Conformal Bootstrap: strong dynamics under siege

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Perturbation Theory

Most of our understanding of physical systems comes from perturbation theory

EX: Anharmonic Oscillator

$$H = \underbrace{\frac{p^2}{2m} + \frac{mw^2}{2}q^2}_{\text{exactly solvable model}} + \underbrace{\lambda q^4}_{\text{small interaction (λ)}}$$

$$\Rightarrow \qquad \mathsf{Observables} = \mathcal{O}_0 + \sum_{n=1}^{\infty} \lambda^n \delta \mathcal{O}_{(n)}$$



however...

- \blacktriangleright The perturbative series is asymptotic: still useful for $\lambda \ll 1$
- In (way too many) interesting cases the perturbation is not small: what do we do then?

Numerical Strong dynamics toolbox

MonteCarlo simulations



► Lattice Field theory



► Hamiltonian Truncation

Theoreticians' Strong dynamics toolbox

 Introduce large parameters in the game: number of fields N, number of dimensions D
 —> solve the theory in the infinite limit

 Study subsectors of a theory with large quantum numbers (spin, electric charge,...)
 —> system becomes semi-classical, i.e. dominated by classical solutions

► Exploit the power of additional symmetries ex: supersymmetry → holomorphicity, localization,...

this talk: study consequences of scale invariance





SCALE INVARIANT SYSTEMS

In a nutshell, scale invariant systems are characterised by the same behaviour at all length scales.

Static configurations \Rightarrow property of self-similarity







dynamical system \Rightarrow "equations" don't depend on scales



Optics \checkmark



Pendulum X

Classical Vs Quantum

Quantum mechanics changes the rules of the game

- classical scale invariance broken at quantum level
- coupling constants depend on the probing energy



 Under special circumstances scale invariance can still be realised at quantum level



Conformal Field Theories in Nature





Conformal Field Theories in Nature



Scale invariance \Rightarrow power law behaviour close to T_c :

$$\langle \textit{Observable}
angle \sim (T-T_c)^{\#}$$

EX:

Correlation Length:

$$\xi \sim (T - T_c)^{-\nu}$$

Heat Capacity: $C \sim (T-T_c)^{2-3
u}$

Magnetic Susceptibility: $\chi \sim (T-T_c)^{(2-\eta)/
u}$

Q: how to compute the critical exponents η , ν ?

An example: the Ising model in 3D





From Scale to Conformal Invariance

In all known unitary theories:

 $\mathsf{Scale Invariance} \Rightarrow \mathsf{Conformal Symmetry}$



Preserves angles but not distances

The AdS/CFT correspondence relates QFT with conformal invariance to string theory in Anti-deSitter background



Usually: Classical gravity \Rightarrow Strongly coupled CFTs

Lately: 2D CFTs \Rightarrow Black holes physics

What are CFTs?

Theories invariant under the conformal algebra SO(D|2) which includes:

- translations
- Lorentz transformations
- dilatations
- "inversion"

They are described by three ingredients:

$$\mathcal{O}_{\Delta,\ell}$$
 spin
imension in energy

2) Interactions between operators:

$$\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k \overbrace{\mathcal{O}_j}^{\mathcal{O}_i} \cdots \underset{k \to 0}{\mathcal{O}_k \sim \sum_k C_{ijk} \mathcal{O}_k} \circ \sum_{\substack{k \to 0 \\ \text{Operator Product Expansion (OPE) coefficients}}$$

d

3) Crossing symmetry constraints: see next slides...

In CFT we are interested in computing correlations functions (observables averaged over states of the theory)

$$\mathcal{O}_{i}(x_{1})...\mathcal{O}_{j}(x_{n})\rangle \longleftrightarrow \sum_{\text{states s}} \langle s|O_{i}(x_{1})...\mathcal{O}_{j}(x_{n})|s\rangle e^{-E_{s}/T}$$
 (1)

fixed by symmetry
$$\begin{cases} \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle & \checkmark \\ \\ \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle & \checkmark \end{cases}$$

encode dynamics

 $\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\mathcal{O}_l(x_4)\rangle$

Use OPE to reduce higher point functions to smaller ones:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}} \rangle^{\mathcal{O}} \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}} \rangle^{\mathcal{O}} \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \rangle \sim \sum_{\mathcal{O}} \rangle^{\mathcal{O}} \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \rangle \rangle \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_1)\mathcal{$$

Crossing symmetry: two expansions must give the same result! (Constraint on spectrum and interactions)

A CFT is an infinite set of primary operators $\mathcal{O}_{\Delta,\ell}$ and OPE coefficients C_{ijk} that satisfy crossing symmetry for all set of four-point functions.

Q: What choices of CFT data are consistent?



- Original formulation was very ambitious: solve exactly CFTs
- Shift of paradigm: "a few" better than "nothing" (quantitative informations can be obtained without solving the theory)
- ▶ "A few" can lead to "all"





Numerical Bootstrap

► Crossing equation for (O(x₁)O(x₂)O(x₃)O(x₄)):







Existence of Λ can be recast into a *convex optimization problem* and checked numerically using linear (or semi-definite) programming algorithms [Rattazzi,Rychkov,Tonni, AV '08] [Poland,Simmons-Duffin, AV '11]



For unfeasible spectra the cone generated by all the vectors $F_{\Delta,\ell}$ is entirely contained in half-space

Look for a Linear functional

$$\Lambda[F_{\Delta,\ell}] \equiv \sum_{n,m}^{N_{\max}} \lambda_{mn} \partial^n \partial^m F_{\Delta,\ell}$$

such that

 $\Lambda[F_{\Delta,\ell}] > 0 \qquad \text{for any } \Delta\ell, \text{ except for} \qquad \Lambda[F_{\Delta_*,\ell_*}] < 0$

- ► *N*_{max}: parametrizes numerical complexity
- ▶ as N_{max} increases, exclusion conditions get stronger

Rules of the game:

- Choose one or more operators $\mathcal{O}_1, \mathcal{O}_2, \dots$
- ► Consider all four point functions containing those operators $< O_1 O_1 O_1 O_1 O_1 >, < O_1 O_1 O_2 O_2 >, ...$
- ► Make assumptions on the operators (and coefficients) appearing in the OPE's O_i × O_j
- ► Check numerically if assumptions are consistent with crossing symmetry
- ▶ If not consistent: no CFT with that operator content

A few applications

Scalars $\sigma \varepsilon$, with OPE: $\sigma \times \sigma \sim 1 + \epsilon + \dots$ $\sigma \times \epsilon \sim \sigma + \dots$ $\epsilon \times \epsilon \sim 1 + \epsilon + \dots$

Q: given Δ_{σ} , how large can Δ_{ε} be? A: study $< \sigma\sigma\sigma\sigma\sigma >, < \sigma\sigma\epsilon\epsilon >, < \epsilon\epsilon\epsilon\epsilon >$



Minimal models (exactly solvable) saturate the bound

Important

No use of Virasoro algebra. Extend the method to 3D right away

Q: given Δ_{σ} , how large can Δ_{ε} be? A: study $< \sigma\sigma\sigma\sigma\sigma >, < \sigma\sigma\epsilon\epsilon >, < \epsilon\epsilon\epsilon\epsilon >$



[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, AV] 2011

Can we narrow down the 3D Ising model?

So far we have assumed anything about the CFT besides unitarity.

The 3D Ising model has only two relevant perturbations

Q: if only σ and ϵ have dimension \leq 3: allowed values for Δ_{σ} , Δ_{ϵ} ?



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So far we have assumed anything about the CFT besides unitarity.

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[Kos,Poland,Simmons-Duffin,AV '15]

Why does it work?

Only a finite number of operators effectively matter

Decoupling of high dimensional operators:

s-channel:
$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}} \bigvee_{e^{-\#\Delta}f_s(x_i)}^{0}$$

t-channel: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}} \bigvee_{e^{-\#\Delta}f_t(x_i)}^{0}$

[Rychkov, Yvernay '15]

[Pappadopulo,Rychkov,Espin,Rattazzi '12]

In the numerical bootstrap functions $f_{s,t} \sim O(1)$

Large dimension operators exponentially suppressed!

Numerical bootstrap gives much more than exclusion plots:

when the allowed region is "small" the whole spectrum can be reconstructed

7.	11	Δ	$\tau = \Lambda = \ell$	f a	1	\mathbb{Z}_2	l	Δ	
142	2	4 190205(18)	2 100205(10)	0.20015041(01)		+	2	3	_
12	2	4.180505(18)	1.63804(88)	0.385(34)		+	4	5.022665(28))
-	1	4.03804(88)	2.112674(10)	0.1353(34) 0.1077052(16)		+	6	7.028488(16))
1	1	6.700778(27)	1.700778(27)	0.1011032(10)		+	8	9.031023(30))
1	6	0.109116(21)	2.08007(25)	0.04191343(88)		+	10	11.0324141(99)
-	2	8.08097(25) 8.747202(56)	2.08097(25)	0.0280902(80)		+	12	13.033286(1	2)
-	6	0.141255(50)	2.0622(20)	0.01101203(13)		+	14	15.033838(1	5)
-	l °	10.0023(29)	1.77075(29)	0.00743(21)		+	16	17.034258(3	4)
1	3	10.11013(30)	1.11013(30)	0.003113(12)		+	18	19.034564(1)	2)
-	10	12.0492(18)	2.0492(18)	0.001940(19)		+	20	21.0347884(8	s4)
-	11	12.787668(92)	1.787008(92)	0.000823634(82)		+	22	23.034983(1)	1)
-	12	14.0383(33)	2.0383(33)	0.0004983(88)		+	24	25.035122(1)	ń
-	1.3	14.80006(51)	1.80006(51)	0.0002150(10)		+	26	27.035249(1)	ń
-	14	16.0305(12)	2.0305(12)	0.0001291(12)		+	28	29.035344(19	9)
-	15	10.81009(16)	1.81009(16)	0.000055870(15)		+	30	31.035452(10	6)
-	16	18.025(11)	2.025(11)	0.0000313(30)		+	32	33 035473(2)	8)
-	17	18.81794(18)	1.81794(18)	0.0000144219(91)		+	34	35.035632(6)	7)
-	18	20.01947(94)	2.01947(94)	8.442(28) · 10 *		+	36	37.035610(4)	ň
-	19	20.8240(11)	1.8240(11)	3.090(54) - 10 -		+	38	39.035638(5)	sí –
-	20	22.0152(36)	2.0152(36)	$2.131(28) \cdot 10^{-6}$		+	40	41 03564(13)	ĩ.
-	21	22.83035(11)	1.83035(11)	$9.5120(13) \cdot 10^{-4}$		-	-		
		the second state (second)							
-	22	24.01143(53)	2.01143(53)	$5.4746(61) \cdot 10^{-7}$	l r	Zo	l	Δ	τ :
-	22 23	24.01143(53) 24.83518(65)	2.01143(53) 1.83518(65)	$5.4746(61) \cdot 10^{-7}$ $2.428(11) \cdot 10^{-7}$	ŀ	Z ₂ +	£ 4	Δ 6.42065(64)	$\tau = 2.4$
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	22 23 24 25	24.01143(53) 24.83518(65) 26.00809(94) 26.8394(13)	2.01143(53) 1.83518(65) 2.00809(94) 1.8394(13)	$5.4746(61) \cdot 10^{-7}$ $2.428(11) \cdot 10^{-7}$ $1.3908(17) \cdot 10^{-7}$ $6.16(18) \cdot 10^{-8}$		Z ₂ + + +	ℓ 4 6 8	Δ 6.42065(64) 8.4957(75) 10.562(12)	τ = 2.4 2.4 2.5
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	22 23 24 25 26 27 28	24.01143(53) 24.83518(65) 26.00809(94) 26.8394(13) 28.0045(17) 28.84330(31) 30.0042(38)	2.01143(53) 1.83518(65) 2.00809(94) 1.8394(13) 2.0045(17) 1.84330(31) 2.0042(38)	$\begin{array}{c} 5.4746(61)\cdot 10^{-7}\\ 2.428(11)\cdot 10^{-7}\\ 1.3908(17)\cdot 10^{-7}\\ 6.16(18)\cdot 10^{-8}\\ 3.523(20)\cdot 10^{-8}\\ 1.5809(50)\cdot 10^{-8}\\ 8.86(18)\cdot 10^{-9}\\ \end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75)	$\tau = 2.4$ 2.4 2.5 2.5 2.6 2.6
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	22 23 24 25 26 27 28 29 30 31 32 33 34	24.01143(53) 24.83518(65) 26.00809(94) 26.8394(13) 28.84330(31) 30.0042(38) 30.84667(23) 31.9996(74) 32.84955(61) 33.99976(28) 34.85245(50) 35.99600(99)	2.01143(53) 1.83518(65) 2.00809(94) 1.8394(13) 2.0045(17) 1.84330(31) 2.0042(38) 1.84667(23) 1.99996(74) 1.84955(61) 1.9996(28) 1.9960(99)	$\begin{array}{c} 5.4746(61)\cdot10^{-7}\\ 2.428(11)\cdot10^{-7}\\ 1.3908(17)\cdot10^{-7}\\ 6.16(18)\cdot10^{-8}\\ 3.523(20)\cdot10^{-8}\\ 1.5809(50)\cdot10^{-8}\\ 8.86(18)\cdot10^{-9}\\ 4.0311(33)\cdot10^{-9}\\ 2.2555(81)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-9}\\ 5.82(11)\cdot10^{-10}\\ 2.669(34)\cdot10^{-10}\\ 2.669(34)\cdot10^{-10}\end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24)	7 : 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 32 33 34 35	$\begin{array}{c} 24.01143(53)\\ 24.83518(65)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 30.0042(38)\\ 30.84667(23)\\ 31.9996(74)\\ 32.84355(61)\\ 33.9976(28)\\ 34.85245(50)\\ 35.99600(99)\\ 36.85548(90) \end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.8394(13)\\ 2.0045(17)\\ 1.84300(31)\\ 2.0042(38)\\ 1.84667(23)\\ 1.99996(74)\\ 1.84965(61)\\ 1.99976(28)\\ 1.85245(50)\\ 1.99900(99)\\ 1.85548(90) \end{array}$	$\begin{array}{c} 5.4746(61)\cdot10^{-7}\\ 2.428(11)\cdot10^{-7}\\ 1.3908(17)\cdot10^{-8}\\ 3.523(20)\cdot10^{-8}\\ 1.5809(50)\cdot10^{-8}\\ 8.86(18)\cdot10^{-9}\\ 2.2555(81)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-9}\\ 5.82(11)\cdot10^{-10}\\ 2.669(34)\cdot10^{-10}\\ 1.374(72)\cdot10^{-10}\\ 1.374(72)\cdot10^{-10}\\ \end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28	$\frac{\Delta}{6.42065(64)} \\ 8.4957(75) \\ 10.562(12) \\ 12.5659(57) \\ 14.633(21) \\ 16.6174(75) \\ 18.678(24) \\ 20.654(22) \\ 22.651(27) \\ 24.671(18) \\ 26.681(20) \\ 28.706(24) \\ 28.706($	7 : 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 32 33 34 35 36	$\begin{array}{c} 24.01143(53)\\ 24.83518(65)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 28.84330(31)\\ 30.084667(23)\\ 31.9996(74)\\ 32.84955(61)\\ 33.9976(25)\\ 34.8524(50)\\ 35.99600(99)\\ 35.99600(99)\\ 35.99600(99)\\ 37.9933(12)\end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.8394(13)\\ 2.0045(17)\\ 1.84330(31)\\ 2.0042(38)\\ 1.84467(23)\\ 1.9996(74)\\ 1.84955(61)\\ 1.9976(28)\\ 1.85245(50)\\ 1.99500(99)\\ 1.85548(90)\\ 1.9939(12)\\ \end{array}$	$\begin{array}{c} 5.4746(61)\cdot10^{-7}\\ 2.428(11)\cdot10^{-7}\\ 1.3908(17)\cdot10^{-7}\\ 5.523(20)\cdot10^{-8}\\ 3.523(20)\cdot10^{-8}\\ 1.5809(50)\cdot10^{-8}\\ 4.0311(33)\cdot10^{-9}\\ 2.2555(81)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-9}\\ 5.82(11)\cdot10^{-10}\\ 2.669(34)\cdot10^{-10}\\ 1.374(72)\cdot10^{-10}\\ 5.94(34)\cdot10^{-11}\\ 4.02(45)\cdot10^{-11}\end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30	$\frac{\Delta}{6.42065(64)}$ 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.709(11)	7 = 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37	$\begin{array}{c} 24.011.43(53)\\ 24.8351.8(53)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 28.84330(31)\\ 30.0042(38)\\ 30.94667(23)\\ 31.99996(74)\\ 33.9976(28)\\ 34.85245(50)\\ 35.99600(99)\\ 36.85548(90)\\ 37.9939(12)\\ 38.85681(49) \end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.83394(13)\\ 2.0045(17)\\ 1.84390(31)\\ 2.0042(38)\\ 1.84667(23)\\ 1.9996(74)\\ 1.84955(61)\\ 1.9976(28)\\ 1.855245(50)\\ 1.99600(99)\\ 1.85548(90)\\ 1.9939(12)\\ 1.85561(49) \end{array}$	$\begin{array}{c} 5.4746(61)\cdot10^{-7}\\ 2.428(11)\cdot10^{-7}\\ 1.3908(17)\cdot10^{-7}\\ 3.523(20)\cdot10^{-8}\\ 3.523(20)\cdot10^{-8}\\ 3.523(20)\cdot10^{-8}\\ 4.0311(33)\cdot10^{-9}\\ 4.0311(33)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-9}\\ 1.0144(28)\cdot10^{-10}\\ 2.2555(81)\cdot10^{-10}\\ 1.374(72)\cdot10^{-10}\\ 1.374(72)\cdot10^{-10}\\ 1.374(45)\cdot10^{-11}\\ 4.02(45)\cdot10^{-11}\\ 1.99(19)\cdot10^{-11}\end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.702(1	7 = 2.4 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38	$\begin{array}{l} 24.01143(53)\\ 24.8518(65)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 28.84330(31)\\ 30.04467(23)\\ 31.99996(74)\\ 32.84955(61)\\ 33.9976(28)\\ 34.85245(50)\\ 35.99600(99)\\ 36.85548(90)\\ 37.9939(12)\\ 38.85691(49)\\ 39.9885(17)\\ \end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.8394(13)\\ 2.0045(17)\\ 1.84330(31)\\ 2.0042(38)\\ 1.84637(23)\\ 1.99996(74)\\ 1.84955(61)\\ 1.9976(28)\\ 1.85245(50)\\ 1.9960(99)\\ 1.85545(90)\\ 1.85545(90)\\ 1.85548(90)\\ 1.9939(12)\\ 1.85691(49)\\ 1.9885(17)\\ \end{array}$	$\begin{array}{c} 5.4746(61)\cdot 10^{-7}\\ 2.428(11)\cdot 10^{-7}\\ 1.30904(17)\cdot 10^{-7}\\ 3.523(22)\cdot 10^{-8}\\ 3.523(22)\cdot 10^{-8}\\ 3.532(23)\cdot 10^{-8}\\ 3.580(18)\cdot 10^{-9}\\ 4.031(33)\cdot 10^{-9}\\ 2.2555(81)\cdot 10^{-9}\\ 2.2555(81)\cdot 10^{-9}\\ 1.0144(28)\cdot 10^{-1}\\ 3.74(12)\cdot 10^{-10}\\ 3.74(12)\cdot 10^{-10}\\ 4.02(45)\cdot 10^{-11}\\ 4.95(18)\cdot 10^{-11}\\ 9.5(18)\cdot 10^{-11}\\ 9.5(18)\cdot 10^{-11}\\ \end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 14.633(21) 16.6174(75) 18.678(24) 22.651(27) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.702(11) 34.718(17) 36.717(16)	$\tau = 2.4$ 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 23 33 34 35 36 37 38 39	$\begin{array}{c} 24.011.43(53)\\ 24.8351.8(53)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 28.045(17)\\ 28.84330(31)\\ 30.042(28)\\ 30.042(28)\\ 30.042(28)\\ 30.042(28)\\ 33.9996(74)\\ 33.9996(74)\\ 33.9996(74)\\ 33.9976(28)\\ 34.85245(50)\\ 35.99600(99)\\ 36.85548(90)\\ 37.9939(12)\\ 38.85691(49)\\ 39.9895(17)\\ 40.8583(11)\\ \end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.8334(13)\\ 2.0045(17)\\ 1.8430(31)\\ 2.0042(38)\\ 1.84667(23)\\ 1.9996(74)\\ 1.84955(61)\\ 1.99976(28)\\ 1.99976(28)\\ 1.99976(28)\\ 1.85245(50)\\ 1.99960(99)\\ 1.85548(90)\\ 1.85548(90)\\ 1.85548(9149)\\ 1.99895(17)\\ 1.8583(11)\\ \end{array}$	$\begin{array}{c} 5.4746(61)\cdot 10^{-7}\\ 2.428(11)\cdot 10^{-7}\\ 1.3090(17)\cdot 10^{-7}\\ 3.532(20)\cdot 10^{-8}\\ 3.532(20)\cdot 10^{-8}\\ 8.580(18)\cdot 10^{-9}\\ 4.031(133)\cdot 10^{-9}\\ 4.031(133)\cdot 10^{-9}\\ 1.0144(28)\cdot 10^{-9}\\ 2.2555(81)\cdot 10^{-10}\\ 2.2555(81)\cdot 10^{-10}\\ 5.94(34)\cdot 10^{-11}\\ 3.74(72)\cdot 10^{-10}\\ 5.94(34)\cdot 10^{-11}\\ 9.5(18)\cdot 10^{-12}\\ 9.5(18)\cdot 10^{-12}\\ 9.5(18)\cdot 10^{-12}\\ \end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 14.633(21) 14.633(21) 14.633(21) 14.633(21) 12.654(22) 22.651(27) 24.671(18) 28.706(24) 30.6923(81) 32.702(11) 32.702(17) 36.717(16) 38.697(17)	7 = 2.4 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6
	22 23 24 25 26 27 28 29 30 31 23 33 34 35 36 37 38 39 40	$\begin{array}{c} 24.011.43(53)\\ 24.8351.8(58)\\ 26.00809(94)\\ 26.8394(13)\\ 28.0045(17)\\ 28.8045(17)\\ 28.84330(31)\\ 30.84667(23)\\ 30.84667(23)\\ 31.99967(74)\\ 32.84955(61)\\ 33.9976(28)\\ 35.99660(99)\\ 35.9960(99)\\ 35.9960(99)\\ 35.855691(49)\\ 39.9895(17)\\ 40.8583(11)\\ 41.9886(15)\\ \end{array}$	$\begin{array}{c} 2.01143(53)\\ 1.83518(65)\\ 2.00809(94)\\ 1.8394(13)\\ 2.0045(17)\\ 1.84330(31)\\ 2.0042(38)\\ 1.84967(23)\\ 1.9996(74)\\ 1.84955(61)\\ 1.9976(28)\\ 1.855245(50)\\ 1.99960(99)\\ 1.85548(90)\\ 1.9980(12)\\ 1.85548(90)\\ 1.9989(12)\\ 1.8558(11)\\ 1.89885(17)\\ 1.9886(15)\\ \end{array}$	$\begin{array}{c} 5.4746(61)\cdot 10^{-7}\\ 2.428(11)\cdot 10^{-7}\\ 1.3908(17)\cdot 10^{-7}\\ 3.523(20)\cdot 10^{-8}\\ 3.523(20)\cdot 10^{-8}\\ 8.56(18)\cdot 10^{-9}\\ 2.255(81)\cdot 10^{-9}\\ 2.255(81)\cdot 10^{-9}\\ 2.698(34)\cdot 10^{-10}\\ 2.698(34)\cdot 10^{-10}\\ 2.698(34)\cdot 10^{-10}\\ 3.74(12)\cdot 10^{-10}\\ 3.74(12)\cdot 10^{-10}\\ 3.74(12)\cdot 10^{-11}\\ 1.99(19)\cdot 10^{-11}\\ 3.7(13)\cdot 10^{-12}\\ 3.7(13$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38	$\frac{\Delta}{6.42065(64)} \\ 8.4957(75) \\ 10.562(12) \\ 12.5659(57) \\ 14.633(21) \\ 14.633(21) \\ 16.6174(75) \\ 18.678(24) \\ 20.654(22) \\ 22.651(27) \\ 24.671(18) \\ 26.681(20) \\ 28.706(24) \\ 30.6923(81) \\ 32.702(11) \\ 34.718(17) \\ 36.717(16) \\ 38.697(17) \\ 40.701(10) \\ 1000 \\ 10$	au = 2.4 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.5 2.6 2
	22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41	$\begin{array}{r} 24.011.43(53)\\ 24.0311.43(53)\\ 24.83518(60,000)\\ 26.8394(13)\\ 28.0445(17)\\ 28.0445(17)\\ 28.84330(31)\\ 30.0442(38)\\ 30.0442(38)\\ 30.0442(38)\\ 30.0442(38)\\ 30.0442(38)\\ 33.9996(74)\\ 33.9996(74)\\ 33.9976(28)\\ 33.9976(28)\\ 33.9976(28)\\ 33.9996(71)\\ 33.99895(17)\\ 39.9895(17)\\ 40.8583(11)\\ 41.9886(15)\\ 42.8007(14)\\ \end{array}$	2.01143(53) 1.83518(65) 2.0089(94) 1.8334(13) 2.0045(17) 1.84330(31) 2.0042(83) 1.99996(74) 1.84955(61) 1.9976(28) 1.99960(99) 1.855245(50) 1.99960(99) 1.855248(90) 1.99391(2) 1.85991(49) 1.9885(17) 1.9885(15) 1.9885	$\begin{array}{c} 5.4746(61)\cdot 10^{-7}\\ 2.428(11)\cdot 10^{-7}\\ 1.3908(17)\cdot 10^{-7}\\ 3.532(20)\cdot 10^{-8}\\ 3.532(20)\cdot 10^{-8}\\ 3.532(20)\cdot 10^{-8}\\ 8.86(18)\cdot 10^{-9}\\ 4.031(133)\cdot 10^{-9}\\ 1.0144(28)\cdot 10^{-9}\\ 2.2555(81)\cdot 10^{-1}\\ 2.2555(81)\cdot 10^{-1}\\ 2.669(34)\cdot 10^{-10}\\ 5.94(13)\cdot 10^{-10}\\ 5.94(13)\cdot 10^{-11}\\ 9.5(18)\cdot 10^{-12}\\ 9.5(18)\cdot 10^{-12}\\ 2.50(66)\cdot 10^{-12}\\ 2.50(66)\cdot 10^{-12}\\ 3.52(13)\cdot 10^{-12}\\ \end{array}$		\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40	$\begin{array}{l} \Delta \\ 6.42065(64) \\ 8.4957(75) \\ 10.562(12) \\ 12.5659(57) \\ 14.633(21) \\ 14.633(21) \\ 16.6174(75) \\ 18.678(24) \\ 20.654(22) \\ 22.651(27) \\ 24.671(18) \\ 26.681(20) \\ 28.706(24) \\ 30.6923(81) \\ 32.702(11) \\ 36.717(16) \\ 36.697(17) \\ 16.727(16) \\ 36.697(17) \\ 16.727(16) \\ 36.697(17) \\ 16.727(16) \\ 36.697(17) \\ 16.727(16) \\ 16.$	7 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6

\mathbb{Z}_2	l	Δ		$\tau = \Delta -$	l	food		fuo	
+	2	3		1		0.32613776	5(45)	0.8891471	(40)
+	4	5.022665(28)		1.022665	(28)	0.069076(4	13)	0.24792(2)))
+	6	7.028488(16)		1.028488	(16)	0.0157416(41)	0.066136(36)
+	8	9.031023(30)		1.031023	(30)	0.0036850(54)	0.017318(30)
+	10	11.0324141(9	99)	1.032414	1(99)	0.00087562	2(13)	0.0044811	(15)
+	12	13.033286(1)	2)	1.033286	(12)	0.00020992	20(37)	0.0011517	4(59)
+	14	15.033838(1)	5)	1.033838	(15)	0.00005063	50(99)	0.0002948	4(56)
+	16	17.034258(3	1)	1.034258	(34)	0.00001228	80(18)	0.0000751	7(18)
+	18	19.034564(1)	2)	1.034564	(12)	2.98935(46	$() \cdot 10^{-6}$	0.0000191	408(89)
+	20	21.0347884(8	§4)	1.034788	4(84)	7.2954(10)	$\cdot 10^{-7}$	4.8632(23)	$) \cdot 10^{-6}$
+	22	23.034983(1	I)	1.034983(11)		$1.78412(27) \cdot 10^{-7}$		1.23201(7)	$2) \cdot 10^{-6}$
+	24	25.035122(1	I)	1.035122(11)		$4.37261(60) \cdot 10^{-8}$		3.1223(15)	$) \cdot 10^{-7}$
+	26	27.035249(1	I)	1.035249(11)		$1.07287(18) \cdot 10^{-8}$		7.8948(42)	$) \cdot 10^{-8}$
+	28	29.035344(19	9)	1.035344	(19)	$2.6409(19) \cdot 10^{-9}$		1.9992(23)	$) \cdot 10^{-8}$
+	30	31.035452(10	5)	1.035452	(16)	6.447(24) ·	10^{-10}	5.003(20)	· 10 ⁻⁹
+	32	33.035473(28	3)	1.035473(28)		$1.640(25) \cdot 10^{-10}$		1.308(21)	· 10 ⁻⁹
+	34	35.035632(6)	7)	1.035632	(67)	$3.58(22) \cdot 1$	10-11	2.90(19) ·	10^{-10}
+	36	37.035610(4)	1)	1.035610	(41)	$1.15(13) \cdot 1$	10-11	$9.6(11) \cdot 1$	0-11
+	38	39.035638(5)	3)	1.035638	(58)	$2.26(71) \cdot 1$	10-12	1.93(60) ·	10-11
+	40	41.03564(13)		1.03564/	13)	$1.7.3(15) \cdot 10$	1-13	63(13).1	0-12
Ŀ	_		_	1.00004(107	1.0(10) . 10	, 	0.0(10) 1	~
7.0	1	Δ	7.0	$= \Delta - \ell$	free	1.0(10) - 10	fun	0.0(10) 1	1
Z2	l 4	Δ 6.42065(64)	τ = 2.4	$= \Delta - \ell$ 2065(64)	foo0	9552(12)	f _{ec} O -0.110	247(54)	
Z ₂ + +	ℓ 4 6	Δ 6.42065(64) 8.4957(75)	τ = 2.4 2.4	$= \Delta - \ell$ 2065(64) 957(75)	foo0 0.001	9552(12) 472(49)	f_{ecO} -0.110 -0.043	247(54) 1(48)	
Z ₂ + + +	ℓ 4 6 8	Δ 6.42065(64) 8.4957(75) 10.562(12)	τ = 2.4 2.5	$= \Delta - \ell$ 2065(64) 957(75) 62(12)	foo0 0.001 0.000 0.000	9552(12) 1472(49) 11084(69)	f _{ee} O -0.110 -0.043 -0.013	247(54) 1(48) 9(11)	
Z ₂ + + + +	ℓ 4 6 8 10	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57)	τ = 2.4 2.5 2.5	$= \Delta - \ell$ 2065(64) 957(75) 62(12) 659(57)	<i>foot</i> 0.001 0.000 0.000 0.000	9552(12) 1472(49) 1084(69) 102598(39)	f_{ecO} -0.110 -0.043 -0.013 -0.004	247(54) 1(48) 9(11) 437(62)	
Z ₂ + + + + +	ℓ 4 6 8 10 12	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21)	$\tau = 2.4$ 2.4 2.5 2.5 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21)	<i>fooc</i> 0.001 0.000 0.000 0.000 6.10(9552(12) 1472(49) 1084(69) 102598(39) $33) \cdot 10^{-6}$	f _{ee} O -0.110 -0.043 -0.013 -0.004 -0.001	247(54) 1(48) 9(11) 437(62) 224(60)	
Z ₂ + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75)	τ = 2.4 2.5 2.5 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75)	<i>food</i> 0.001 0.000 0.000 0.000 6.10(1.417	9552(12) 1472(49) 1084(69) 102598(39) $33) \cdot 10^{-6}$ $7(34) \cdot 10^{-6}$	<i>fee0</i> -0.110 -0.043 -0.013 -0.004 -0.001 -0.000	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54)	
Z ₂ + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24)	$\tau = 2.4$ 2.4 2.5 2.5 2.6 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75) 78(24)	<i>foot</i> 0.001 0.000 0.000 0.000 6.10(1.417 3.547	$1.5(10)^{-1}$ 9552(12) 1472(49) 1084(69) 002598(39) $33) \cdot 10^{-6}$ $7(34) \cdot 10^{-6}$ $7(59) \cdot 10^{-7}$	<i>f_{eeO}</i> -0.110 -0.043 -0.013 -0.004 -0.001 -0.000 -0.000	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64)	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22)	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75) 78(24) 54(22)	<i>food</i> 0.001 0.000 0.000 0.000 6.10(1.417 3.547 7.99(9552(12) 472(49) 1084(69) 102598(39) $33) \cdot 10^{-6}$ $(34) \cdot 10^{-6}$ $(59) \cdot 10^{-7}$ $90) \cdot 10^{-8}$	f _{ee} ∂ -0.110 -0.043 -0.013 -0.004 -0.001 -0.000 -0.000 -0.000	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64) 0284(26)	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27)	$\tau = 2.4$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75) 78(24) 54(22) 51(27)	<i>fore</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83($1.5(10)^{-1}$ 9552(12) 1472(49) 1084(69) 102598(39) $33) \cdot 10^{-6}$ $('34) \cdot 10^{-6}$ $('59) \cdot 10^{-7}$ $90) \cdot 10^{-8}$ $13) \cdot 10^{-8}$	f _{ee} ∂ -0.110 -0.043 -0.013 -0.004 -0.000 -0.000 -0.000 -7.58(s	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 00972(64) 00972(64) 00972(64) 0284(26) $47) \cdot 10^{-6}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18)	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75) 78(24) 54(22) 51(27) 71(18)	<i>fore</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55)	9552(12) 1472(49) 1084(69) 1084(69) $33) \cdot 10^{-6}$ $('34) \cdot 10^{-6}$ $('59) \cdot 10^{-7}$ $90) \cdot 10^{-8}$ $13) \cdot 10^{-8}$ $72) \cdot 10^{-9}$	f_{ecO} -0.1100 -0.043 -0.013 -0.0044 -0.0013 -0.0004 -0.00000 -0.00000 -0.00000 -0.00000 -0.0000000 -0.00000000000000000000000000000000000	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64) 0284(26) $47) \cdot 10^{-6}$ $19) \cdot 10^{-6}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20)	$\tau = 2.4$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6	$\Delta - \ell$ 2065(64) 957(75) 62(12) 659(57) 33(21) 174(75) 78(24) 54(22) 51(27) 71(18) 81(20)	<i>food</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168	$150(10)^{-1}$ (1) $105(10)^{-1}$ (1) 102598(29) $33) \cdot 10^{-6}$ $(34) \cdot 10^{-6}$ $(59) \cdot 10^{-7}$ $90) \cdot 10^{-8}$ $13) \cdot 10^{-8}$ $72) \cdot 10^{-9}$ $5(29) \cdot 10^{-9}$	f_{ecO} -0.110 -0.043 -0.013 -0.004 -0.0012 -0.0000 -0.0000 -0.0000 -7.58(4 -2.09(1 -5.67(1)	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0072(64) 0284(26) 0284(26) 0284(26) 10^{-6} $19) \cdot 10^{-6}$ $17) \cdot 10^{-7}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24)	$\tau =$ 2.4 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6	$\frac{-\Delta - \ell}{2065(64)}$ $\frac{-\Delta - \ell}{957(75)}$ $\frac{62(12)}{659(57)}$ $\frac{33(21)}{174(75)}$ $\frac{78(24)}{78(22)}$ $\frac{54(22)}{51(27)}$ $\frac{51(27)}{71(18)}$ $\frac{81(20)}{06(24)}$	<i>food</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168 2.81($15(10)^{-1}$ 9552(12) 11084(69) 002598(39) $33) \cdot 10^{-6}$ $(34) \cdot 10^{-6}$ $(59) \cdot 10^{-7}$ $90) \cdot 10^{-8}$ $13) \cdot 10^{-8}$ $12) \cdot 10^{-9}$ $(29) \cdot 10^{-9}$ $17) \cdot 10^{-10}$	f_{ecO} -0.110 -0.043 -0.013 -0.004 -0.0013 -0.0001 -0.0000 -0.0000 -7.58(4 -2.09(1 -5.67(1 -1.49(1	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64) 0284(26) 0284(26) 0284(26) $19) \cdot 10^{-6}$ $19) \cdot 10^{-6}$ $17) \cdot 10^{-7}$ $11) \cdot 10^{-7}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 28.706(24)	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6	$\frac{-\Delta - \ell}{2065(64)}$ $\frac{-\Delta - \ell}{957(75)}$ $\frac{62(12)}{659(57)}$ $\frac{33(21)}{174(75)}$ $\frac{78(24)}{54(22)}$ $\frac{54(22)}{51(27)}$ $\frac{51(27)}{71(18)}$ $\frac{81(20)}{006(24)}$ $\frac{923(81)}{923(81)}$	<i>food</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168 2.81(6.69)	$(15)(10)^{-1}$ $(15)(10)^{-1}$ (15)(10)(10)(10)(10)(10)(10)(10)(10)(10)(10	f_{ecO} -0.1107 -0.043 -0.0132 -0.0044 -0.00132 -0.0000 -0.0000 -0.0000 -7.58(4 -2.09(1) -5.67(1) -1.49(1) -4.162	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64) 0284(26) $47) \cdot 10^{-6}$ $19) \cdot 10^{-6}$ $17) \cdot 10^{-7}$ $(11) \cdot 10^{-7}$ $(88) \cdot 10^{-8}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 30	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.702(11)	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.7 2.6 2.6 2.6 2.6 2.6 2.7 2.6 2.7 2.6 2.7 2.6 2.7	$\frac{-\Delta - \ell}{2065(64)}$ $\frac{-\Delta - \ell}{957(75)}$ $\frac{62(12)}{659(57)}$ $\frac{33(21)}{174(75)}$ $\frac{78(24)}{51(27)}$ $\frac{51(27)}{71(18)}$ $\frac{81(20)}{06(24)}$ $\frac{923(81)}{02(11)}$	<i>food</i> 0.000 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168 2.81(6.69(1.62)	$135(20)^{-1}$ $135(20)^{-1}$ 1472(49) 1084(69) 102598(39) $33) \cdot 10^{-6}$ $(59) \cdot 10^{-8}$ $7(59) \cdot 10^{-8}$ $7(2) \cdot 10^{-9}$ $17) \cdot 10^{-10}$ $16) \cdot 10^{-11}$ $16) \cdot 10^{-11}$	f_{eeO} -0.1100 -0.043 -0.003 -0.004 -0.0001 -0.0000 -0.0000 -7.58(i -2.09(i) -5.67(i) -1.49(i) -1.49(i) -1.49(i) -1.066	247(54) 1(48) 9(11) 437(62) 224(60) 3791(54) 0972(64) 0284(26) $47) \cdot 10^{-6}$ $17) \cdot 10^{-7}$ $11) \cdot 10^{-7}$ $11) \cdot 10^{-8}$ $(88) \cdot 10^{-8}$ $(59) \cdot 10^{-8}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.702(11) 32.702(11) 34.718(17)	$\tau = 2.4$ 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6	$\frac{-\Delta - \ell}{2065(64)}$ $\frac{-\Delta - \ell}{957(75)}$ $\frac{62(12)}{62(12)}$ $\frac{659(57)}{33(21)}$ $\frac{174(75)}{78(24)}$ $\frac{54(22)}{54(22)}$ $\frac{54(22)}{54(27)}$ $\frac{81(20)}{06(24)}$ $\frac{923(81)}{923(81)}$ $\frac{02(11)}{18(17)}$	<i>ford</i> 0.001 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168 2.81(6.69(1.62(4.15($\begin{array}{c} 9552(12)\\ 9552(12)\\ 4172(49)\\ 1084(69)\\ 002598(39)\\ 33)\cdot 10^{-6}\\ (59)\cdot 10^{-7}\\ 90)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-6}\\ (29)\cdot 10^{-9}\\ 17)\cdot 10^{-10}\\ 36)\cdot 10^{-11}\\ 42)\cdot 10^{-12}\\ 42)\cdot 10^{-12}\end{array}$	f_{eeO} -0.1100 -0.043 -0.003 -0.0001 -0.0000 -0.0000 -0.0000 -7.58(4) -2.09(1) -1.49(1) -4.162 -1.066 -2.83(1)	$\begin{array}{c} 2247(54) \\ (148) \\ 9(11) \\ 437(62) \\ 224(60) \\ 972(64) \\ 0284(26) \\ 47) \cdot 10^{-6} \\ 19) \cdot 10^{-6} \\ 17) \cdot 10^{-7} \\ 11) \cdot 10^{-7} \\ (88) \cdot 10^{-9} \\ (59) \cdot 10^{-8} \\ 8) \cdot 10^{-9} \end{array}$	
Z_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 30 32 34	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 22.651(27) 22.651(27) 22.651(27) 22.651(27) 24.671(18) 28.706(24) 30.6923(81) 32.702(11) 34.718(17) 36.717(16)	$\overline{\tau} =$ 2.4 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.7 2.7 2.7 2.7	$\begin{array}{l} 1.00094(\\ \hline 1.00094(\\ \hline 2005(64)\\ 957(75)\\ 62(12)\\ 659(57)\\ 33(21)\\ 174(75)\\ 54(22)\\ 51(27)\\ 77(18)\\ 81(20)\\ 006(24)\\ 923(81)\\ 02(11)\\ 18(17)\\ 17(16) \end{array}$	$f_{\sigma\sigma\mathcal{O}}$ 0.001 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000 0.0000 0.0000 0.00000 0.00000 0.0000 0.000	$\begin{array}{c} 9552(12)\\ 9552(12)\\ 472(49)\\ 11084(69)\\ 002598(33)\cdot 10^{-6}\\ (34)\cdot 10^{-6}\\ (34)\cdot 10^{-8}\\ 90)\cdot 10^{-8}\\ 72)\cdot 10^{-9}\\ (12)\cdot 10^{-9}\\ (12)\cdot 10^{-10}\\ 36)\cdot 10^{-11}\\ 16)\cdot 10^{-11}\\ 16)\cdot 10^{-11}\\ 42)\cdot 10^{-12}\\ 77)\cdot 10^{-13}\\ \end{array}$	f_{eeO} -0.1100 -0.043 -0.0132 -0.0044 -0.0012 -0.00012 -0.000012 -0.000012 -0.000012 -0.000012 -0.000012 -0.000	$\begin{array}{c} 247(54) \\ 1(48) \\ 9(11) \\ 437(62) \\ 224(60) \\ 3791(54) \\ 0972(64) \\ 00284(26) \\ 17) \cdot 10^{-6} \\ 19) \cdot 10^{-6} \\ 17) \cdot 10^{-7} \\ (88) \cdot 10^{-8} \\ (59) \cdot 10^{-8} \\ (59) \cdot 10^{-9} \\ 18) \cdot 10^{-9} \\ 59) \cdot 10^{-10} \end{array}$	
Z_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36	Δ 6.42065(64) 8.4957(75) 10.562(12) 12.5659(57) 14.633(21) 16.6174(75) 18.678(24) 20.654(22) 22.651(27) 24.671(18) 26.681(20) 28.706(24) 30.6923(81) 32.702(11) 34.718(17) 36.717(16) 38.697(17)	$\overline{\tau} =$ 2.4 2.4 2.5 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.6 2.7 2.7 2.7 2.7 2.6	$\frac{1.0004}{2065(64)} = \frac{\Delta - \ell}{2065(64)}$ $\frac{2065(64)}{957(75)}$ $\frac{62(12)}{659(57)}$ $\frac{33(21)}{174(75)}$ $\frac{78(24)}{54(22)}$ $\frac{51(27)}{71(18)}$ $\frac{81(20)}{09(24)}$ $\frac{906(24)}{923(81)}$ $\frac{906(24)}{17(16)}$ $\frac{97(17)}{17(16)}$	<i>ford</i> 0.001 0.000 0.000 0.000 0.000 6.10(1.417 3.547 7.99(1.83(4.55(1.168 2.81(6.69(1.62(4.15(9.44(2.40($\begin{array}{c} 133(10)^{-11}\\ 9552(12)\\ 1054(69)\\ 1058(69)\\ 33)\cdot 10^{-6}\\ (34)\cdot 10^{-6}\\ (59)\cdot 10^{-7}\\ 90)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-9}\\ (29)\cdot 10^{-9}\\ (29)\cdot 10^{-9}\\ (29)\cdot 10^{-9}\\ 17)\cdot 10^{-10}\\ 16)\cdot 10^{-11}\\ 16)\cdot 10^{-11}\\ 16)\cdot 10^{-11}\\ 39)\cdot 10^{-13}\\ \end{array}$	f_{eeO} -0.1100 -0.043 -0.0130 -0.0044 -0.00130 -0.000000 -0.00000000 -0.0000000 -0.00000000000000000000000000000000000	$\begin{array}{c} 247(54) \\ 1(48) \\ 9(11) \\ 437(62) \\ 224(60) \\ 3791(54) \\ 0072(64) \\ 00284(26) \\ 10) \cdot 10^{-6} \\ 17) \cdot 10^{-6} \\ 19) \cdot 10^{-6} \\ 18) \cdot 10^{-7} \\ (88) \cdot 10^{-8} \\ (59) \cdot 10^{-8} \\ 18) \cdot 10^{-9} \\ 19) \cdot 10^{-10} \end{array}$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 30 32 34 36 38	$\begin{array}{c} \Delta \\ 6.42065(64) \\ 8.4957(75) \\ 110.562(12) \\ 112.5659(57) \\ 114.633(21) \\ 116.6174(75) \\ 118.678(24) \\ 22.651(27) \\ 24.671(18) \\ 22.651(27) \\ 24.671(18) \\ 30.9923(81) \\ 32.702(11) \\ 34.718(17) \\ 36.717(16) \\ 38.697(17) \\ 40.701(19) \end{array}$	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.7 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.6 2.7 2.7 2.6 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.6 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.6 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7	$\begin{split} & \frac{150004}{1000000000000000000000000000000000$	$\begin{array}{c} f_{\sigma\sigma\mathcal{O}}\\ 0.001\\ 0.000\\ 0.00$	$\begin{array}{c} 9552(12)\\ 9552(12)\\ 11084(68)\\ 002598(69)\\ 002598(69)\\ 002598(69)\\ 002598(69)\\ 002598(69)\\ 002598(69)\\ 002598(69)\\ 002598(10)\\ (33)\cdot 10^{-6}\\ (33)\cdot 10^{-6}\\ (33)\cdot 10^{-6}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ (23)\cdot 10^{-3}\\ (39)\cdot 10^{-13}\\ 16)\cdot 10^{-14}\\ 42)\cdot 10^{-12}\\ 12)\cdot 10^{-14}\\ 12)\cdot 10^{-$	f_{icO} -0.110 -0.013 -0.013 -0.003 -0.000 -0.000 -0.000 -0.000 -7.58(4 -2.09(1) -5.67(1) -1.49(1) -1.49(2) -1.49(2) -1.49(2) -1.49(2) -2.33(3) -7.33(3) -2.12(2) -5.2(15)	$\begin{array}{c} 234(13) & 1 \\ \hline 247(54) \\ 1(48) \\ 9(11) \\ 437(62) \\ 224(60) \\ 3791(54) \\ 0972(64) \\ 19) & 10^{-6} \\ 19) & 10^{-6} \\ 19) & 10^{-7} \\ 11) & 10^{-7} \\ 11) & 10^{-7} \\ 11) & 10^{-7} \\ 11) & 10^{-7} \\ 10) & 10^{-8} \\ 10) & 10^{-9} \\ 10) & 10^{-9} \\ 10) & 10^{-9} \\ 10) & 10^{-9} \\ 10) & 10^{-10} \\ 10) & 10^{-10} \\ 10) & 10^{-11} \\ 10) & 10^{-1$	
\mathbb{Z}_2 + + + + + + + + + + + + + + + + + + +	ℓ 4 6 8 10 12 14 16 18 20 22 24 26 30 32 34 36 38 40	$\begin{array}{c} \Delta \\ 6.42065(64) \\ 8.4957(75) \\ 110.562(12) \\ 112.5659(57) \\ 114.633(21) \\ 116.6174(75) \\ 20.654(22) \\ 22.651(27) \\ 22.651(27) \\ 22.651(27) \\ 22.651(27) \\ 22.6681(20) \\ 23.706(24) \\ 30.6923(81) \\ 32.702(11) \\ 34.718(17) \\ 36.6717(16) \\ 38.697(17) \\ 40.701(19) \\ 42.726(18) \end{array}$	$\tau =$ 2.4 2.5 2.5 2.6 2.6 2.6 2.6 2.6 2.7 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.7 2.7 2.6 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7 2.7	$\begin{array}{l} 1.50040(\\ = \Delta - \ell \\ 2065(64) \\ 957(75) \\ 62(12) \\ 659(57) \\ 33(21) \\ 174(75) \\ 78(24) \\ 54(22) \\ 51(27) \\ 71(18) \\ 81(20) \\ 906(24) \\ 923(81) \\ 02(11) \\ 18(17) \\ 17(16) \\ 97(17) \\ 17(16) \\ 97(17) \\ 101(19) \\ 26(18) \end{array}$	$f_{\sigma\sigma\mathcal{O}}$ 0.001 0.000 0.000 0.000 0.000 0.000 0.000 1.417 3.543 3.543 4.551 1.688 2.814 6.699 1.422 4.156	$\begin{array}{c} 9552(12)\\ 9552(12)\\ 472(49)\\ 1084(69)\\ 33)\cdot 10^{-6}\\ (34)\cdot 10^{-6}\\ (34)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-8}\\ 13)\cdot 10^{-9}\\ (29)\cdot 10^{-9}\\ (29)\cdot 10^{-9}\\ 10^{-11}\\ 42)\cdot 10^{-11}\\ 42)\cdot 10^{-12}\\ 39)\cdot 10^{-13}\\ 39)\cdot 10^{-13}\\ 7)\cdot 10^{-14}\\ 9)\cdot 10^{-14}\\ \end{array}$	f_{ecO} -0.1100 -0.0133 -0.0013 -0.0013 -0.0001 -0.0000 -0.0000 -0.0000 -7.58(4) -2.09(1) -5.67(1) -1.49(1) -1.49(2) -1.68(4) -1.28(3) -1.28(4) -	$\begin{array}{c} 237(54) \\ 1(48) \\ 9(11) \\ 347(62) \\ 224(60) \\ 3791(54) \\ 0072(64) \\ 00$	

\mathbb{Z}_2	l	Δ	$\tau = \Delta - \ell$	faaO	fue
+	0	3.82968(23)	3.82968(23)	0.053012(55)	1.5360(16)
+	2	5.50915(44)	3.50915(44)	0.0105745(42)	0.69023(49)
+	4	7.38568(28)	3.38568(28)	0.00237745(44)	0.22975(10)
+	6	9.32032(34)	3.32032(34)	0.00055657(42)	0.06949(11)
+	8	11.2751(24)	3.2751(24)	0.00013251(91)	0.01980(15)
+	10	13.2410(10)	3.2410(10)	0.00003234(15)	0.005459(39)
+	12	15.2301(64)	3.2301(64)	$7.64(14) \cdot 10^{-6}$	0.001538(22)
+	14	17.1944(55)	3.1944(55)	$1.930(46) \cdot 10^{-6}$	0.000386(14)
+	16	19.1950(62)	3.1950(62)	$4.568(72) \cdot 10^{-7}$	0.0001107(16)
+	18	21.1720(23)	3.1720(23)	$1.153(27) \cdot 10^{-7}$	0.00002798(33)
+	20	23.167(10)	3.167(10)	$2.74(11) \cdot 10^{-8}$	$7.45(52) \cdot 10^{-6}$
+	22	25.163(10)	3.163(10)	$6.88(22) \cdot 10^{-9}$	$1.937(51) \cdot 10^{-6}$
+	24	27.1491(82)	3.1491(82)	$1.716(45) \cdot 10^{-9}$	$4.92(42) \cdot 10^{-7}$
+	26	29.1460(53)	3.1460(53)	$4.183(78) \cdot 10^{-10}$	$1.347(62) \cdot 10^{-7}$
+	28	31.1306(52)	3.1306(52)	$1.056(50) \cdot 10^{-10}$	$3.35(10) \cdot 10^{-8}$
+	30	33.126(12)	3.126(12)	$2.54(10) \cdot 10^{-11}$	$8.35(42) \cdot 10^{-9}$
+	32	35.1299(77)	3.1299(77)	$6.71(17) \cdot 10^{-12}$	$2.36(13) \cdot 10^{-9}$
+	34	37.1174(64)	3.1174(64)	$1.39(14) \cdot 10^{-12}$	$4.87(48) \cdot 10^{-10}$
+	36	39.1079(78)	3.1079(78)	$4.84(56) \cdot 10^{-13}$	$1.70(17) \cdot 10^{-10}$
+	38	41.101(29)	3.101(29)	$8.4(28) \cdot 10^{-14}$	$2.5(11) \cdot 10^{-11}$
+	40	43.102(18)	3.102(18)	$2.63(64) \cdot 10^{-14}$	$9.0(26) \cdot 10^{-12}$
+	42	45.116(27)	3.116(27)	$7.9(22) \cdot 10^{-15}$	$3.42(95) \cdot 10^{-12}$

[Simmons-Duffin,'16]

The solution, even if approximate, contains important info:

- \blacktriangleright operators needed to satisfy crossing (low dim + $\Delta \ell$ small)
- how fast operators are suppressed
- ▶ how analytic structures are reproduced in $\sum_{O} C_{O}^{2}$

Study known spectra of interacting CFTs \Rightarrow infer general structure of CFTs

An application to high energy physics

Conformal Window of SU(N) gauge theories with N_f fermions in the fundamental



At the IR fixed point:

- Global Symmetry: $U(1)_V \times SU(N_f)_L \times SU(N_f)_R$
- "Mesons" $M_i^{\bar{j}}$: scalars in $\overline{\Box} \times \Box$
- Bootstrap $\langle M_i^{\bar{j}} M_k^{\bar{l}} M_p^{\bar{q}} M_r^{\bar{s}} \rangle$

Can we get any constraint on the mass anomalous dimension $\gamma_M \equiv 3 - \Delta_M$?

At the IR fixed point:

- IR fixed point is stable: no relevant perturbation which is singlet under symmetry group
- The smallest dimension singlet $\sim Tr[F_{\mu\nu}F^{\mu\nu}]$ must be irrelevant

Bound on first singlet in the OPE $M_i^{\bar{j}} \times M_k^{\bar{l}}$:



 $\Rightarrow \gamma_M \leq 1.79$ [Nakayama, Yu] 2016

Case study: $N_f = 8$, symmetry enhancement

- ► Evidences of fixed point from Lattice simulation with Wilson fermions
- ► Lattice action only implements a sub-group of the global symmetry
- Enhancement to larger global symmetry in IR

Symmetry breaking operators are irrelevant

Bound on the lowest dimension $\overline{\Box} \times \Box$ operator in the OPE $M_i^{\bar{j}} \times M_k^{\bar{l}}$:



 $\Rightarrow \gamma_M \leq 1.31$ [Nakayama, Yu] 2016 Scale invariant systems are the building blocks of quantum systems CFT's in $D \ge 3$ (neglected for many years) are now under siege

- ▶ Numerical techniques allow to precisely determine CFT data
- Other complementary approaches available

We have only scratched the surface: are we on the verge of a "CFT eight-fold way"?