	$f_i \bar{f}_i \to \gamma \pi^0_{tc}$
365	$f_i \bar{f}_i \to \gamma \pi'^0_{tc}$
366	$f_i \bar{f}_i \to Z^0 \pi^0_{tc}$
367	$f_i \bar{f}_i \to Z^0 \pi'^0_{tc}$
368	$f_i \bar{f}_i \to W^\pm \pi^\mp_{tc}$
370	$f_i \bar{f}_j \to W^\pm_L Z^0_L$
371	$f_i \bar{f}_j \to W^\pm_L \pi^0_{tc}$
372	$f_i \bar{f}_j \to \pi^\pm_{tc} Z^0_L$
373	$f_i \bar{f}_j \to \pi^\pm_{tc} \pi^0_{tc}$
374	$f_i \bar{f}_j \to \gamma \pi^\pm_{tc}$
375	$f_i \bar{f}_j \to Z^0 \pi^\pm_{tc}$
376	$f_i \bar{f}_j \to W^\pm \pi^0_{tc}$
377	$f_i \bar{f}_j \to W^\pm \pi'^0_{tc}$
381	$q_i q_j \to q_i q_j$
382	$q_i \bar{q}_i \to q_k \bar{q}_k$
383	$q_i \bar{q}_i \to gg$
384	$f_i g \to f_i g$
385	$gg \to q_k \bar{q}_k$
386	$gg \to gg$
387	$f_i \bar{f}_i \to Q_k \bar{Q}_k$
388	$gg \to Q_k \bar{Q}_k$

No.	Subprocess
	Compositeness:
146	$e\gamma \to e^*$
147	$dg \to d^*$
148	$ug \to u^*$
167	$q_i q_j \to d^* q_k$
168	$q_i q_j \to u^* q_k$
169	$q_i \bar{q}_i \to e^\pm e^{*\mp}$
165	$f_i \bar{f}_i (\to \gamma^*/Z^0) \to f_k \bar{f}_k$
166	$f_i \bar{f}_j (\to W^\pm) \to f_k \bar{f}_l$
	Extra Dimensions:
391	$f\bar{f} \to G^*$
392	$gg \to G^*$
393	$q\bar{q} \to gG^*$
394	$qg \to qG^*$
395	$gg \to gG^*$
	Left–right symmetry:
341	$\ell_i \ell_j \to H^{\pm\pm}_L$
342	$\ell_i \ell_j \to H^{\pm\pm}_R$
343	$\ell^\pm_i \gamma \to H^{\pm\pm}_L e^\mp$
344	$\ell^\pm_i \gamma \to H^{\pm\pm}_R e^\mp$
345	$\ell^\pm_i \gamma \to H^{\pm\pm}_L \mu^\mp$
346	$\ell^\pm_i \gamma \to H^{\pm\pm}_R \mu^\mp$
347	$\ell^\pm_i \gamma \to H^{\pm\pm}_L \tau^\mp$
348	$\ell^\pm_i \gamma \to H^{\pm\pm}_R \tau^\mp$
349	$f_i \bar{f}_i \to H^{++}_L H^{--}_L$
350	$f_i \bar{f}_i \to H^{++}_R H^{--}_R$
351	$f_i f_j \to f_k f_l H^{\pm\pm}_L$
352	$f_i f_j \to f_k f_l H^{\pm\pm}_R$
353	$f_i \bar{f}_i \to Z^0_R$
354	$f_i \bar{f}_j \to W^\pm_R$
	SUSY:
201	$f_i \bar{f}_i \to \tilde{e}_L \tilde{e}^*_L$
202	$f_i \bar{f}_i \to \tilde{e}_R \tilde{e}^*_R$
203	$f_i \bar{f}_i \to \tilde{e}_L \tilde{e}^*_R +$
204	$f_i \bar{f}_i \to \tilde{\mu}_L \tilde{\mu}^*_L$
205	$f_i \bar{f}_i \to \tilde{\mu}_R \tilde{\mu}^*_R$
206	$f_i \bar{f}_i \to \tilde{\mu}_L \tilde{\mu}^*_R +$
207	$f_i \bar{f}_i \to \tilde{\tau}_1 \tilde{\tau}^*_1$
208	$f_i \bar{f}_i \to \tilde{\tau}_2 \tilde{\tau}^*_2$
209	$f_i \bar{f}_i \to \tilde{\tau}_1 \tilde{\tau}^*_2 +$

No.	Subprocess
210	$f_i \bar{f}_j \to \tilde{\ell}_L \tilde{\nu}^*_\ell +$
211	$f_i \bar{f}_j \to \tilde{\tau}_1 \tilde{\nu}^*_\tau +$
212	$f_i \bar{f}_j \to \tilde{\tau}_2 \tilde{\nu}^*_\tau +$
213	$f_i \bar{f}_i \to \tilde{\nu}_\ell \tilde{\nu}^*_\ell$
214	$f_i \bar{f}_i \to \tilde{\nu}_\tau \tilde{\nu}^*_\tau$
216	$f_i \bar{f}_i \to \tilde{\chi}_1 \tilde{\chi}_1$
217	$f_i \bar{f}_i \to \tilde{\chi}_2 \tilde{\chi}_2$
218	$f_i \bar{f}_i \to \tilde{\chi}_3 \tilde{\chi}_3$
219	$f_i \bar{f}_i \to \tilde{\chi}_4 \tilde{\chi}_4$
220	$f_i \bar{f}_i \to \tilde{\chi}_1 \tilde{\chi}_2$
221	$f_i \bar{f}_i \to \tilde{\chi}_1 \tilde{\chi}_3$
222	$f_i \bar{f}_i \to \tilde{\chi}_1 \tilde{\chi}_4$
223	$f_i \bar{f}_i \to \tilde{\chi}_2 \tilde{\chi}_3$
224	$f_i \bar{f}_i \to \tilde{\chi}_2 \tilde{\chi}_4$
225	$f_i \bar{f}_i \to \tilde{\chi}_3 \tilde{\chi}_4$
226	$f_i \bar{f}_i \to \tilde{\chi}^\pm_1 \tilde{\chi}^\mp_1$
227	$f_i \bar{f}_i \to \tilde{\chi}^\pm_2 \tilde{\chi}^\mp_2$
228	$f_i \bar{f}_i \to \tilde{\chi}^\pm_1 \tilde{\chi}^\mp_2$
229	$f_i \bar{f}_j \to \tilde{\chi}_1 \tilde{\chi}^\pm_1$
230	$f_i \bar{f}_j \to \tilde{\chi}_2 \tilde{\chi}^\pm_1$
231	$f_i \bar{f}_j \to \tilde{\chi}_3 \tilde{\chi}^\pm_1$
232	$f_i \bar{f}_j \to \tilde{\chi}_4 \tilde{\chi}^\pm_1$
233	$f_i \bar{f}_j \to \tilde{\chi}_1 \tilde{\chi}^\pm_2$
234	$f_i \bar{f}_j \to \tilde{\chi}_2 \tilde{\chi}^\pm_2$
235	$f_i \bar{f}_j \to \tilde{\chi}_3 \tilde{\chi}^\pm_2$
236	$f_i \bar{f}_j \to \tilde{\chi}_4 \tilde{\chi}^\pm_2$
237	$f_i \bar{f}_i \to \tilde{g} \tilde{\chi}_1$
238	$f_i \bar{f}_i \to \tilde{g} \tilde{\chi}_2$
239	$f_i \bar{f}_i \to \tilde{g} \tilde{\chi}_3$
240	$f_i \bar{f}_i \to \tilde{g} \tilde{\chi}_4$
241	$f_i \bar{f}_j \to \tilde{g} \tilde{\chi}^\pm_1$
242	$f_i \bar{f}_j \to \tilde{g} \tilde{\chi}^\pm_2$
243	$f_i \bar{f}_i \to \tilde{g}\tilde{g}$
244	$gg \to \tilde{g}\tilde{g}$
246	$f_i g \to \tilde{q}_{iL} \tilde{\chi}_1$
247	$f_i g \to \tilde{q}_{iR} \tilde{\chi}_1$
248	$f_i g \to \tilde{q}_{iL} \tilde{\chi}_2$
249	$f_i g \to \tilde{q}_{iR} \tilde{\chi}_2$

No.	Subprocess
250	$f_i g \to \tilde{q}_{iL} \tilde{\chi}_3$
251	$f_i g \to \tilde{q}_{iR} \tilde{\chi}_3$
252	$f_i g \to \tilde{q}_{iL} \tilde{\chi}_4$
253	$f_i g \to \tilde{q}_{iR} \tilde{\chi}_4$
254	$f_i g \to \tilde{q}_{jL} \tilde{\chi}^\pm_1$
256	$f_i g \to \tilde{q}_{jL} \tilde{\chi}^\pm_2$
258	$f_i g \to \tilde{q}_{iL} \tilde{g}$
259	$f_i g \to \tilde{q}_{iR} \tilde{g}$
261	$f_i \bar{f}_i \to \tilde{t}_1 \tilde{t}^*_1$
262	$f_i \bar{f}_i \to \tilde{t}_2 \tilde{t}^*_2$
263	$f_i \bar{f}_i \to \tilde{t}_1 \tilde{t}^*_2 +$
264	$gg \to \tilde{t}_1 \tilde{t}^*_1$
265	$gg \to \tilde{t}_2 \tilde{t}^*_2$
271	$f_i f_j \to \tilde{q}_{iL} \tilde{q}_{jL}$
272	$f_i f_j \to \tilde{q}_{iR} \tilde{q}_{jR}$
273	$f_i f_j \to \tilde{q}_{iL} \tilde{q}_{jR} +$
274	$f_i \bar{f}_j \to \tilde{q}_{iL} \tilde{q}^*_{jL}$
275	$f_i \bar{f}_j \to \tilde{q}_{iR} \tilde{q}^*_{jR}$
276	$f_i \bar{f}_j \to \tilde{q}_{iL} \tilde{q}^*_{jR} +$
277	$f_i \bar{f}_i \to \tilde{q}_{jL} \tilde{q}^*_{jL}$
278	$f_i \bar{f}_i \to \tilde{q}_{jR} \tilde{q}^*_{jR}$
279	$gg \to \tilde{q}_{iL} \tilde{q}^*_{iL}$
280	$gg \to \tilde{q}_{iR} \tilde{q}^*_{iR}$
281	$bq_i \to \tilde{b}_1 \tilde{q}_{iL}$
282	$bq_i \to \tilde{b}_2 \tilde{q}_{iR}$
283	$bq_i \to \tilde{b}_1 \tilde{q}_{iR} +$
284	$b\bar{q}_i \to \tilde{b}_1 \tilde{q}^*_{iL}$
285	$b\bar{q}_i \to \tilde{b}_2 \tilde{q}^*_{iR}$
286	$b\bar{q}_i \to \tilde{b}_1 \tilde{q}^*_{iR} +$
287	$f_i \bar{f}_i \to \tilde{b}_1 \tilde{b}^*_1$
288	$f_i \bar{f}_i \to \tilde{b}_2 \tilde{b}^*_2$
289	$gg \to \tilde{b}_1 \tilde{b}^*_1$
290	$gg \to \tilde{b}_2 \tilde{b}^*_2$
291	$bb \to \tilde{b}_1 \tilde{b}_1$
292	$bb \to \tilde{b}_2 \tilde{b}_2$
293	$bb \to \tilde{b}_1 \tilde{b}_2$
294	$bg \to \tilde{b}_1 \tilde{g}$
295	$bg \to \tilde{b}_2 \tilde{g}$
296	$b\bar{b} \to \tilde{b}_1 \tilde{b}^*_2 +$

Process simulation

Many specialized processes already available in Pythia8/Herwig++

but, processes usually only implemented in lowest non-trivial order ...

Need external programs that ...

1. include higher order loop corrections or, alternatively, do kinematic dependent rescaling

3. allow matching of higher order ME generators [otherwise need to trust parton shower description …]

5. provide correct spin correlations often absent in PS ...[e.g. top produced unpolarized, while t ➝ bW ➝ blν decay correct]

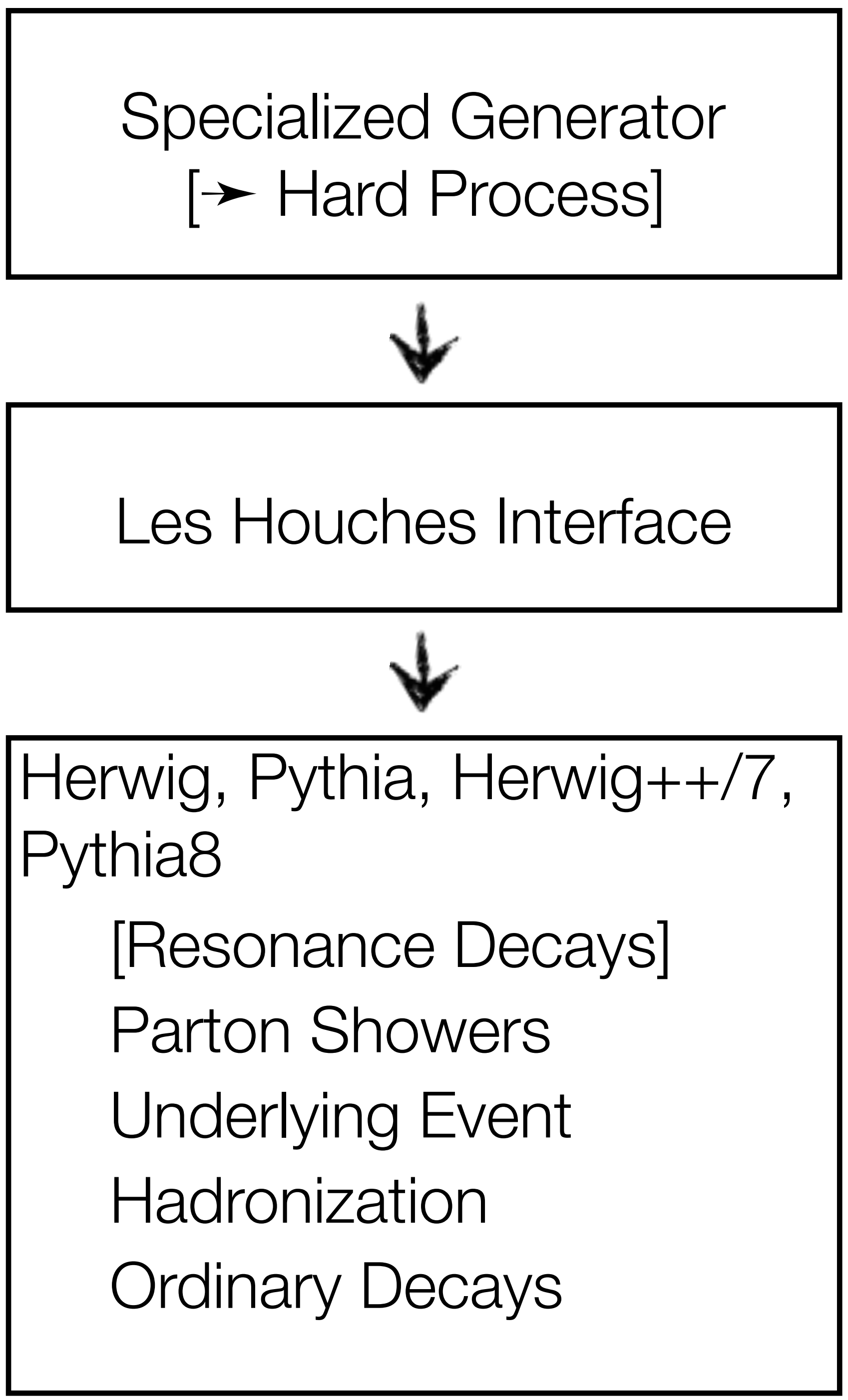
7. simulate newly available physics scenarios …[appear quickly; need for many specialised generators]

Les Houches Accord ...

Specifies how parton-level information about the hard process and sequential decays can be encoded and passed on to a general-purpose generator.

Les Houches: regular annual meeting between theoreticians and experimentalists on MC generator developments.

Specialised Generators [some examples]



AcerMC : ttbb, .sinlr top

ALPGEN : W/Z + ≤ 6j,
nW + mZ + kH + ≤ 3j, ...

AMEGIC++ : generic LO

CompHEP : generic LO

GRACE : generic LO
[+Bases/Spring] [+ some NLO loops]

GR@PPA : bbbb

MadCUP : W/Z+ ≤ 3j, ttbb

HELAS & MadGraph : generic LO

MCFM : NLO W/Z+ ≤ 2j,
WZ, WH, H+ ≤ 1j

O'Mega & WHIZARD : generic LO

VECBOS : W/Z+ ≤ 4j

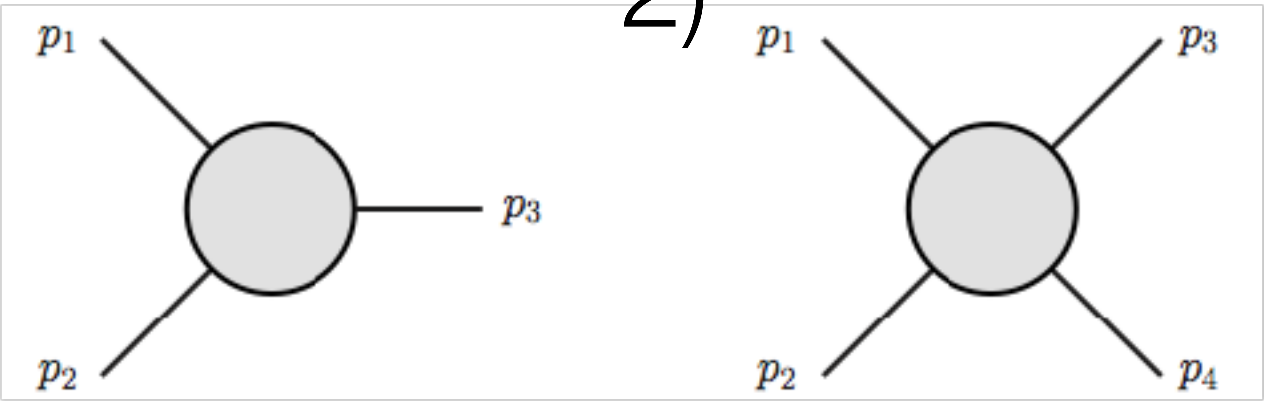
HRES : Higgs boson production @NNLO

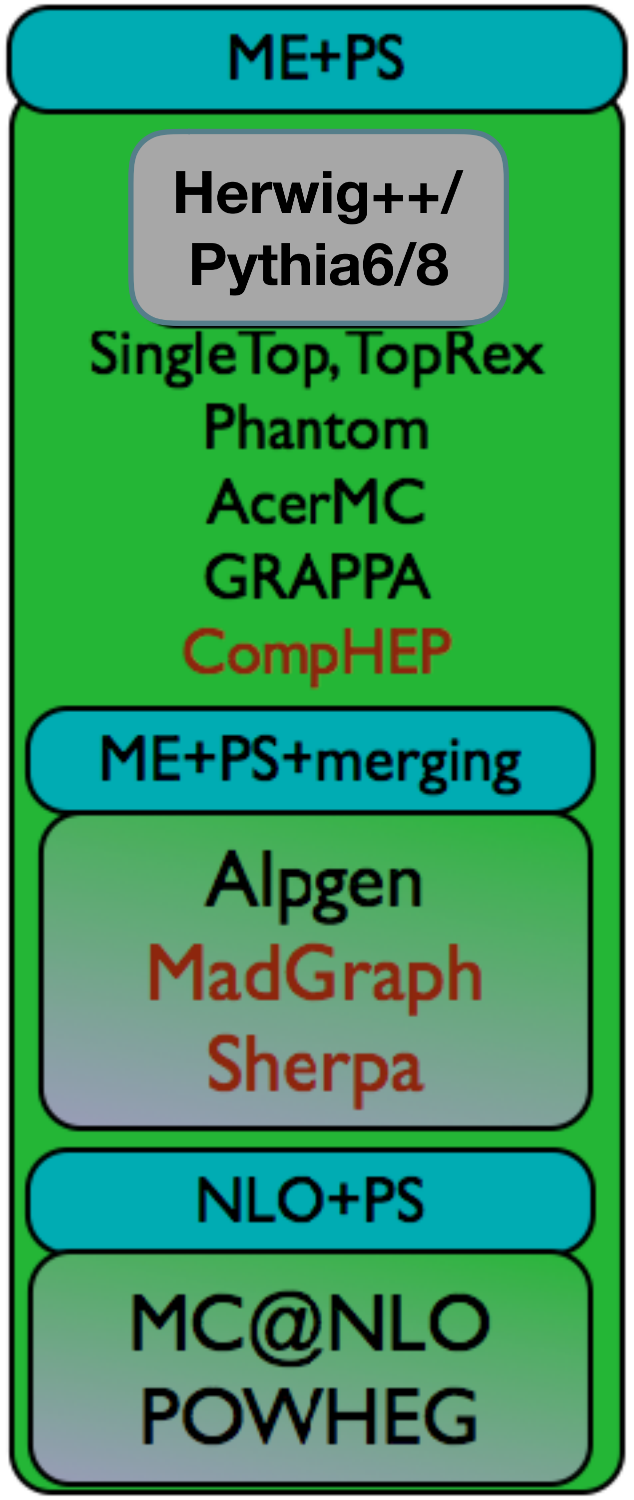
DYNNLO : W/Z production @NNLO

Type I : Leading order matrix element & leading log parton shower

LO ME for hard processes
[2→1 or 2→2]

1) 2)

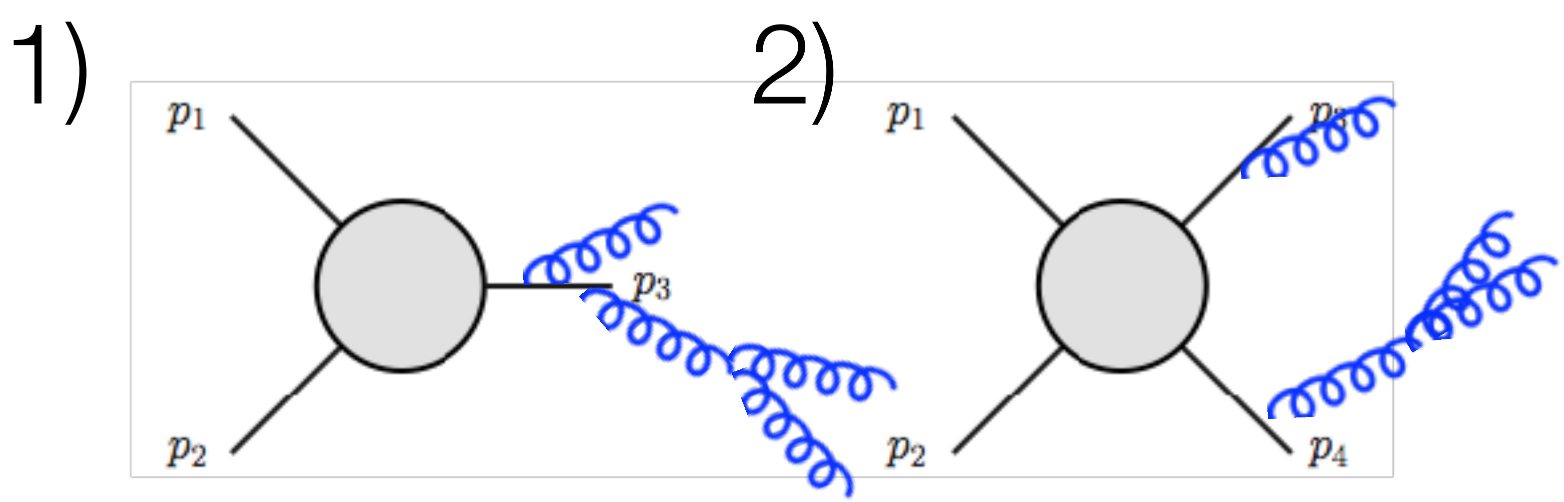


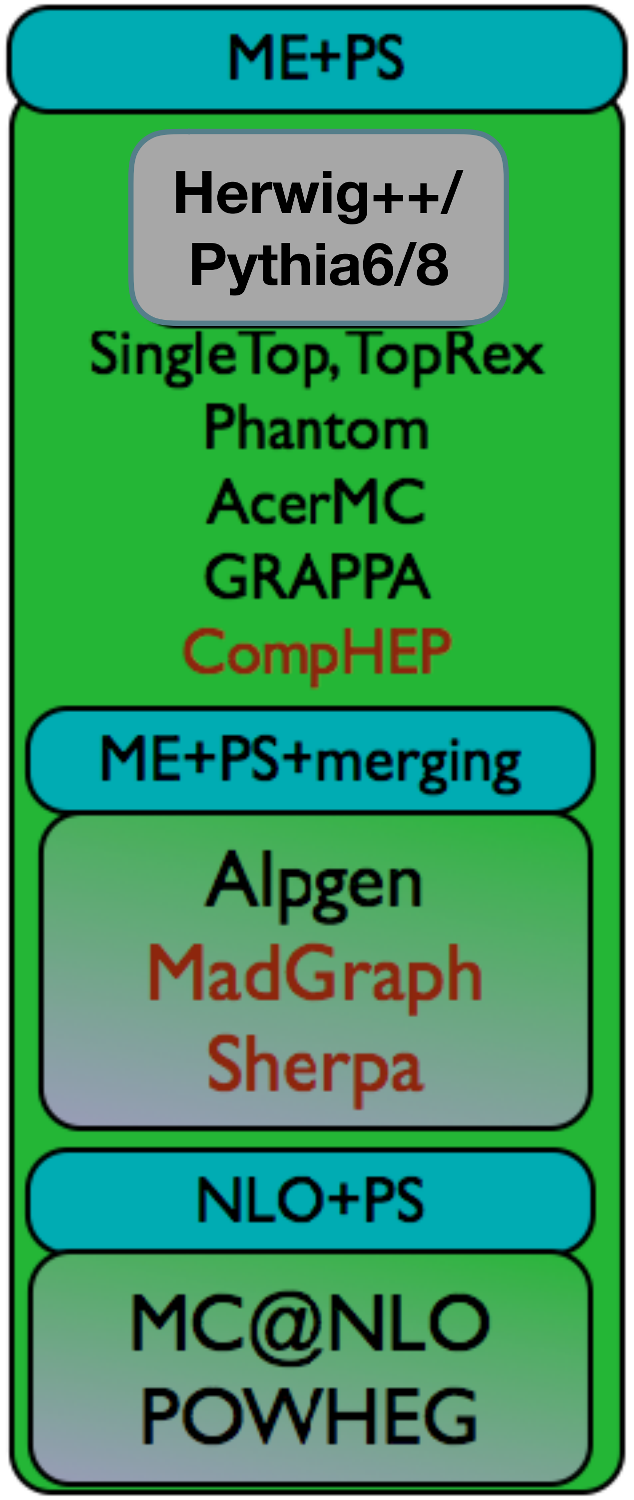


[F. Maltoni]

Type I : Leading order matrix element & leading log parton shower

LO ME for hard processes
[2→1 or 2→2]





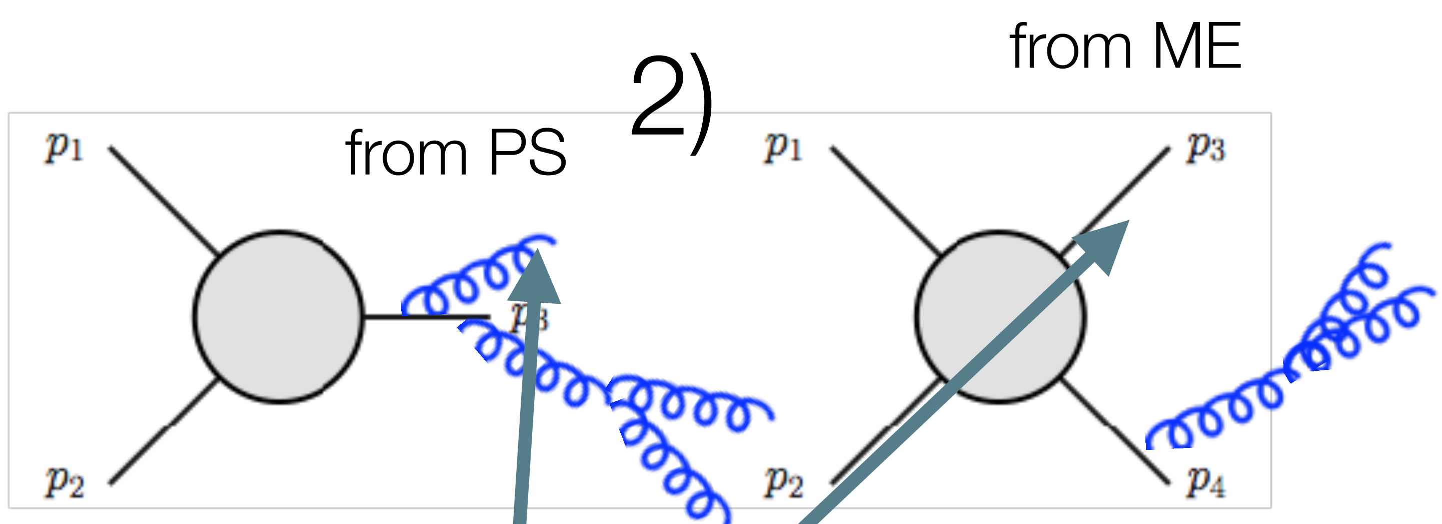
- Parton Shower: attaches gluons at each leg.
- only in soft-collinear approxmation
- typically underestimate large angle/hard emission
- 1) or 2) at ME (different generations, different accuracy: cannot be combined

[F. Maltoni]

Type 2 : Leading order matrix element & leading log parton shower + merging

LO ME for hard processes
[2→1 or 2→2]

1)



- Type 1 can be improved using 1) + 2)
- use ME calculation for hard/large angle jets
- but needs to remove double-counting: merging (CKKW, MLM)
- very good description of high jet multiplicity kinematics

ME+PS

Herwig++/
Pythia6/8

SingleTop, TopRex
Phantom
AcerMC
GRAPPA
CompHEP

ME+PS+merging

Alpgen
MadGraph
Sherpa

NLO+PS

MC@NLO
POWHEG

Merging @LO

MLM matching (simplified)

1) define matching cuts:
for example $p_T^J > 20$ GeV, $\Delta R=0.4$

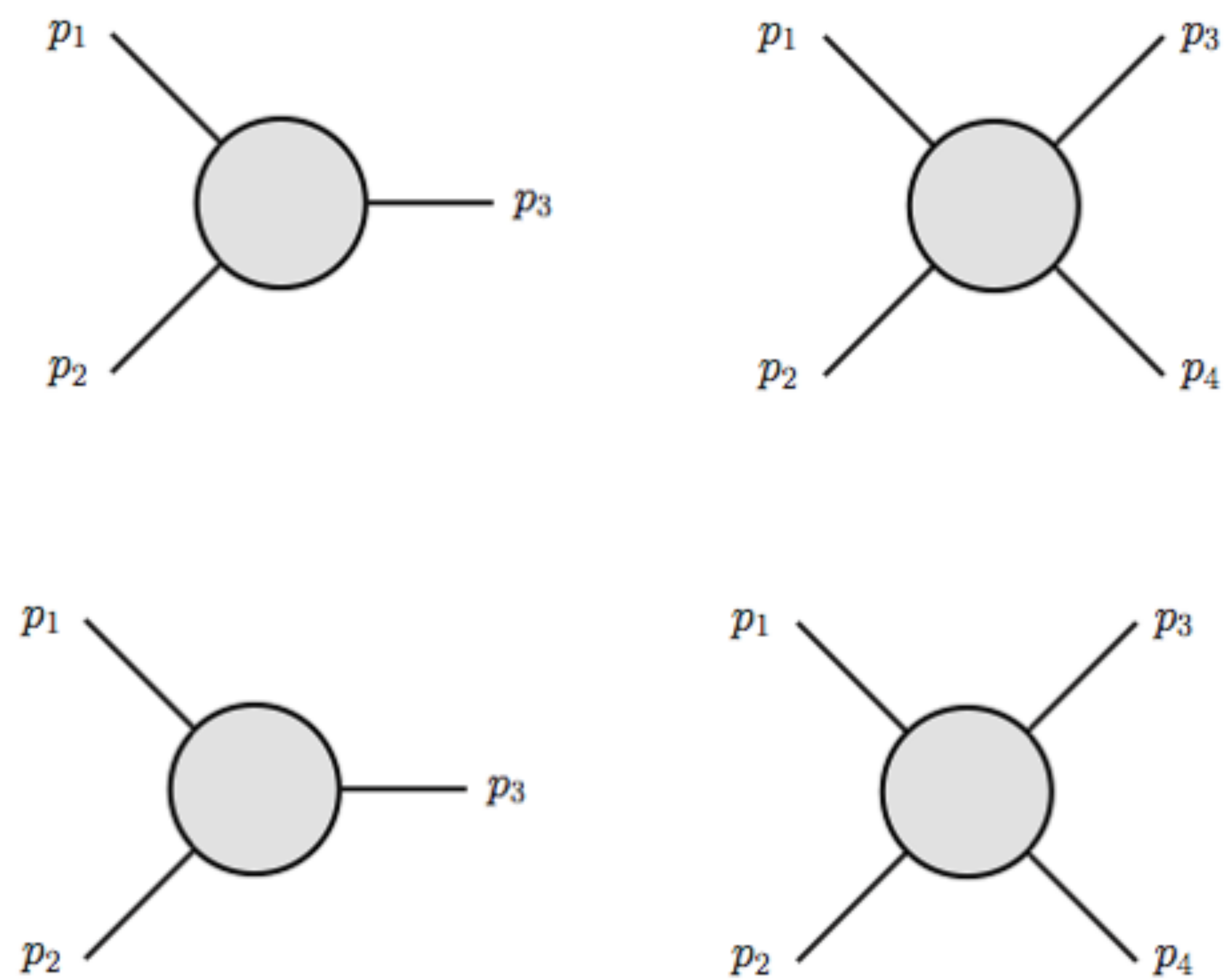
Merging @LO

MLM matching (simplified)

1) define matching cuts:
for example $p_T^J > 20$ GeV, $\Delta R=0.4$

2) generate ME with 1, 2, …n jets

1 parton 2 partons



Merging @LO

MLM matching (simplified)

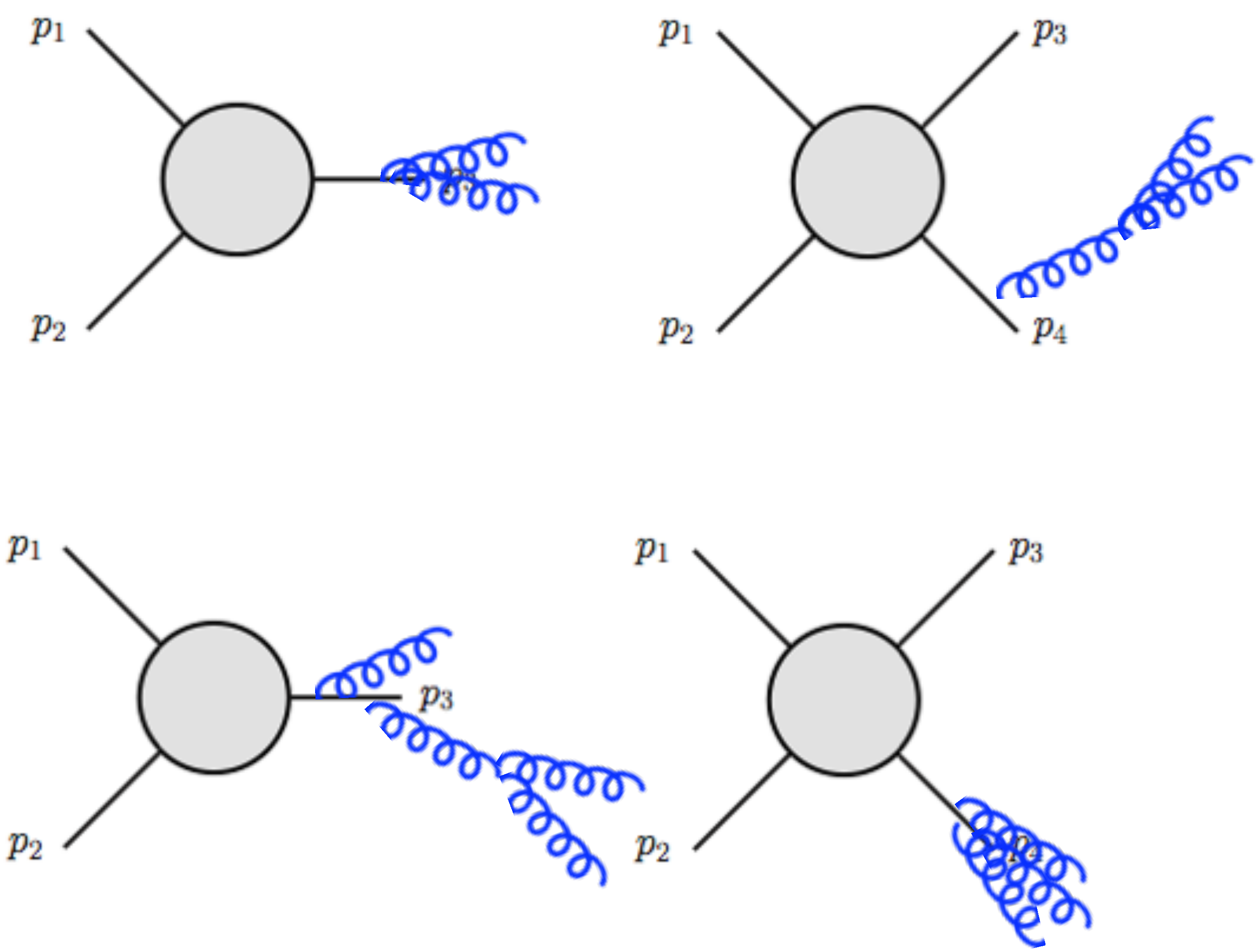
1) define matching cuts:
for example $p_T^J > 20$ GeV, $\Delta R=0.4$

2) generate ME with 1, 2, …n jets

3) shower all events

1 parton

2 partons



Merging @LO

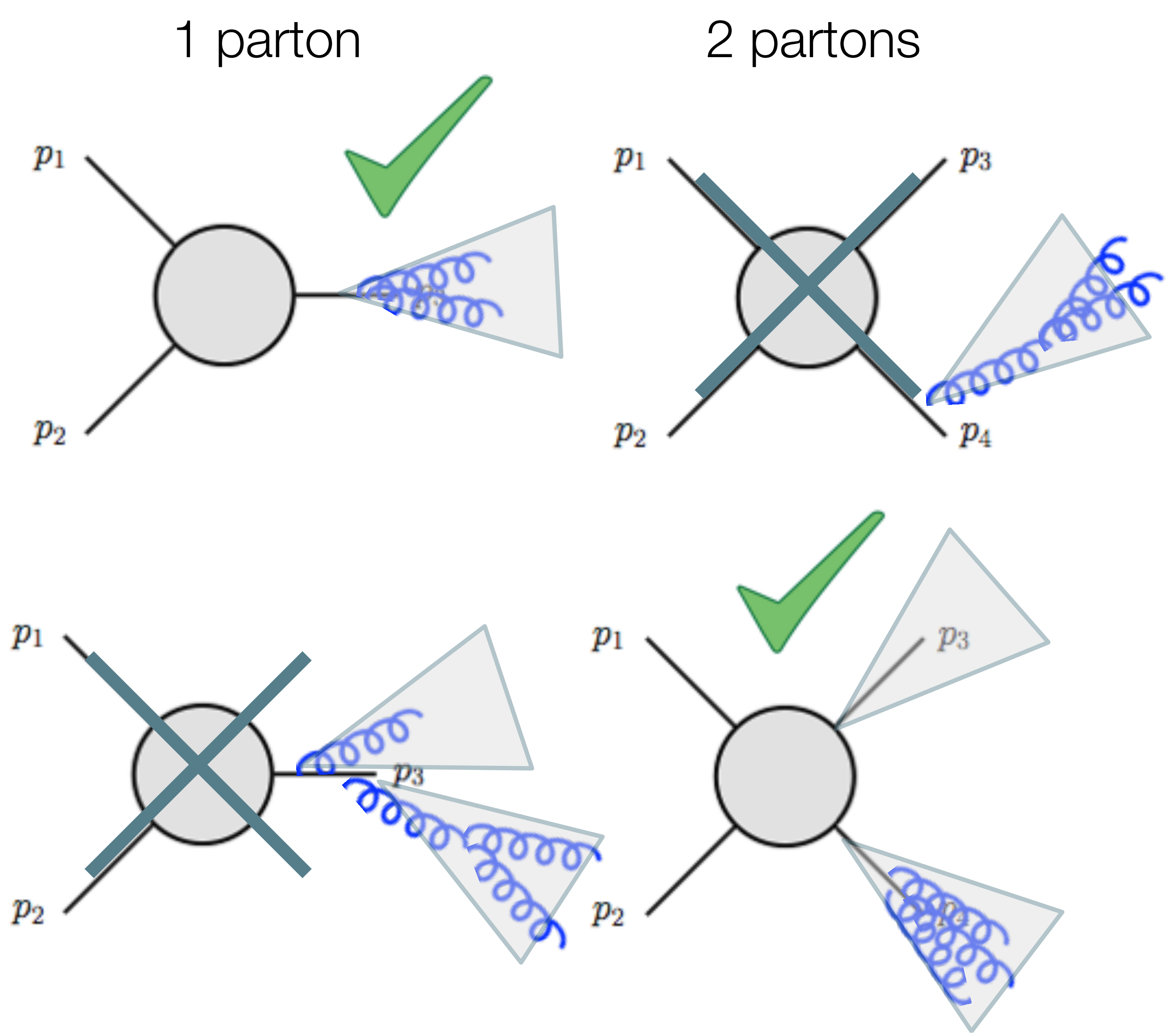
MLM matching (simplified)

1) define matching cuts:
for example $p_T^J > 20$ GeV, $\Delta R=0.4$

2) generate ME with 1, 2, …n jets

3) shower all events

4) select only events where jets above the p_T threshold match with final partons



Merging @LO

MLM matching (simplified)

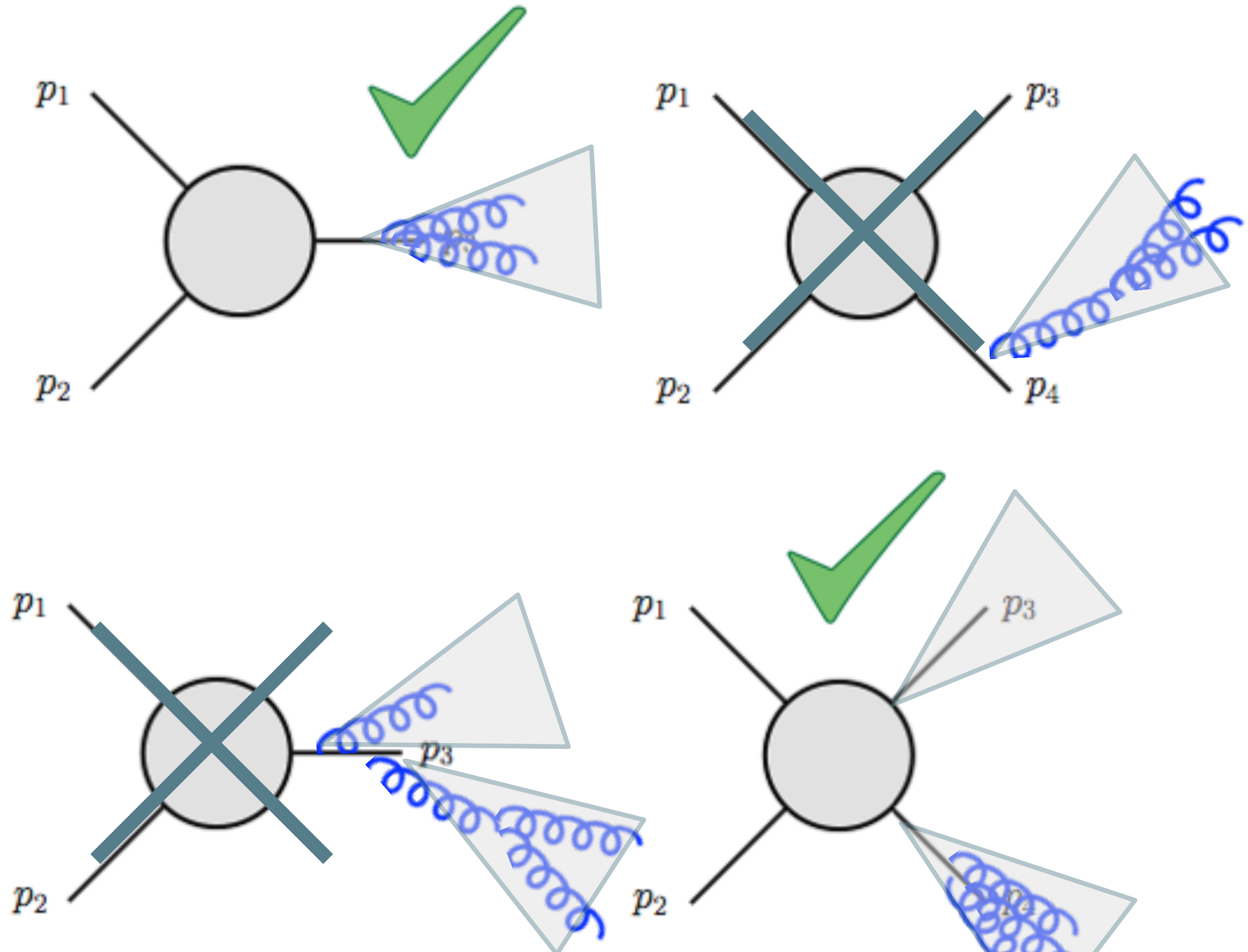
1) define matching cuts:
for example p_T^J > 20 GeV, ΔR=0.4

2) generate ME with 1, 2, …n jets

3) shower all events

4) select only events where jets above the p_T threshold match with final partons

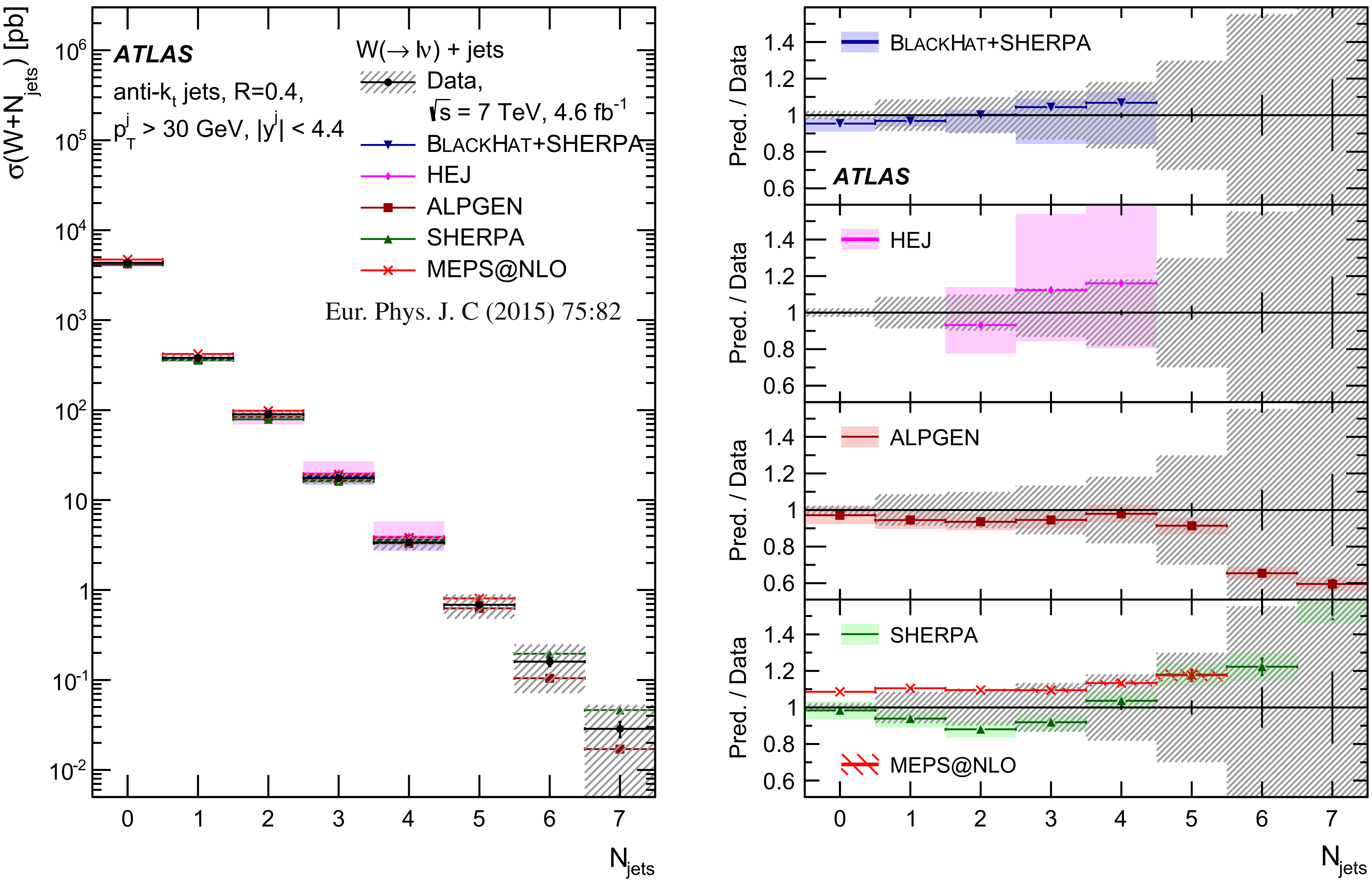
1 parton | 2 partons



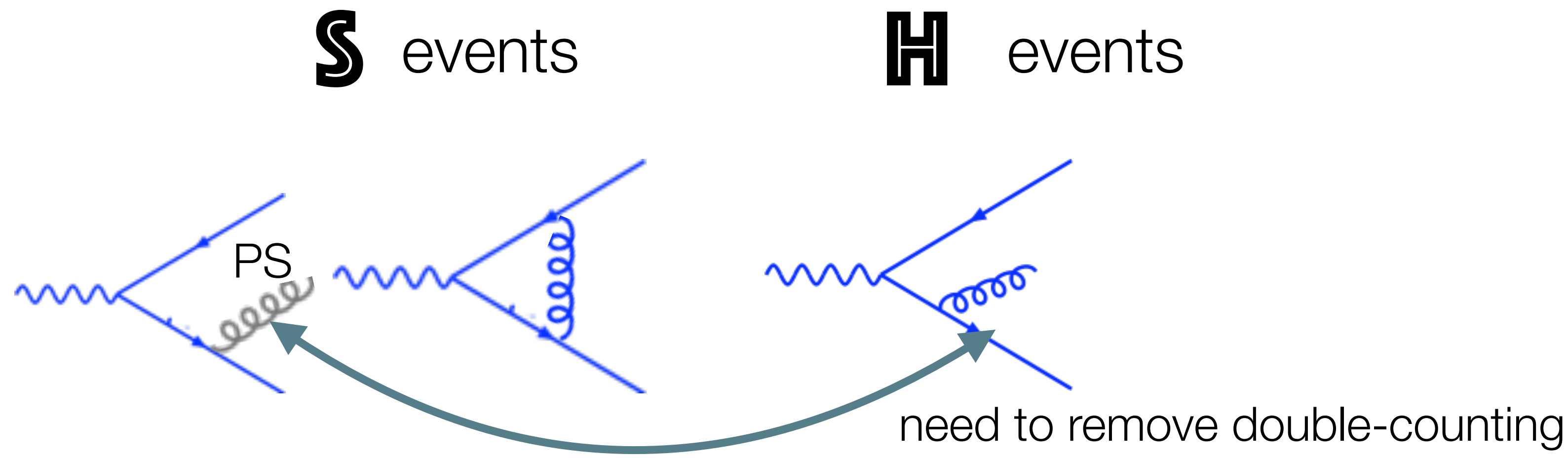
Consequences:

all jets with p_T > 20 GeV and ΔR>0.4 to other jets come from ME
collinear and soft jets come from PS
Use each of them where they are best.

W+jets distributions

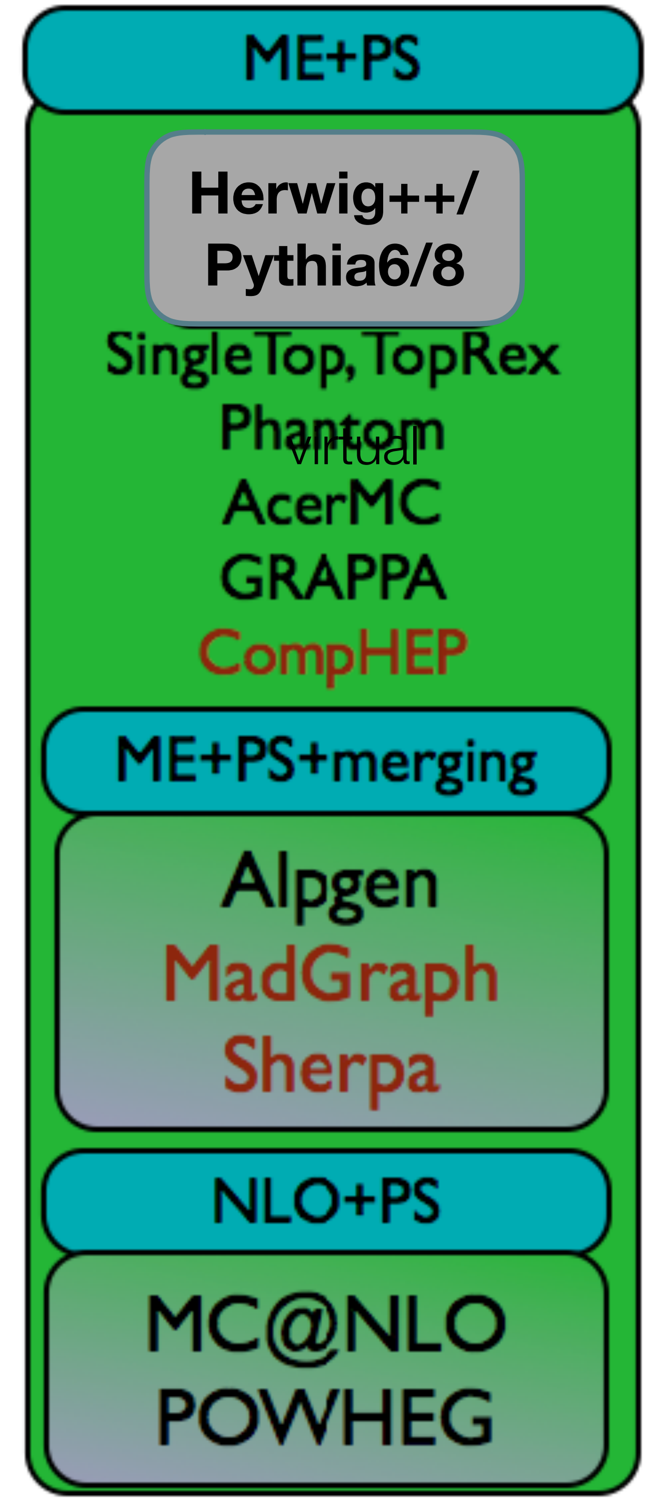


Type III : Next-to-leading order ME & leading-log parton shower

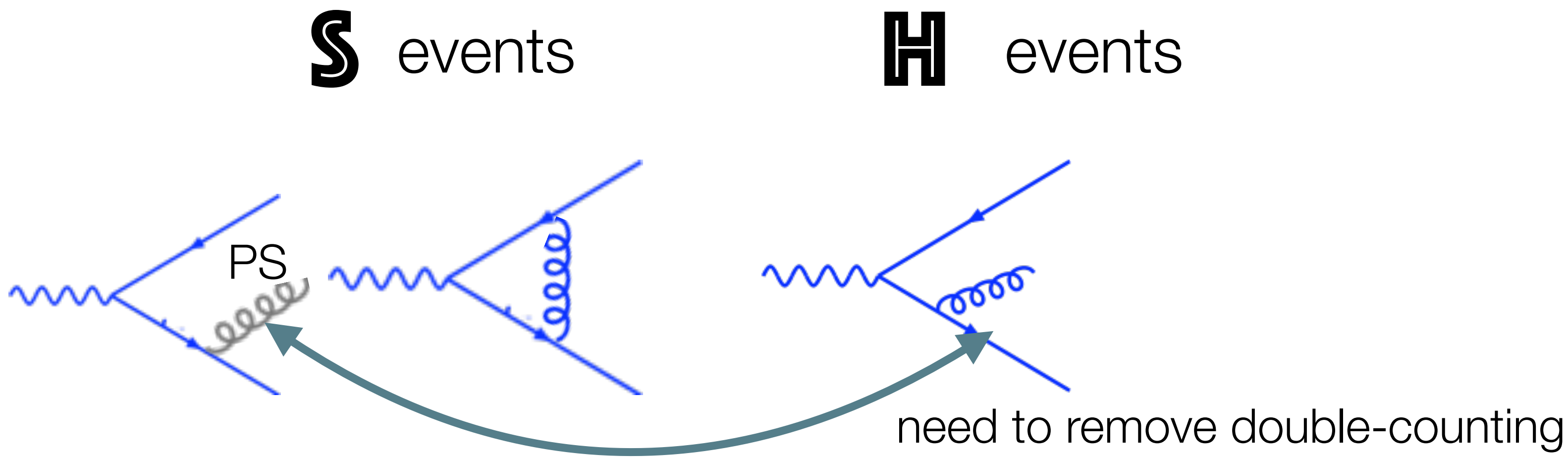


hard processes simulated at NLO accuracy including real & virtual corrections ...

improved description of cross sections & kinematic distributions



Type III : Next-to-leading order ME & leading-log parton shower



hard processes simulated at NLO accuracy including real & virtual corrections ...

improved description of cross sections & kinematic distributions

2 Matching methods:

1. Powheg

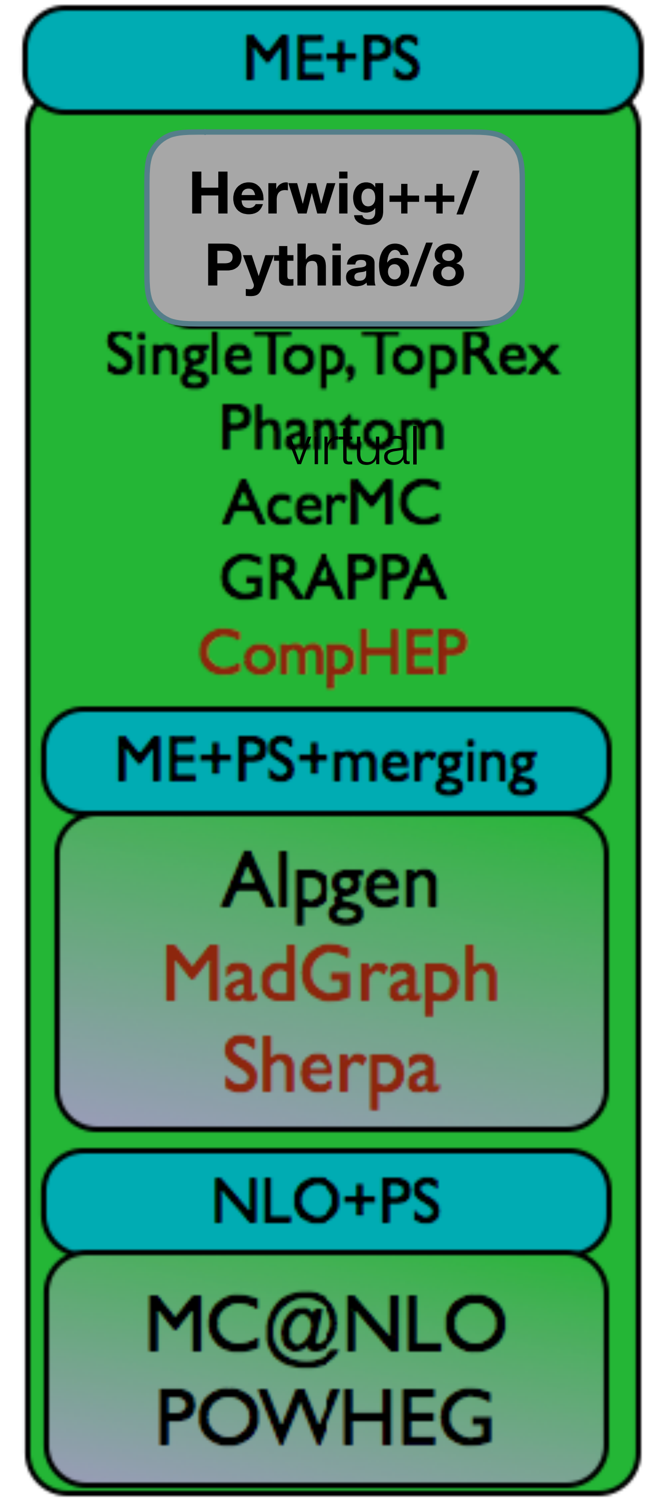
Truncated showers:

1) first emission produced by the ME;
2) don't allow the PS to produce patrons harder than the first emission;
3) not exact at NLO (containes unbalanced higher order terms)

2. MC@NLO:

$$|ME|^2 = |ME + PS - PS(\text{up to } \alpha_s^2)|^2$$

+ Result is exact at NLO…
- produce some negative weights, need retuning for each PS



[F. Maltoni]

Merging @NLO (quite new, going to be used at 13 TeV)

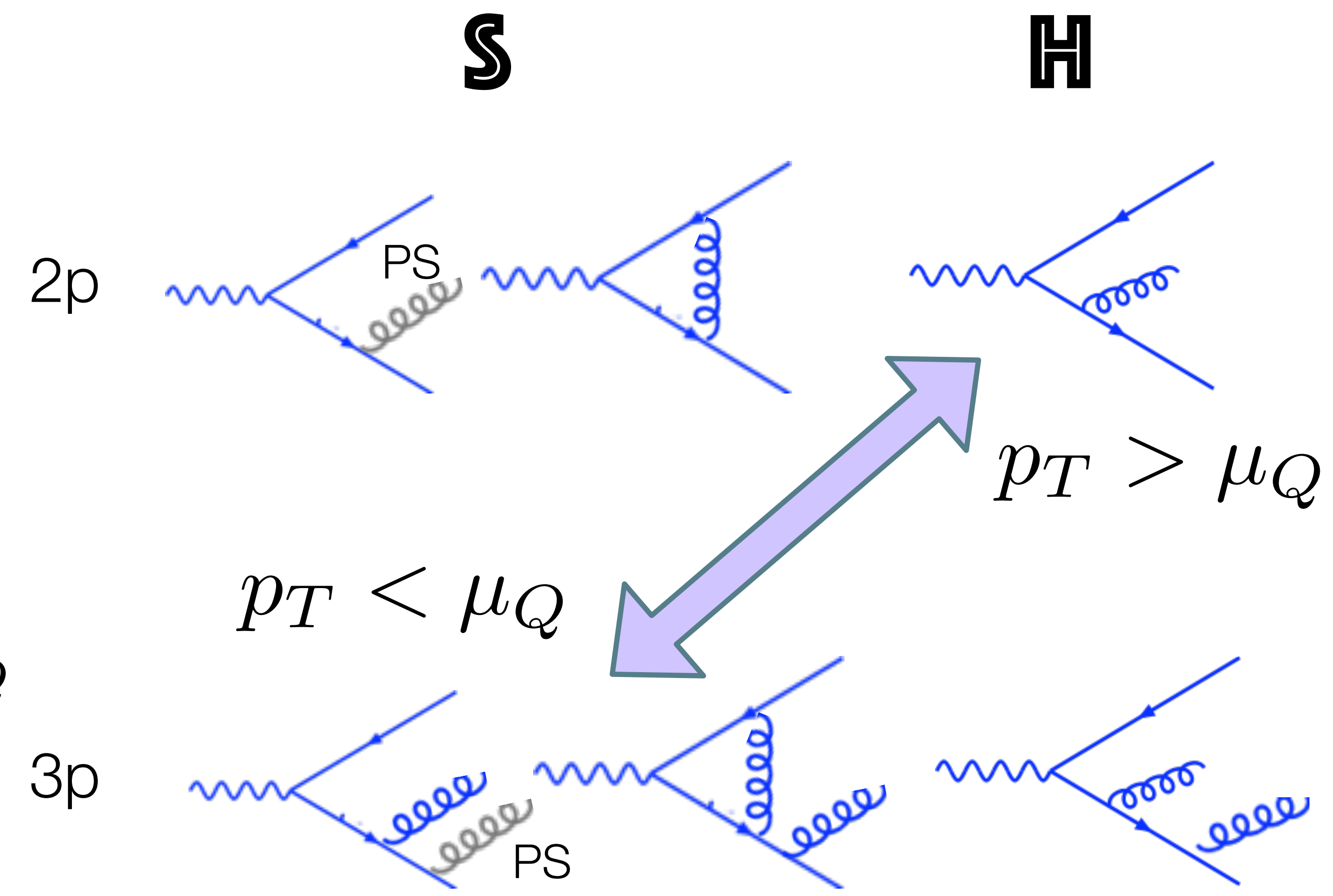
JHEP12(2012)061

FxFx (Frederix-Frixione) merging

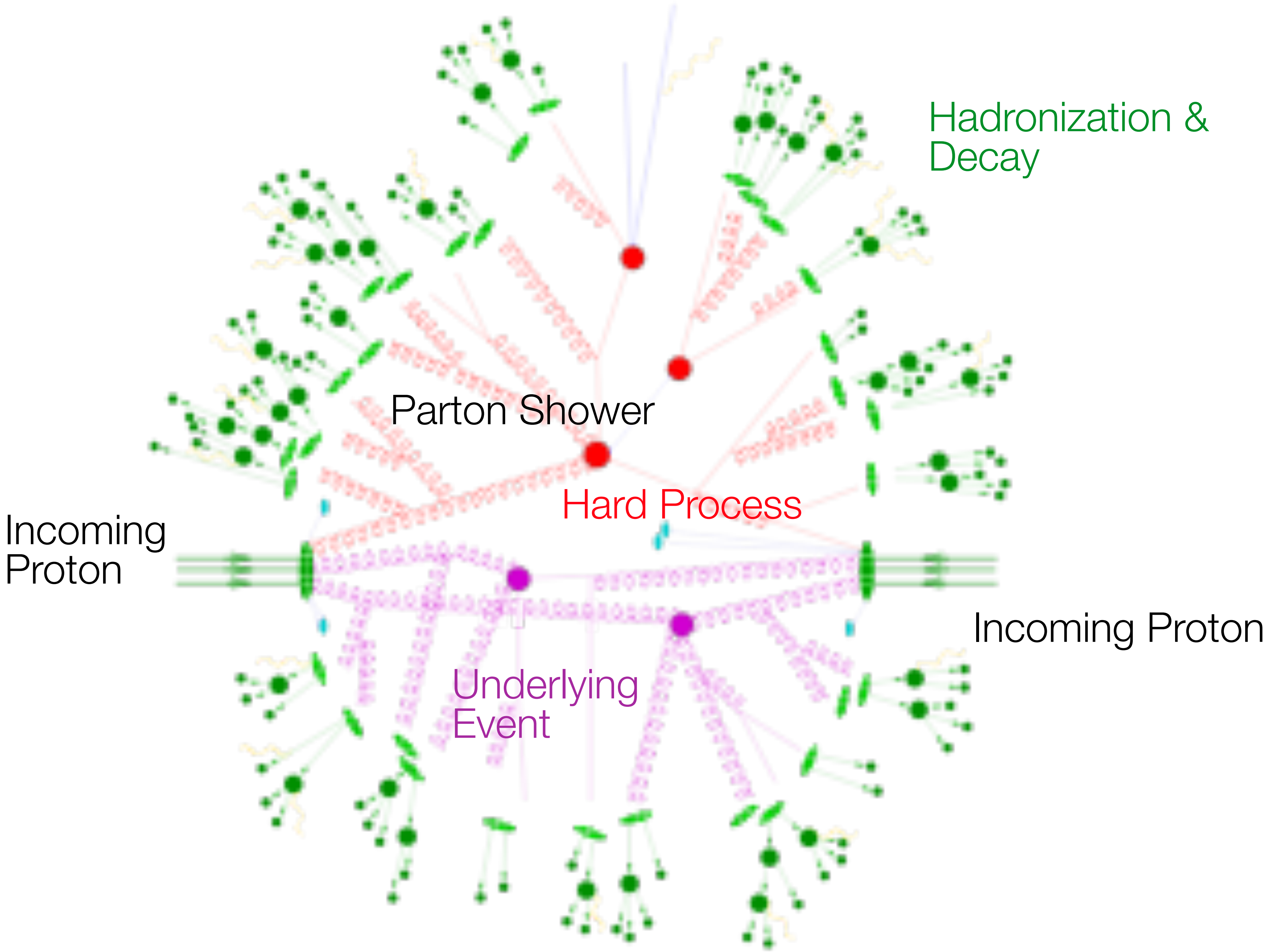
1) define a matching scale μ_Q;

2) don't allow 𝕊 events with $p_T > \mu_Q$ (those will be provided by ℍ events of n-1 partons NLO real emission); the restriction is imposed both at ME and on the shower starting scale $\mu < \mu_Q$

3) treat the obtained events as LO ones and apply an LO-style merging (this allow to produce smoother distributions)



Let's recap



[T. Gleisberg et al., JHEP02 (2004) 056]

From partons to color neutral hadrons

Fragmentation:

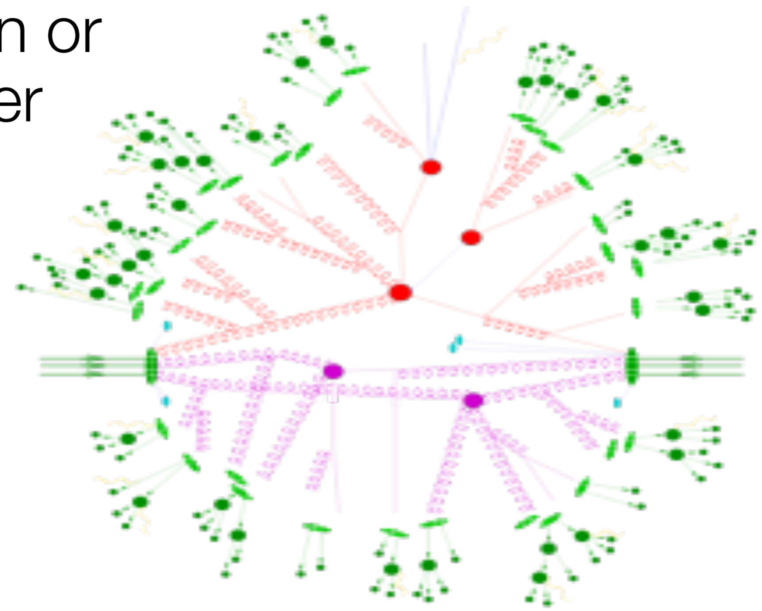
Parton splitting into other partons
[QCD: re-summation of leading-logs]
["Parton shower"]

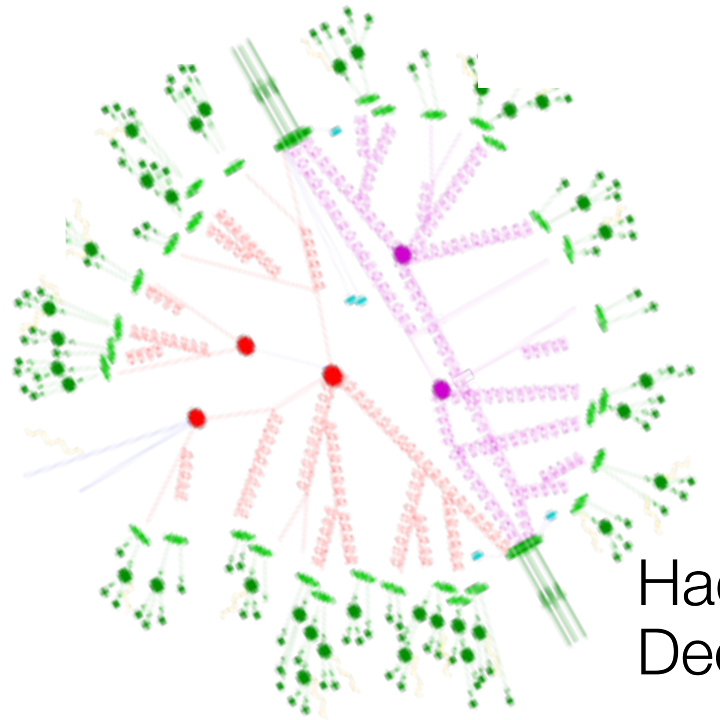
Hadronization:

Parton shower forms hadrons
[non-perturbative, only models]

Decay of unstable hadrons
[perturbative QCD, electroweak theory]

Fragmentation or Parton Shower





Hadronization & Decays

Non-perturbative transition from partons to hadrons ...

[Modelling relies on phenomenological models available]

Models based on MC simulations
very successful:

Generation of complete final states ...
[Needed by experimentalists in detector simulation]

Caveat: tunable ad-hoc parameters

Most popular MC models:

Pythia/8 : Lund string model
Herwig/++ : Cluster model

Independent fragmentation of each parton

Simplest approach:
[Field, Feynman, Nucl. Phys. B136 (1978) 1]

Start with original quark

Generate quark-antiquark pairs from vacuum

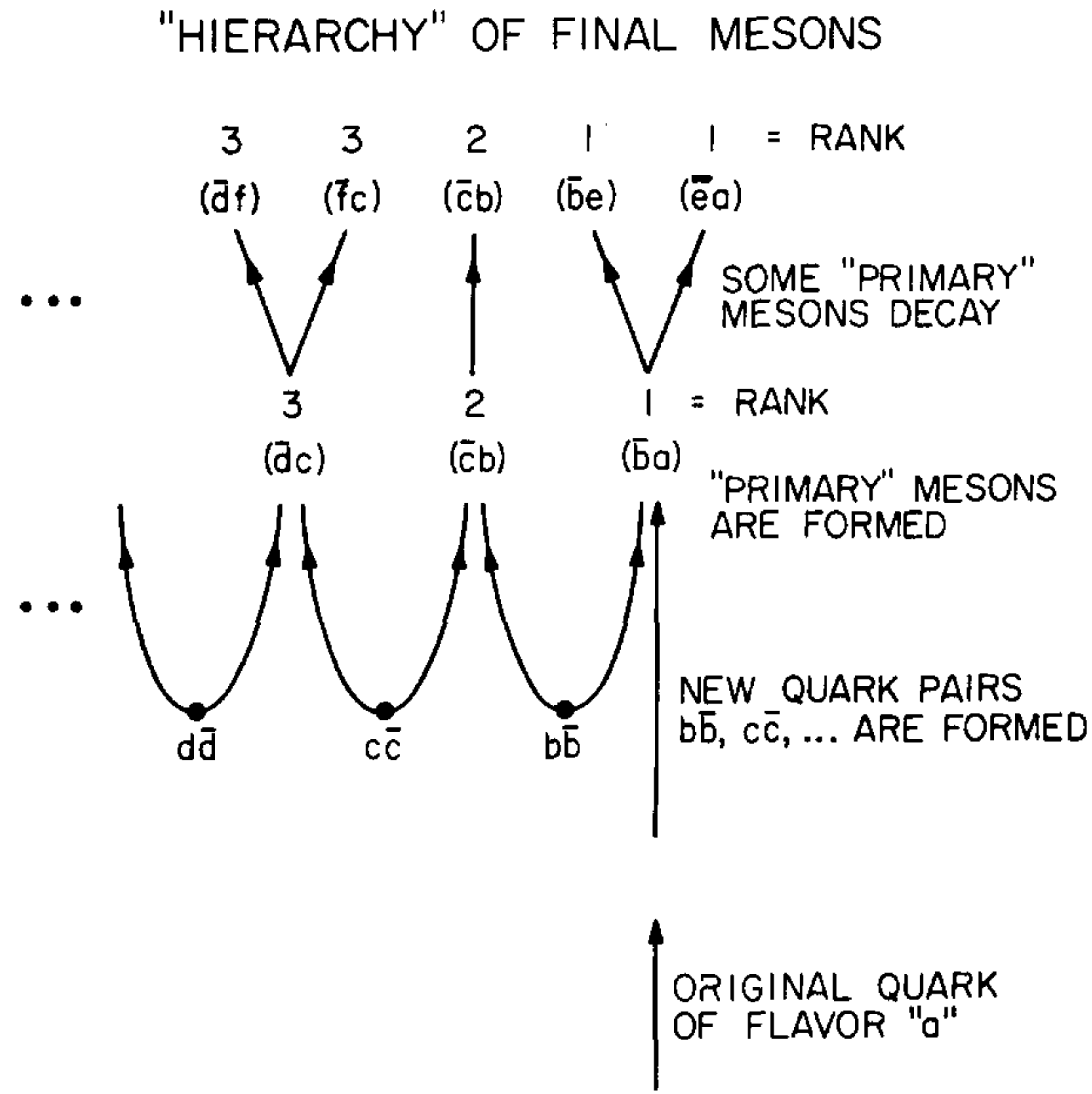
➛ form "primary meson" with energy fraction z

Continue with leftover quark with energy fraction 1–z

Stop at low energies (cut-off)

Include flavour non-perturbative fragmentation functions D(z)

D(z): probability to find a meson/hadror with energy fraction z in jet ...



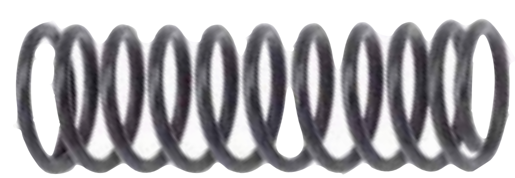
Lund String Model

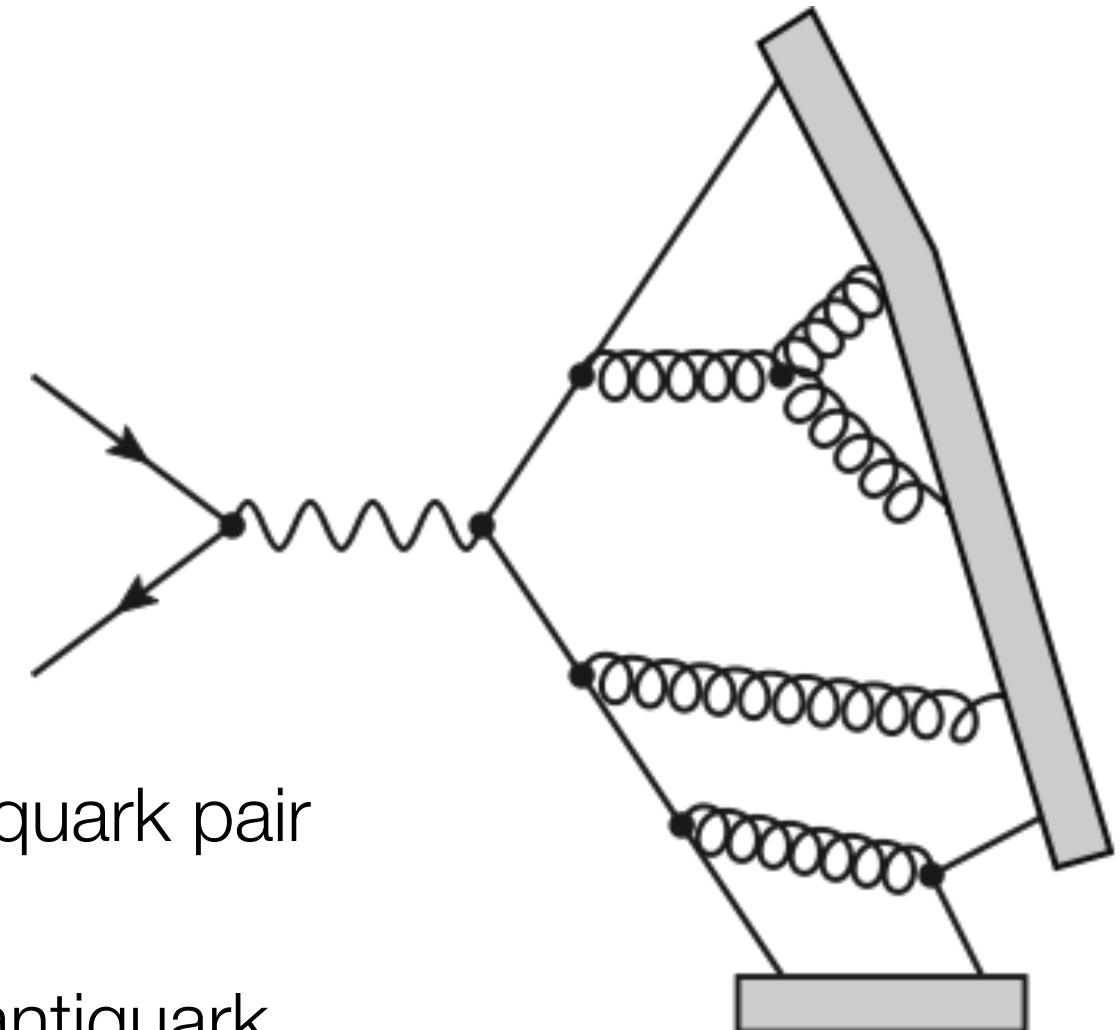
[Andersson et al., Phys. Rep. 97 (1983) 31]

QCD potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s(1/r^2)}{r} + kr$$

neglected





After: Ellis et al.,
QCD and Collider Physics

String formation between initial quark-antiquark pair

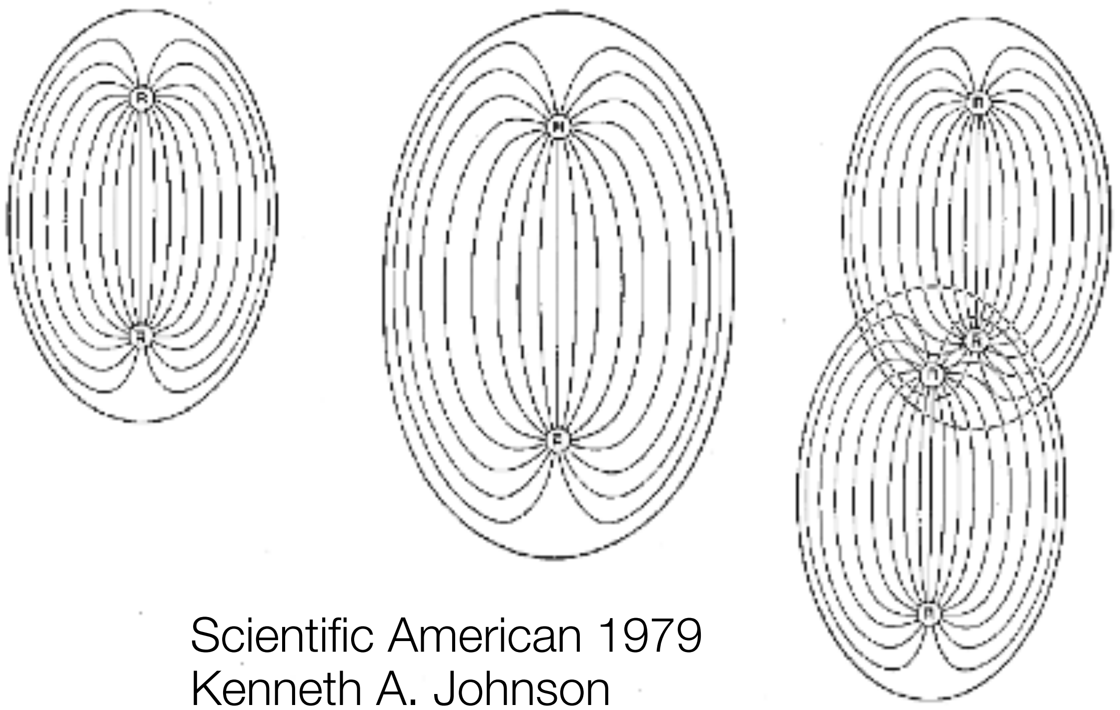
- String breaks up if potential energy large enough to produce a new quark-antiquark pair
- Gluons = 'kinks' in string
- At low energy: hadron formation
- Very widely used ...
  [default in Pythia 6/8]

Lund String Model

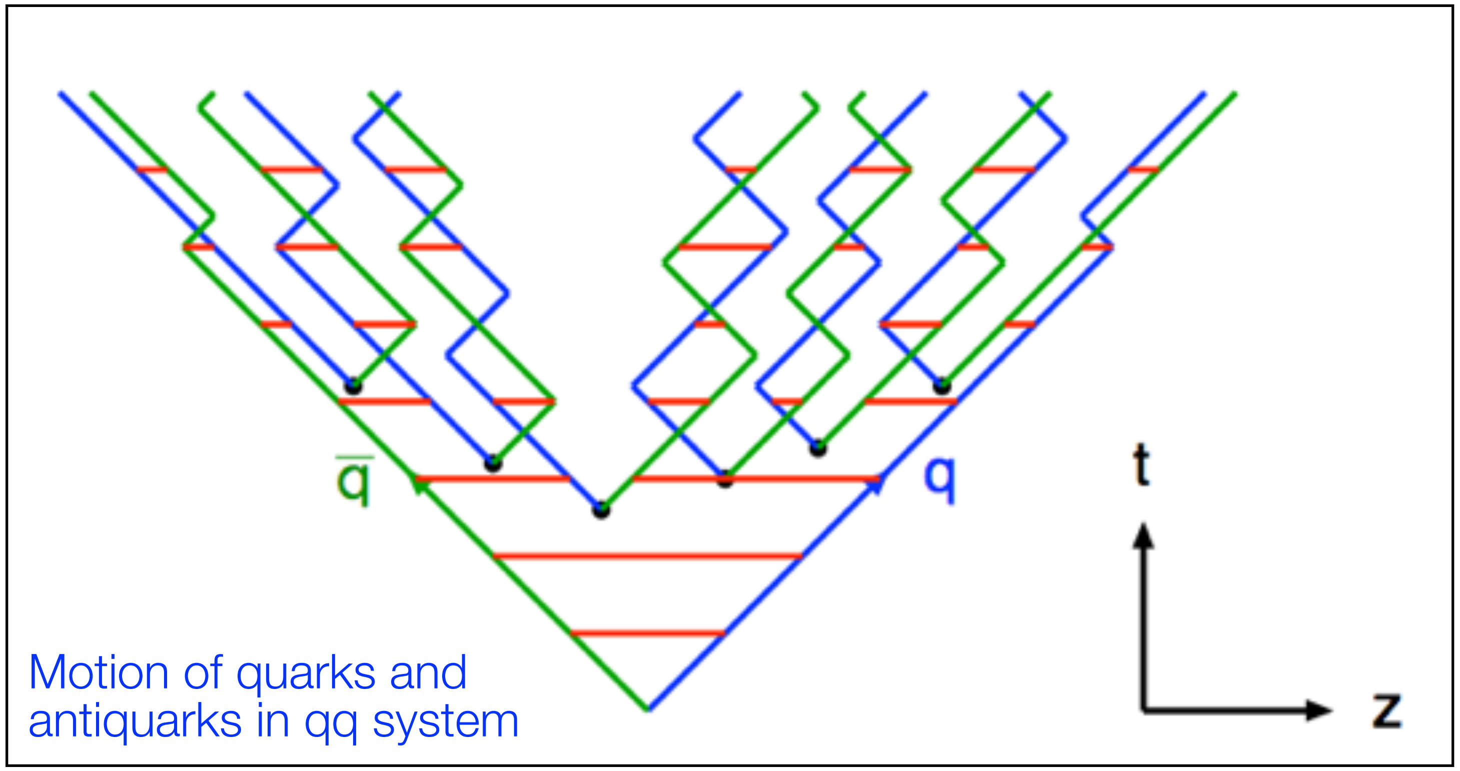
Repeated string breaks for large system with pure V(r) = κ·r, i.e. neglect Coulomb part

$$\left|\frac{dE}{dz}\right| = \left|\frac{dp_z}{dz}\right| = \left|\frac{dE}{dt}\right| = \left|\frac{dp_z}{dt}\right| = \kappa$$

Energy-momentum quantities can be read from space-time quantities ...



Scientific American 1979
Kenneth A. Johnson



Simple but powerful picture of hadron production

[with extensions to massive quarks, baryons, ...]

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right)$$
$$\propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

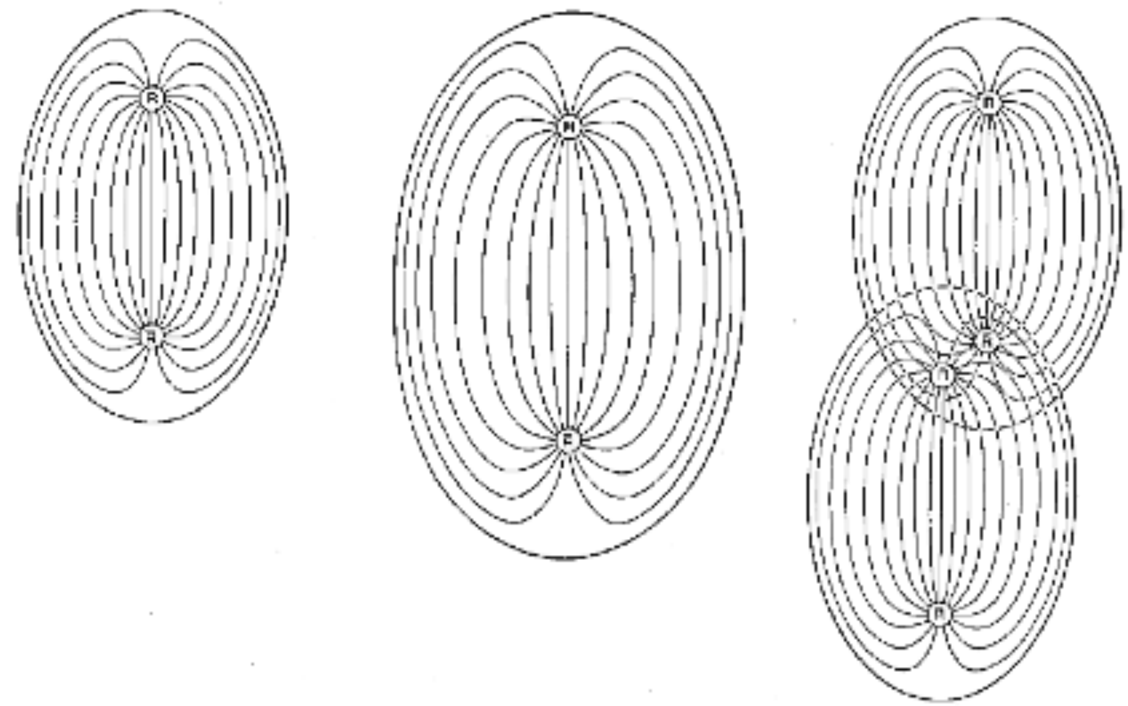
Yields: Common Gaussian p⊥ spectrum
Heavy quark suppression

Lund String Model

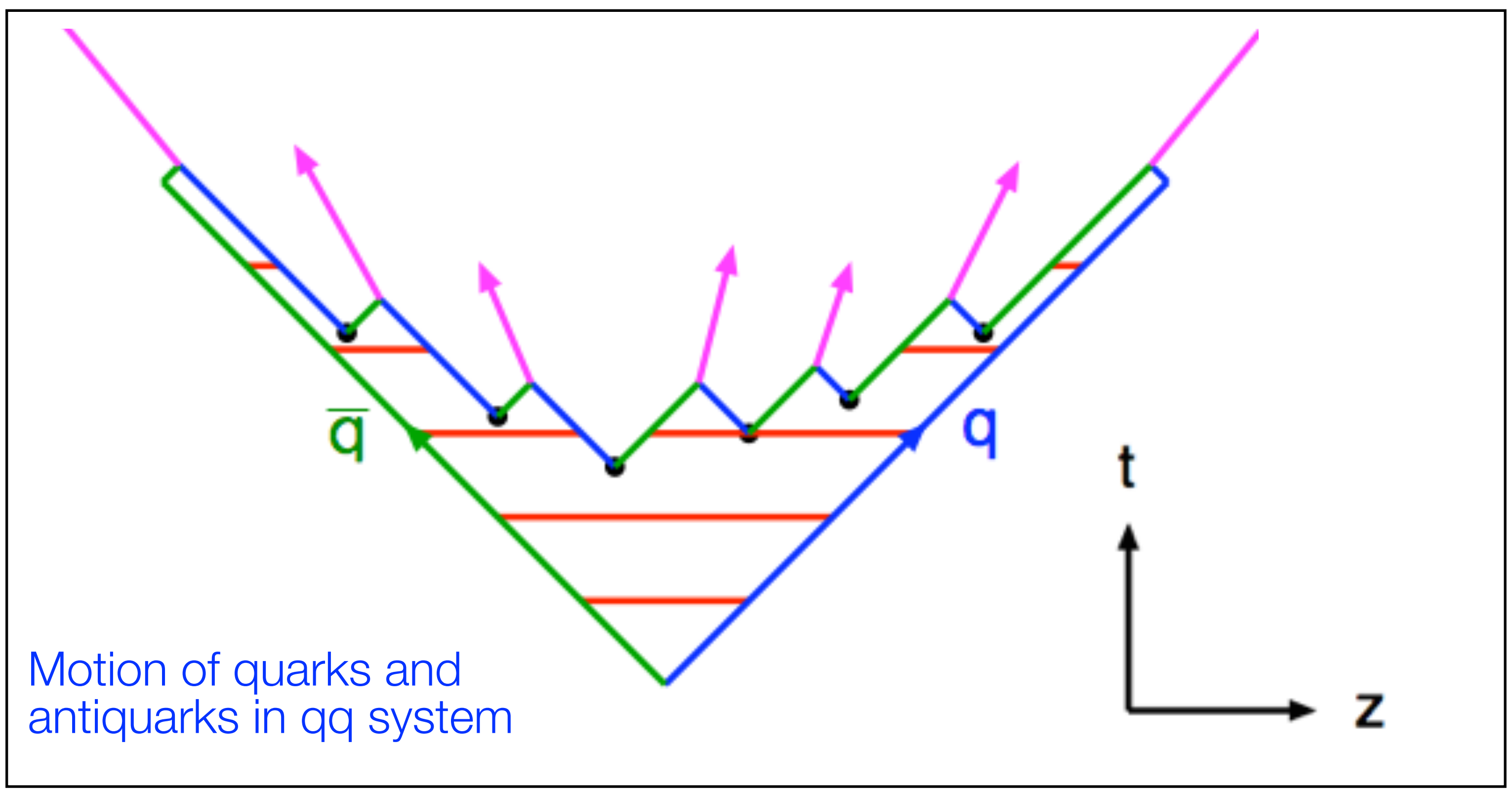
Repeated string breaks for large system with pure V(r) = κ·r, i.e. neglect Coulomb part

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

Energy-momentum quantities can be read from space-time quantities ...



Kenneth A. Johnson



Simple but powerful picture of hadron production

[with extensions to massive quarks, baryons, ...]

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right)$$
$$\propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

Yields: Common Gaussian p⊥ spectrum
Heavy quark suppression

Cluster Model

[Webber, Nucl. Phys. B238 (1984) 492]

Color flow confined during hadronisation process

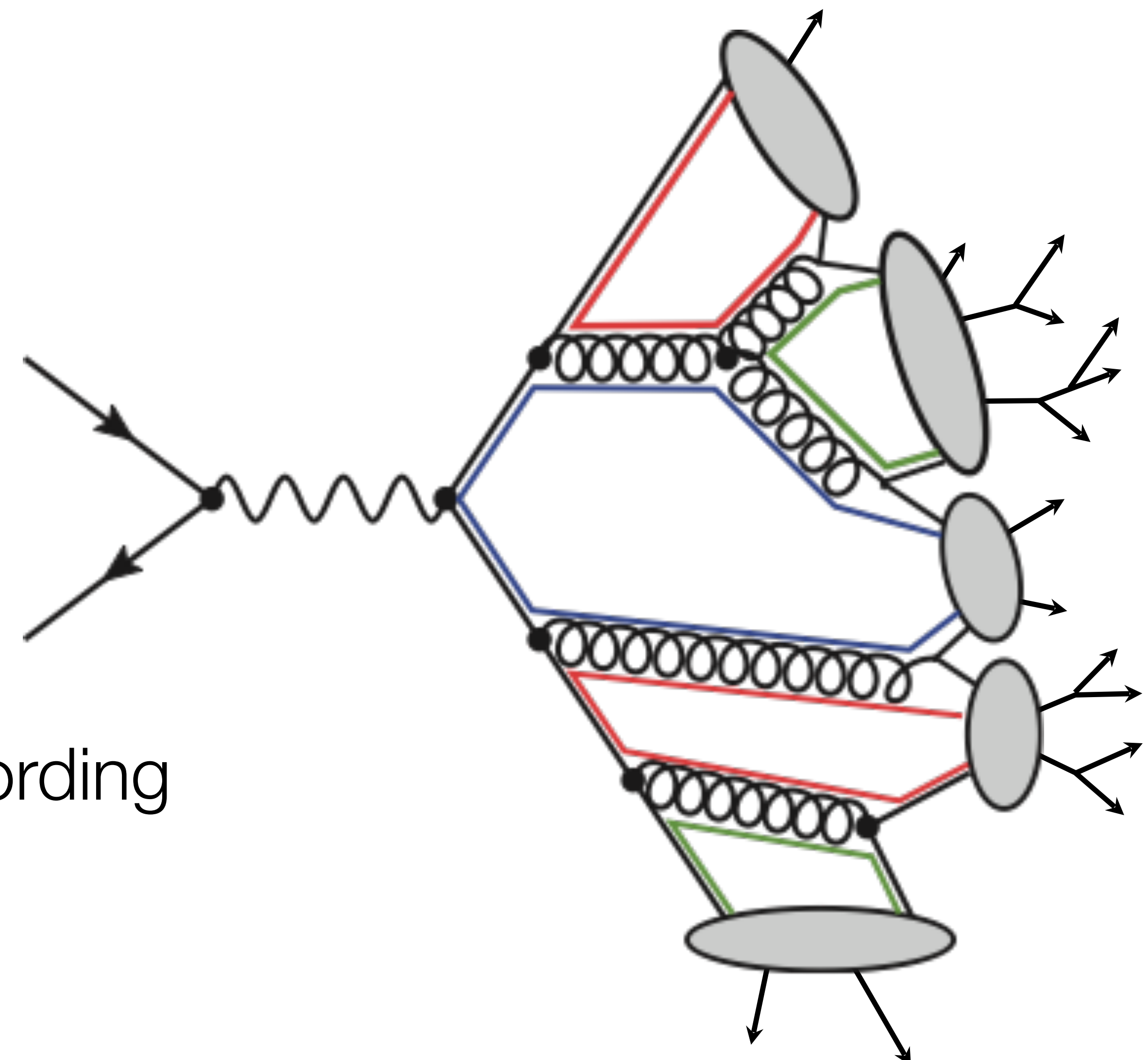
➢ Formation of color-neutral parton clusters

Gluons (color-anticolor) split to quark-antiquark pairs

Clusters decay into 2 hadrons according to phase-space, i.e. isotropically

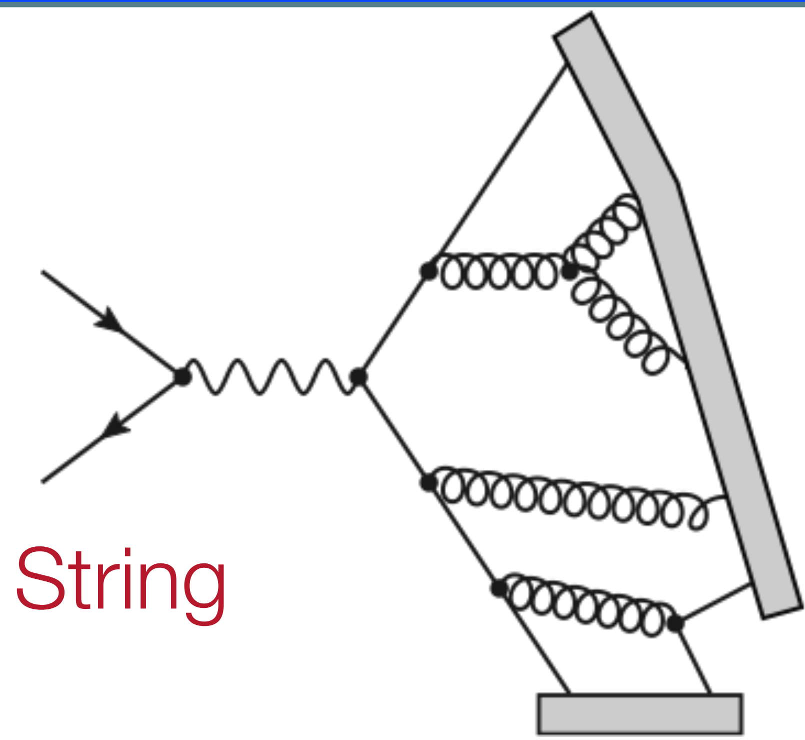
➢ no free tuning parameters parton clusters

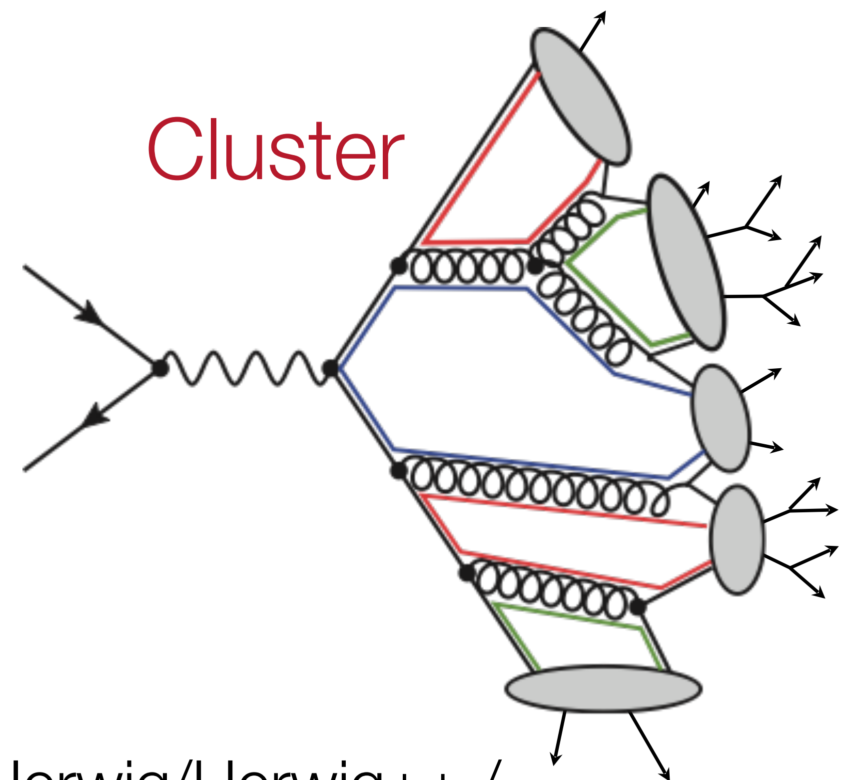
Very widely used ...
[default in Herwig/Herwig++]



After: Ellis et al.,
QCD and Collider Physics

Hadronisation models summary





Model	Pythia6/8 (string)	Herwig/Herwig++ / Sherpa(cluster)
Energy-mom. picture	powerful	simple
	predictive	unpredictive
Parameters	few	many
Flavour composition	messy	simple
	unpredictive	in-between
Parameters	many	few

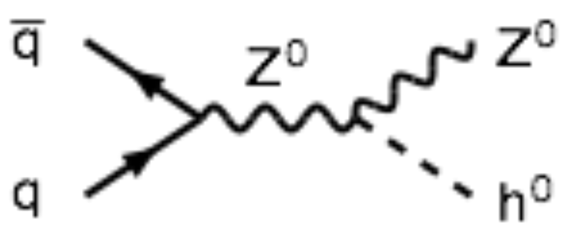
[from G.Herten,T.Sjöstrand]

Structure of basic generator process [by order of consideration]

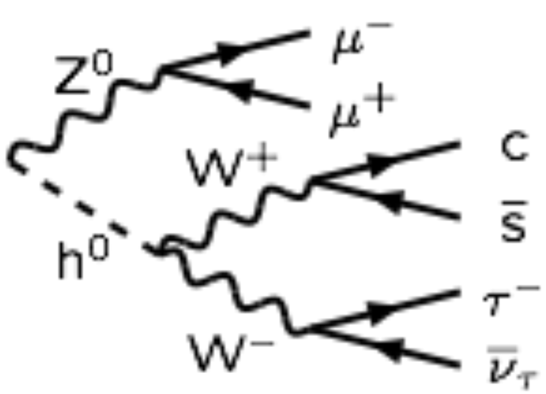
From the 'simple' to the 'complex' or
from 'calculable' at large scales to 'modelled; at small

Matrix elements (ME)

1. Hard subprocess:
   $|M|^2$, Breit Wigners, PDFs

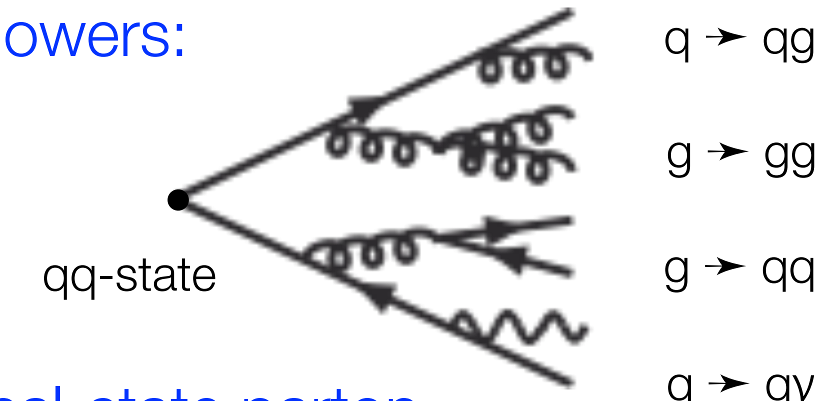


2. Resonance decays:
   Includes particle correlations

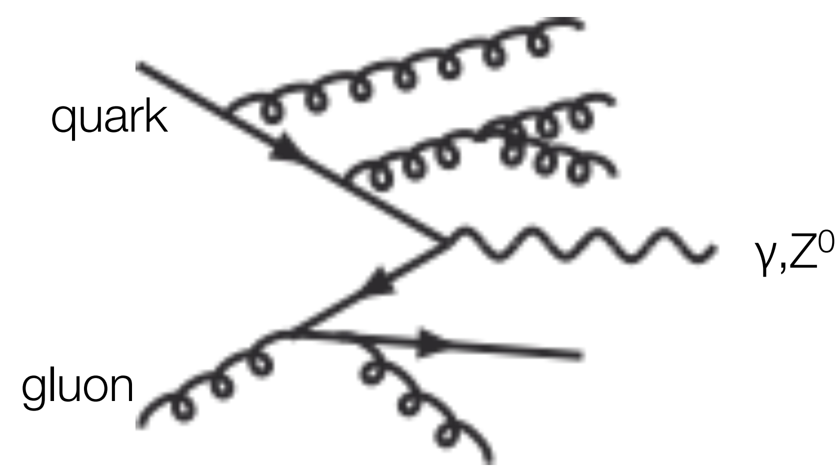


Parton Shower (PS)

3. Final-state parton
   showers:



4. Final-state parton
   showers:



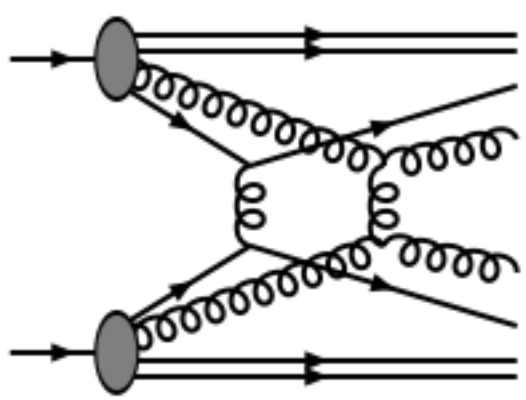
[from G.Herten]

Conclusions: Structure of basic generator process

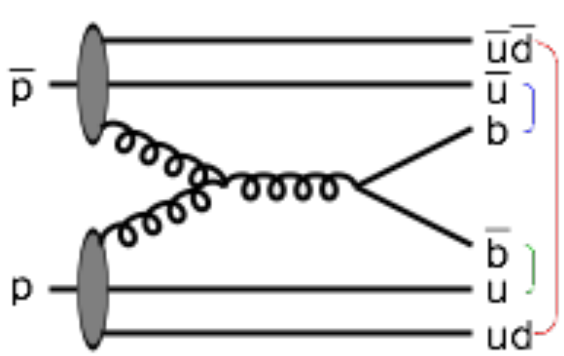
From the 'simple' to the 'complex' or
from 'calculable' at large scales to 'modelled; at small

Underlying Event (UE)

5. Multi-parton interaction:

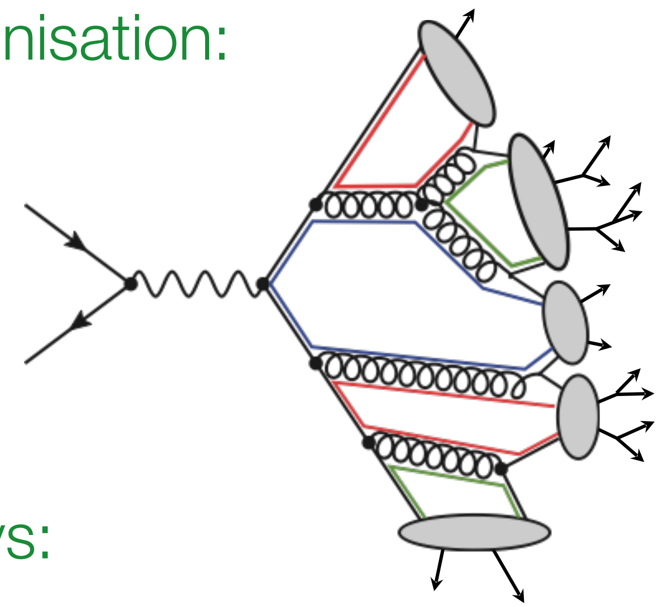


6. Beam remnants:

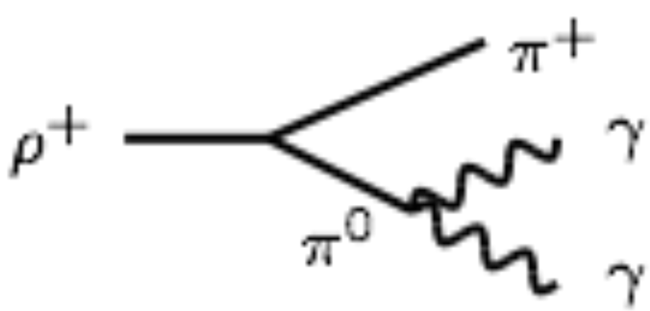


Stable Particle State

7. Hadronisation:



8. Decays:



[from G.Herten]

The DGLAP evolution equation is said to resum large collinear logarithms. So where are these logarithsm, and where is the resummation?

Let's perform the integration of the DGLAP equation and expand the result:

$$
\begin{aligned}
f(x,t) &= f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \int_x^1 \frac{dz}{z} P(z)\, q\left(\frac{x}{z}, t'\right) \\
&= f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \Bigg\{ f_0\left(\frac{x}{z}\right) + \\
&\qquad + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') \left[f_0\left(\frac{x}{zz'}\right) + \ldots \right] \Bigg\} \\
&= f_0(x) + \frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \int_x^1 \frac{dz}{z} P(z)\, f_0\left(\frac{x}{z}\right) + \\
&\qquad + \frac{1}{2!} \left[\frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right)\right]^2 \int_x^1 \frac{dz}{z} P(z) \int_{x/z}^1 \frac{dz'}{z'} P(z')\, f_0\left(\frac{x}{zz'}\right) + \ldots
\end{aligned}
$$

As suggested by the last step, it is indeed a resummation of all terms proportional to $\left[\frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right)\right]^n$.

[from J.Alwall]