Conformal Bootstrap:
strong dynamics under siege

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Perturbation Theory

Most of our understanding of physical systems comes from perturbation theory

EX: Anharmonic Oscillator

$$H = \underbrace{\frac{p^2}{2m} + \frac{mw^2}{2}q^2}_{\text{exactly solvable model}} + \underbrace{\lambda q^4}_{\text{small interaction }(\lambda)}$$

$$\Rightarrow \qquad \text{Observables} = \mathcal{O}_0 + \sum_{n=1}^{\infty} \lambda^n \delta\mathcal{O}_{(n)}$$

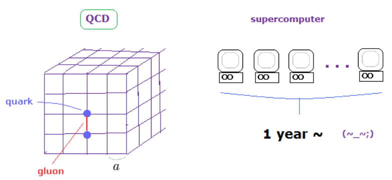
however...

- The perturbative series is asymptotic: still useful for $\lambda \ll 1$
- In (way too many) interesting cases the perturbation is not small: what do we do then?

Numerical Strong dynamics toolbox

- MonteCarlo simulations

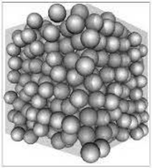
- Lattice Field theory



- Hamiltonian Truncation

Theoreticians' Strong dynamics toolbox

- Introduce large parameters in the game:
  number of fields N, number of dimensions D
  $\longrightarrow$ solve the theory in the infinite limit



- Study subsectors of a theory with large quantum numbers (spin, electric charge,...)
  $\longrightarrow$ system becomes semi-classical, i.e. dominated by classical solutions

- Exploit the power of additional symmetries
  ex: supersymmetry $\longrightarrow$ holomorphicity, localization,...

this talk: study consequences of scale invariance

SCALE INVARIANT SYSTEMS

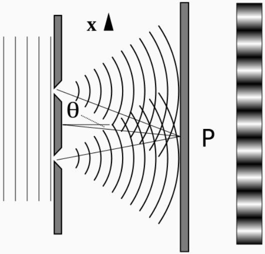
Scale invariance (an appetizer)

In a nutshell, scale invariant systems are characterised by the same behaviour at all length scales.

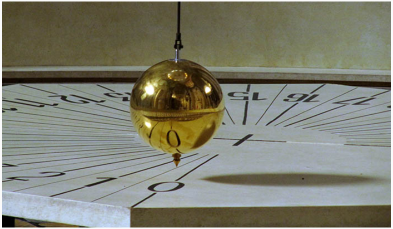
Static configurations $\Rightarrow$ property of self-similarity

Scale invariance & dynamics

dynamical system $\Rightarrow$ "equations" don't depend on scales



Optics ✓

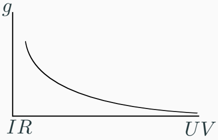


Pendulum X

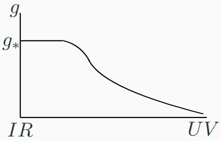
Classical Vs Quantum

Quantum mechanics changes the rules of the game

- classical scale invariance broken at quantum level
- coupling constants depend on the probing energy

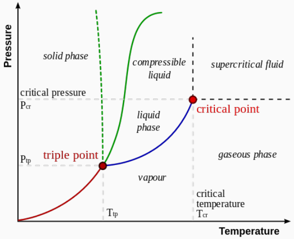


- Under special circumstances scale invariance can still be realised at quantum level

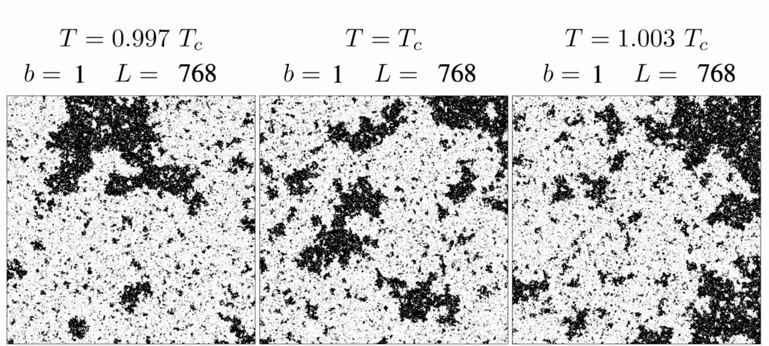


Conformal Field Theories in Nature

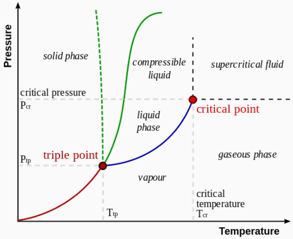
Vapour-liquid transition in water



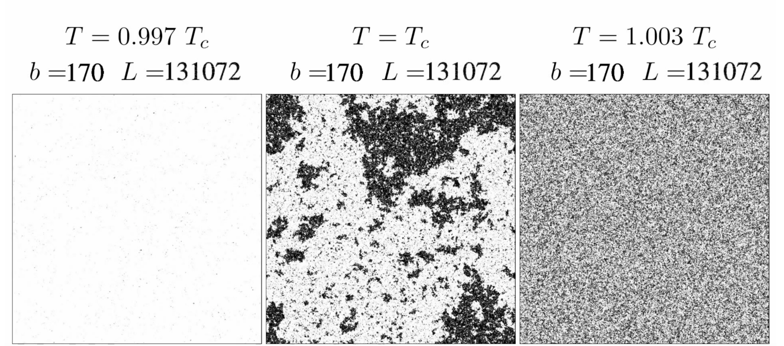
Real simulation close to the critical temperature: ■ = vapour □ =liquid



Vapour-liquid transition in water



Real simulation close to the critical temperature: ■ = vapour □ =liquid



An example: the Ising model in 3D

Scale invariance $\Rightarrow$ power law behaviour close to T_c:

$$\langle Observable \rangle \sim (T - T_c)^{\#}$$

EX:

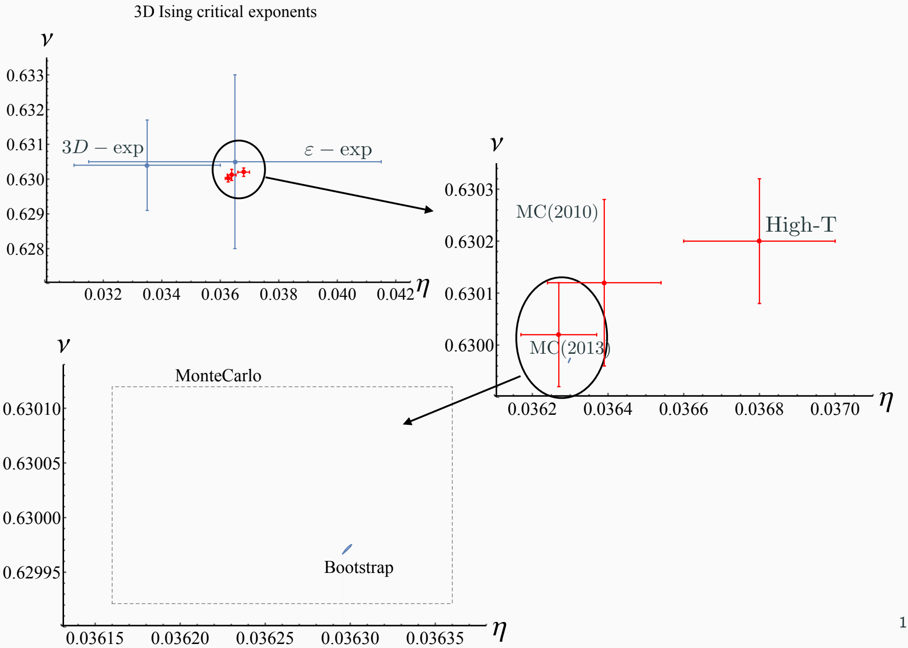
Correlation Length: $\xi \sim (T - T_c)^{-\nu}$

Heat Capacity: $C \sim (T - T_c)^{2-3\nu}$

Magnetic Susceptibility: $\chi \sim (T - T_c)^{(2-\eta)/\nu}$

Q: how to compute the critical exponents η, ν?

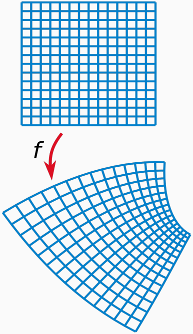
An example: the Ising model in 3D



From Scale to Conformal Invariance

In all known unitary theories:

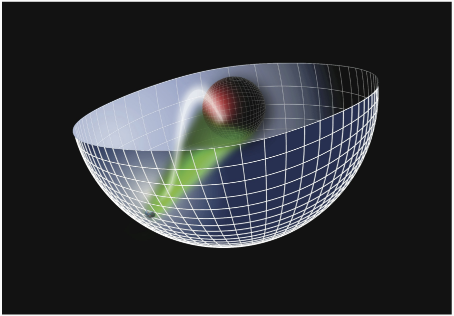
Scale Invariance $\Rightarrow$ Conformal Symmetry



Preserves angles but not distances

From boiling water to quantum gravity

The AdS/CFT correspondence relates QFT with conformal invariance to string theory in Anti-deSitter background



Usually: Classical gravity $\Rightarrow$ Strongly coupled CFTs

Lately: 2D CFTs $\Rightarrow$ Black holes physics

What are CFTs?

Theories invariant under the *conformal algebra $SO(D|2)$* which includes:

- translations
- Lorentz transformations
- dilatations
- "inversion"

They are described by three ingredients:

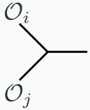
1) Spectrum: infinite set of operators $\mathcal{O}_{\Delta,\ell}$

spin

dimension in energy

2) Interactions between operators:

$$\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k \quad \mathcal{O}_k \sim \sum_k C_{ijk}\mathcal{O}_k$$



Operator Product Expansion (OPE) coefficients

3) Crossing symmetry constraints: see next slides...

The power of conformal invariance

In CFT we are interested in computing correlations functions (observables averaged over states of the theory)

$$\langle \mathcal{O}_i(x_1)...\mathcal{O}_j(x_n)\rangle \longleftrightarrow \sum_{\text{states s}} \langle s|O_i(x_1)...\mathcal{O}_j(x_n)|s\rangle e^{-E_s/T} \tag{1}$$

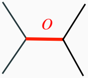
$$\text{fixed by symmetry} \quad \begin{cases} \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle & \checkmark \\ \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)O_k(x_3)\rangle & \checkmark \end{cases}$$

$$\text{encode dynamics} \quad \begin{cases} \langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)O_k(x_3)O_l(x_4)\rangle \\ \ldots \end{cases}$$

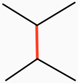
Four point functions

Use OPE to reduce higher point functions to smaller ones:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}}$$



$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}}$$



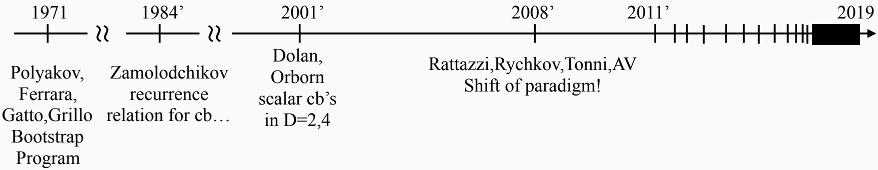
Crossing symmetry: two expansions must give the same result!
(Constraint on spectrum and interactions)

Definition of a CFT:

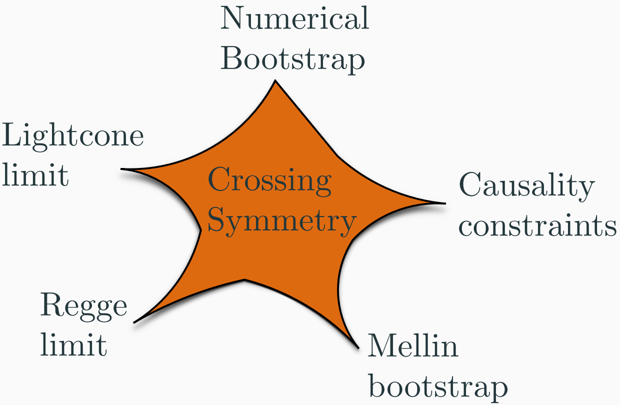
A CFT is an infinite set of primary operators $\mathcal{O}_{\Delta,\ell}$ and OPE coefficients C_{ijk} that satisfy crossing symmetry for all set of four-point functions.

Q: What choices of CFT data are consistent?

A bit of history



- Original formulation was very ambitious: solve exactly CFTs
- Shift of paradigm: "a few" better than "nothing"
  (quantitative informations can be obtained without solving the theory)
- "A few" can lead to "all"


Numerical
Bootstrap
Lightcone
limit
Crossing
Symmetry
Causality
constraints
Regge
limit
Mellin
bootstrap

are $\{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_n\}$ part of a CFT?

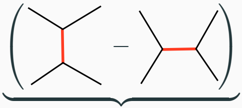
Conformal bootstrap

No

Maybe

- Crossing equation for $\langle O(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$:

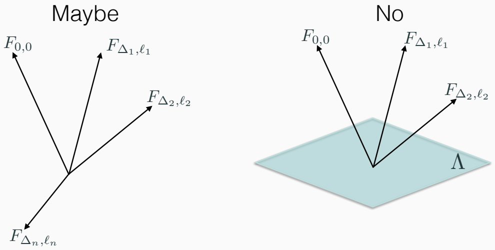
$$\sum_{\Delta,\ell} C^2_{\Delta,\ell}$$



$$= 0$$

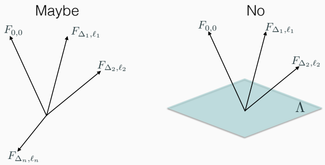
Known functions $F_{\Delta,\ell}$

- Unitarity: $C^2_{\Delta,\ell} \geq 0$



Existence of Λ can be recast into a *convex optimization problem* and checked numerically using linear (or semi-definite) programming algorithms

[Rattazzi,Rychkov,Tonni, AV '08] [Poland,Simmons-Duffin, AV '11]



For unfeasible spectra the cone generated by all the vectors $F_{\Delta,\ell}$ is entirely contained in half-space

Look for a Linear functional

$$\Lambda[F_{\Delta,\ell}] \equiv \sum_{n,m}^{N_{\max}} \lambda_{mn} \partial^n \partial^m F_{\Delta,\ell}$$

such that

$$\Lambda[F_{\Delta,\ell}] > 0 \qquad \text{for any } \Delta\ell\text{, except for} \qquad \Lambda[F_{\Delta_*,\ell_*}] < 0$$

- $N_{\max}$: parametrizes numerical complexity
- as $N_{\max}$ increases, exclusion conditions get stronger

Numerical Bootstrap

Rules of the game:

- Choose one or more operators $\mathcal{O}_1, \mathcal{O}_2, ...$
- Consider all four point functions containing those operators $< \mathcal{O}_1\mathcal{O}_1\mathcal{O}_1\mathcal{O}_1 >, < \mathcal{O}_1\mathcal{O}_1\mathcal{O}_2\mathcal{O}_2 >, ...$
- Make assumptions on the operators (and coefficients) appearing in the OPE's $\mathcal{O}_i \times \mathcal{O}_j$
- Check numerically if assumptions are consistent with crossing symmetry
- If not consistent: no CFT with that operator content

A few applications

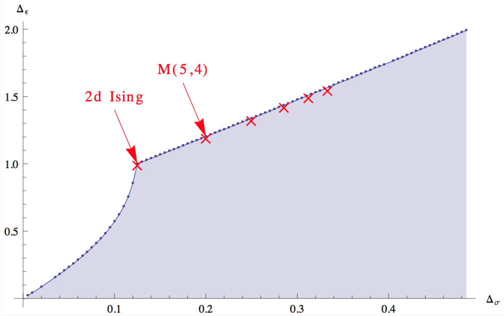
Comparison with 2D results

Scalars σ ε, with OPE:

$$\begin{aligned} \sigma \times \sigma &\sim 1 + \epsilon + \ldots. \\ \sigma \times \epsilon &\sim \sigma + \ldots. \\ \epsilon \times \epsilon &\sim 1 + \epsilon + \ldots. \end{aligned}$$

Q: given Δ_σ, how large can Δ_ε be?

A: study $< \sigma\sigma\sigma\sigma >, < \sigma\sigma\epsilon\epsilon >, < \epsilon\epsilon\epsilon\epsilon >$



[Rychkov, AV 09]

Minimal models (exactly solvable) saturate the bound

Important

No use of Virasoro algebra. Extend the method to 3D right away

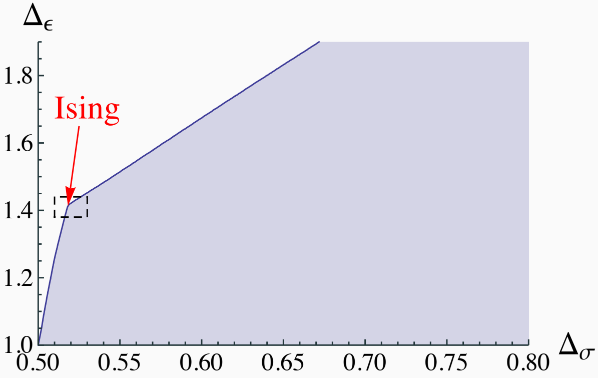
An application in 3D

Scalars σ ε, with OPE:

$$\sigma \times \sigma \sim 1 + \epsilon + ...$$
$$\sigma \times \epsilon \sim \sigma + ...$$
$$\epsilon \times \epsilon \sim 1 + \epsilon + ...$$

Q: given Δ_σ, how large can Δ_ε be?

A: study $<\sigma\sigma\sigma\sigma>, <\sigma\sigma\epsilon\epsilon>, <\epsilon\epsilon\epsilon\epsilon>$



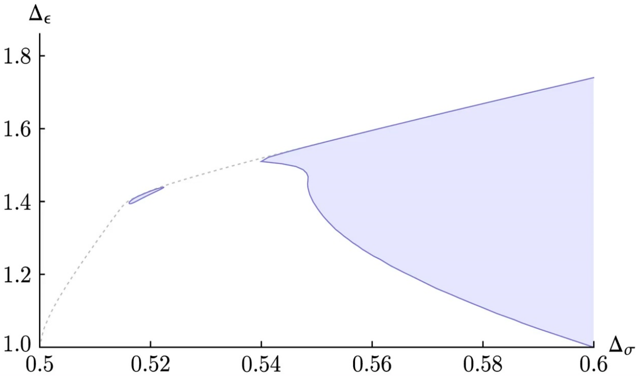
[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV] 2011

Can we narrow down the 3D Ising model?

So far we have assumed anything about the CFT besides unitarity.

The 3D Ising model has only two relevant perturbations

Q: if only σ and ϵ have dimension ≤ 3: allowed values for Δ_σ, Δ_ϵ?



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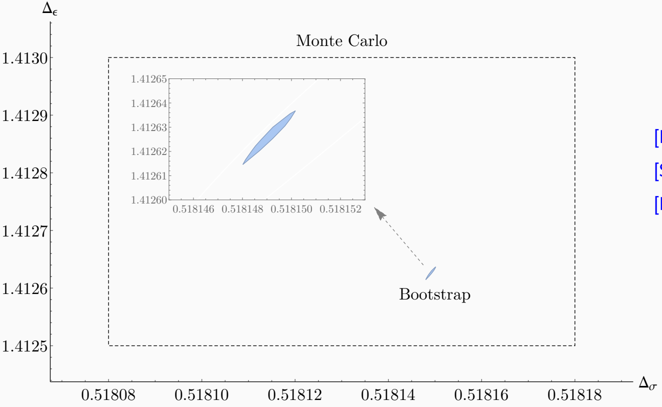
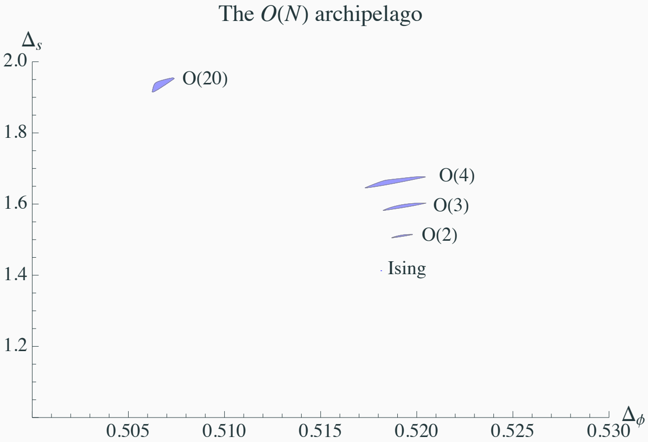


[Kos,Poland,Simmons-Duffin '14]

[Simmons-Duffin '15]

[Kos,Poland,Simmons-Duffin,AV '16]

An island for any N

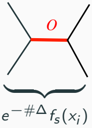


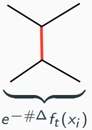
[Kos,Poland,Simmons-Duffin,AV '15]

Why does it work?

Only a finite number of operators effectively matter

Decoupling of high dimensional operators:

s-channel: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}}$ 

t-channel: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}}$ 

[Rychkov,Yvernay '15]

[Pappadopulo,Rychkov,Espin,Rattazzi '12]

In the numerical bootstrap functions $f_{s,t} \sim O(1)$

Large dimension operators exponentially suppressed!

Data mining & CFT learning

Numerical bootstrap gives much more than exclusion plots:

when the allowed region is "small" the whole spectrum can be reconstructed

$\mathbb{Z}_2$	ℓ	Δ	$\tau=\Delta-\ell$	$f_{\sigma\epsilon\mathcal{O}}$
$-$	2	4.180305(18)	2.180305(18)	0.38915941(81)
$-$	3	4.63804(88)	1.63804(88)	0.1385(34)
$-$	4	6.112674(19)	2.112674(19)	0.1077052(16)
$-$	5	6.709778(27)	1.709778(27)	0.04191549(88)
$-$	6	8.08097(25)	2.08097(25)	0.0286902(80)
$-$	7	8.747293(56)	1.747293(56)	0.01161255(13)
$-$	8	10.0623(29)	2.0623(29)	0.00745(21)
$-$	9	10.77075(36)	1.77075(36)	0.003115(12)
$-$	10	12.0492(18)	2.0492(18)	0.001940(19)
$-$	11	12.787668(92)	1.787668(92)	0.000823634(82)
$-$	12	14.0383(33)	2.0383(33)	0.0004983(88)
$-$	13	14.80006(51)	1.80006(51)	0.0002150(10)
$-$	14	16.0305(12)	2.0305(12)	0.0001291(12)
$-$	15	16.81009(16)	1.81009(16)	0.000055870(15)
$-$	16	18.025(11)	2.025(11)	0.0000313(30)
$-$	17	18.81794(18)	1.81794(18)	0.0000144219(91)
$-$	18	20.01947(94)	2.01947(94)	$8.442(28)\cdot 10^{-6}$
$-$	19	20.8246(11)	1.8246(11)	$3.690(54)\cdot 10^{-6}$
$-$	20	22.0152(36)	2.0152(36)	$2.131(28)\cdot 10^{-6}$
$-$	21	22.83035(11)	1.83035(11)	$9.5120(13)\cdot 10^{-7}$
$-$	22	24.01143(53)	2.01143(53)	$5.4746(61)\cdot 10^{-7}$
$-$	23	24.83518(65)	1.83518(65)	$2.428(11)\cdot 10^{-7}$
$-$	24	26.00809(94)	2.00809(94)	$1.3908(17)\cdot 10^{-7}$
$-$	25	26.8394(13)	1.8394(13)	$6.16(18)\cdot 10^{-8}$
$-$	26	28.0045(17)	2.0045(17)	$3.523(20)\cdot 10^{-8}$
$-$	27	28.84330(31)	1.84330(31)	$1.5809(50)\cdot 10^{-8}$
$-$	28	30.0042(38)	2.0042(38)	$8.86(18)\cdot 10^{-9}$
$-$	29	30.84667(23)	1.84667(23)	$4.0311(33)\cdot 10^{-9}$
$-$	30	31.99996(74)	1.99996(74)	$2.2555(81)\cdot 10^{-9}$
$-$	31	32.84955(61)	1.84955(61)	$1.0144(28)\cdot 10^{-9}$
$-$	32	33.9976(28)	1.9976(28)	$5.82(11)\cdot 10^{-10}$
$-$	33	34.85245(50)	1.85245(50)	$2.669(34)\cdot 10^{-10}$
$-$	34	35.99600(99)	1.99600(99)	$1.374(72)\cdot 10^{-10}$
$-$	35	36.85548(90)	1.85548(90)	$5.94(34)\cdot 10^{-11}$
$-$	36	37.9939(12)	1.9939(12)	$4.02(45)\cdot 10^{-11}$
$-$	37	38.85691(49)	1.85691(49)	$1.99(19)\cdot 10^{-11}$
$-$	38	39.9895(17)	1.9895(17)	$9.5(18)\cdot 10^{-12}$
$-$	39	40.8583(11)	1.8583(11)	$3.7(13)\cdot 10^{-12}$
$-$	40	41.9886(15)	1.9886(15)	$2.50(96)\cdot 10^{-12}$
$-$	41	42.8607(14)	1.8607(14)	$1.32(24)\cdot 10^{-12}$
$-$	42	43.9915(21)	1.9915(21)	$7.9(19)\cdot 10^{-13}$

$\mathbb{Z}_2$	ℓ	Δ	$\tau=\Delta-\ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
$+$	2	3	1	0.32613776(45)	0.8891471(40)
$+$	4	5.022665(28)	1.022665(28)	0.069076(43)	0.24792(20)
$+$	6	7.028488(16)	1.028488(16)	0.0157416(41)	0.066136(36)
$+$	8	9.031023(30)	1.031023(30)	0.0036850(54)	0.017318(30)
$+$	10	11.0324141(99)	1.0324141(99)	0.00087562(13)	0.0044811(15)
$+$	12	13.033286(12)	1.033286(12)	0.000209920(37)	0.00115174(59)
$+$	14	15.033838(15)	1.033838(15)	0.000050650(99)	0.00029484(56)
$+$	16	17.034258(34)	1.034258(34)	0.000012280(18)	0.00007517(18)
$+$	18	19.034564(12)	1.034564(12)	$2.98935(46)\cdot 10^{-6}$	0.0000191408(89)
$+$	20	21.0347884(84)	1.0347884(84)	$7.2954(10)\cdot 10^{-7}$	$4.8632(23)\cdot 10^{-6}$
$+$	22	23.034983(11)	1.034983(11)	$1.78412(27)\cdot 10^{-7}$	$1.23201(72)\cdot 10^{-6}$
$+$	24	25.035122(11)	1.035122(11)	$4.37261(60)\cdot 10^{-8}$	$3.1223(15)\cdot 10^{-7}$
$+$	26	27.035249(11)	1.035249(11)	$1.07287(18)\cdot 10^{-8}$	$7.8948(42)\cdot 10^{-8}$
$+$	28	29.035344(19)	1.035344(19)	$2.6409(19)\cdot 10^{-9}$	$1.9992(23)\cdot 10^{-8}$
$+$	30	31.035452(16)	1.035452(16)	$6.447(24)\cdot 10^{-10}$	$5.003(20)\cdot 10^{-9}$
$+$	32	33.035473(28)	1.035473(28)	$1.640(25)\cdot 10^{-10}$	$1.308(21)\cdot 10^{-9}$
$+$	34	35.035632(67)	1.035632(67)	$3.58(22)\cdot 10^{-11}$	$2.90(19)\cdot 10^{-10}$
$+$	36	37.035610(41)	1.035610(41)	$1.15(13)\cdot 10^{-11}$	$9.6(11)\cdot 10^{-11}$
$+$	38	39.035638(58)	1.035638(58)	$2.26(71)\cdot 10^{-12}$	$1.93(60)\cdot 10^{-11}$
$+$	40	41.03564(13)	1.03564(13)	$7.3(15)\cdot 10^{-13}$	$6.3(13)\cdot 10^{-12}$

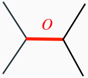
$\mathbb{Z}_2$	ℓ	Δ	$\tau=\Delta-\ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
$+$	4	6.42065(64)	2.42065(64)	0.0019552(12)	$-0.110247(54)$
$+$	6	8.4957(75)	2.4957(75)	0.000472(49)	$-0.0431(48)$
$+$	8	10.562(12)	2.562(12)	0.0001084(69)	$-0.0139(11)$
$+$	10	12.5659(57)	2.5659(57)	0.00002598(39)	$-0.004437(62)$
$+$	12	14.633(21)	2.633(21)	$6.10(33)\cdot 10^{-6}$	$-0.001224(60)$
$+$	14	16.6174(75)	2.6174(75)	$1.417(34)\cdot 10^{-6}$	$-0.0003791(54)$
$+$	16	18.678(24)	2.678(24)	$3.547(59)\cdot 10^{-7}$	$-0.0000972(64)$
$+$	18	20.654(22)	2.654(22)	$7.99(90)\cdot 10^{-8}$	$-0.0000284(26)$
$+$	20	22.651(27)	2.651(27)	$1.83(13)\cdot 10^{-8}$	$-7.58(47)\cdot 10^{-6}$
$+$	22	24.671(18)	2.671(18)	$4.55(72)\cdot 10^{-9}$	$-2.09(19)\cdot 10^{-6}$
$+$	24	26.681(20)	2.681(20)	$1.168(29)\cdot 10^{-9}$	$-5.67(17)\cdot 10^{-7}$
$+$	26	28.706(24)	2.706(24)	$2.81(17)\cdot 10^{-10}$	$-1.49(11)\cdot 10^{-7}$
$+$	28	30.6923(81)	2.6923(81)	$6.69(36)\cdot 10^{-11}$	$-4.162(88)\cdot 10^{-8}$
$+$	30	32.702(11)	2.702(11)	$1.62(16)\cdot 10^{-11}$	$-1.066(59)\cdot 10^{-8}$
$+$	32	34.718(17)	2.718(17)	$4.15(42)\cdot 10^{-12}$	$-2.83(18)\cdot 10^{-9}$
$+$	34	36.717(16)	2.717(16)	$9.44(77)\cdot 10^{-13}$	$-7.33(59)\cdot 10^{-10}$
$+$	36	38.697(17)	2.697(17)	$2.40(39)\cdot 10^{-13}$	$-2.12(34)\cdot 10^{-10}$
$+$	38	40.701(19)	2.701(19)	$5.4(17)\cdot 10^{-14}$	$-5.2(15)\cdot 10^{-11}$
$+$	40	42.726(18)	2.726(18)	$1.59(49)\cdot 10^{-14}$	$-1.55(48)\cdot 10^{-11}$
$+$	42	44.729(15)	2.729(15)	$4.2(12)\cdot 10^{-15}$	$-4.4(11)\cdot 10^{-12}$

$\mathbb{Z}_2$	ℓ	Δ	$\tau=\Delta-\ell$	$f_{\sigma\sigma\mathcal{O}}$	$f_{\epsilon\epsilon\mathcal{O}}$
$+$	0	3.82968(23)	3.82968(23)	0.053012(55)	1.5360(16)
$+$	2	5.50915(44)	3.50915(44)	0.0105745(42)	0.69023(49)
$+$	4	7.38568(28)	3.38568(28)	0.00237745(44)	0.22975(10)
$+$	6	9.32032(34)	3.32032(34)	0.00055657(42)	0.06949(11)
$+$	8	11.2751(24)	3.2751(24)	0.00013251(91)	0.01980(15)
$+$	10	13.2410(10)	3.2410(10)	0.00003234(15)	0.005459(39)
$+$	12	15.2301(64)	3.2301(64)	$7.64(14)\cdot 10^{-6}$	0.001538(22)
$+$	14	17.1944(55)	3.1944(55)	$1.930(46)\cdot 10^{-6}$	0.000386(14)
$+$	16	19.1950(62)	3.1950(62)	$4.568(72)\cdot 10^{-7}$	0.0001107(16)
$+$	18	21.1720(23)	3.1720(23)	$1.153(27)\cdot 10^{-7}$	0.00002798(33)
$+$	20	23.167(10)	3.167(10)	$2.74(11)\cdot 10^{-8}$	$7.45(52)\cdot 10^{-6}$
$+$	22	25.163(10)	3.163(10)	$6.88(22)\cdot 10^{-9}$	$1.937(51)\cdot 10^{-6}$
$+$	24	27.1491(82)	3.1491(82)	$1.716(45)\cdot 10^{-9}$	$4.92(42)\cdot 10^{-7}$
$+$	26	29.1460(53)	3.1460(53)	$4.183(78)\cdot 10^{-10}$	$1.347(62)\cdot 10^{-7}$
$+$	28	31.1306(52)	3.1306(52)	$1.056(50)\cdot 10^{-10}$	$3.35(10)\cdot 10^{-8}$
$+$	30	33.126(12)	3.126(12)	$2.54(10)\cdot 10^{-11}$	$8.35(42)\cdot 10^{-9}$
$+$	32	35.1299(77)	3.1299(77)	$6.71(17)\cdot 10^{-12}$	$2.36(13)\cdot 10^{-9}$
$+$	34	37.1174(64)	3.1174(64)	$1.39(14)\cdot 10^{-12}$	$4.87(48)\cdot 10^{-10}$
$+$	36	39.1079(78)	3.1079(78)	$4.84(56)\cdot 10^{-13}$	$1.70(17)\cdot 10^{-10}$
$+$	38	41.101(29)	3.101(29)	$8.4(28)\cdot 10^{-14}$	$2.5(11)\cdot 10^{-11}$
$+$	40	43.102(18)	3.102(18)	$2.63(64)\cdot 10^{-14}$	$9.0(26)\cdot 10^{-12}$
$+$	42	45.116(27)	3.116(27)	$7.9(22)\cdot 10^{-15}$	$3.42(95)\cdot 10^{-12}$

[Simmons-Duffin,'16]

Data mining & CFT learning

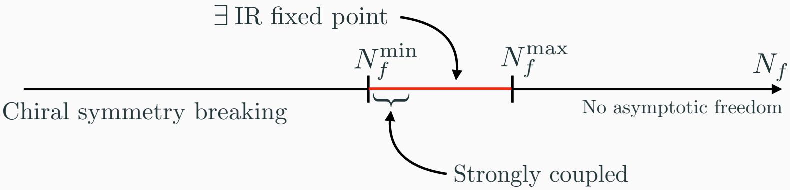
The solution, even if approximate, contains important info:

- operators needed to satisfy crossing (low dim + $\Delta - \ell$ small)
- how fast operators are suppressed
- how analytic structures are reproduced in $\sum_{\mathcal{O}} C_{\mathcal{O}}^2$ 

Study known spectra of interacting CFTs $\Rightarrow$ infer general structure of CFTs

An application to high energy physics

Conformal Window of $SU(N)$ gauge theories with N_f fermions in the fundamental



At the IR fixed point:

- Global Symmetry: $U(1)_V \times SU(N_f)_L \times SU(N_f)_R$
- "Mesons" $M_i{}^{\bar{j}}$: scalars in $\overline{\square} \times \square$
- Bootstrap $\langle M_i{}^{\bar{j}} M_k{}^{\bar{l}} M_p{}^{\bar{q}} M_r{}^{\bar{s}} \rangle$

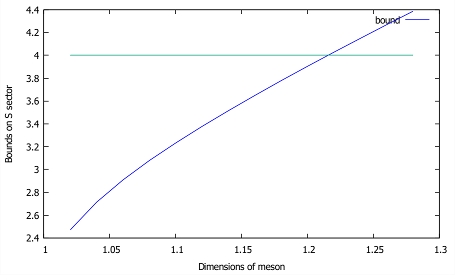
Can we get any constraint on the mass anomalous dimension $\gamma_M \equiv 3 - \Delta_M$?

Case study: $N_f = 8$: stability

At the IR fixed point:

- IR fixed point is stable: no relevant perturbation which is singlet under symmetry group
- The smallest dimension singlet $\sim Tr[F_{\mu\nu}F^{\mu\nu}]$ must be irrelevant

Bound on first singlet in the OPE $M_i^{\bar{j}} \times M_k^{\bar{l}}$:



$\Rightarrow \gamma_M \leq 1.79$

[Nakayama, Yu] 2016

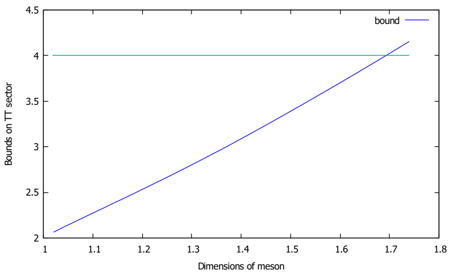
Case study: $N_f = 8$, symmetry enhancement

- Evidences of fixed point from Lattice simulation with Wilson fermions
- Lattice action only implements a sub-group of the global symmetry
- Enhancement to larger global symmetry in IR

$$\Updownarrow$$

Symmetry breaking operators are irrelevant

Bound on the lowest dimension $\overline{\square\square} \times \square\square$ operator in the OPE $M_i{}^{\bar{j}} \times M_k{}^{\bar{l}}$:



$\Rightarrow \gamma_M \leq 1.31$

[Nakayama, Yu] 2016

Conclusions

Scale invariant systems are the building blocks of quantum systems

CFT's in $D \geq 3$ (neglected for many years) are now under siege

- Numerical techniques allow to precisely determine CFT data
- Other complementary approaches available

We have only scratched the surface:
are we on the verge of a "CFT eight-fold way"?