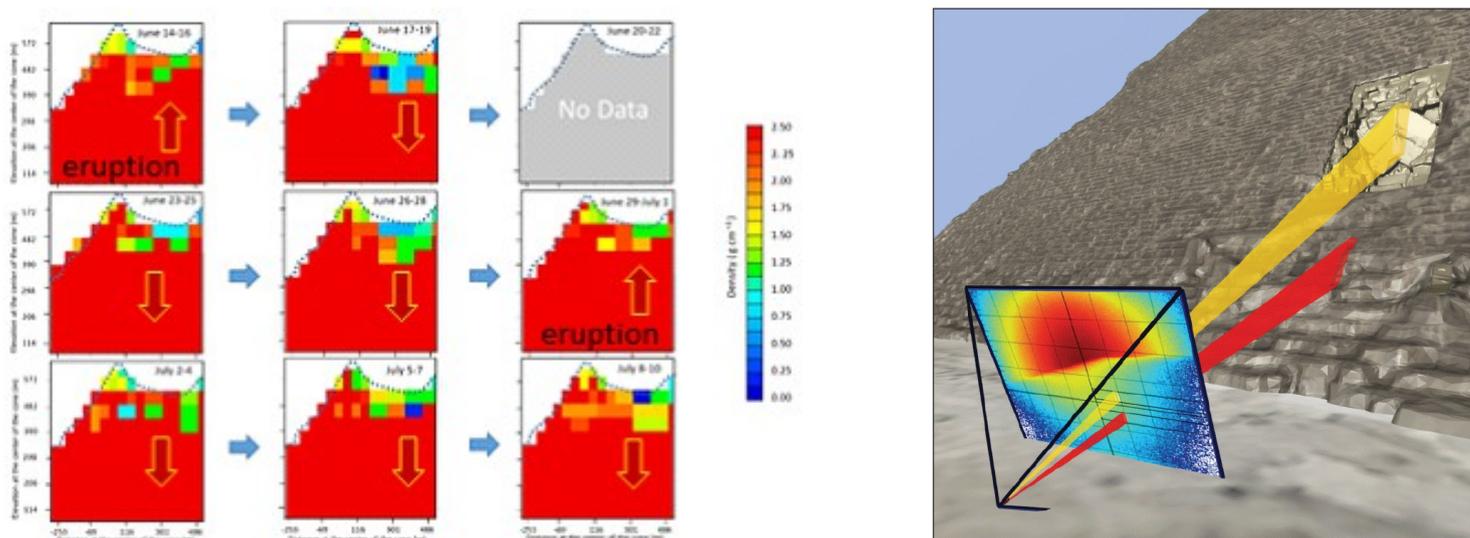


Cosmic rays as an imaging tool



Andrea Giammanco

Centre for Cosmology, Particle Physics and Phenomenology
UCL, Louvain-la-Neuve, Belgium



Muo-what??



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Muography

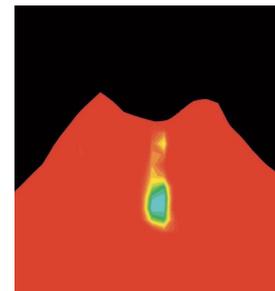
From Wikipedia, the free encyclopedia

Muography is an [imaging](#) technique that produces a projectional image of a target volume by recording [elementary particles](#), called [muons](#), either electronically or chemically with materials that are sensitive to charged particles such as nuclear emulsions. [Cosmic rays](#) from outer space generate muons in the Earth's atmosphere as a result of [nuclear reactions](#) between primary cosmic rays and atmospheric nuclei. They are highly penetrative and millions of muons pass through our bodies every day.

Muography utilizes muons by tracking the number of muons that pass through the target volume to determine the density of the inaccessible internal structure. Muography is a technique similar in principle to [radiography](#) (imaging with [X-rays](#)) but capable of surveying much larger objects. Since muons are less likely to interact, stop and decay in low density matter than high density matter, a larger number of muons will travel through the low density regions of target objects in comparison to higher density regions. The apparatuses record the trajectory of each event to produce a muogram that displays the matrix of the resulting numbers of transmitted muons after they have passed through hectometer to kilometer-sized objects. The internal structure of the object, imaged in terms of density, is displayed by converting muograms to muographs.



m



km

Scale

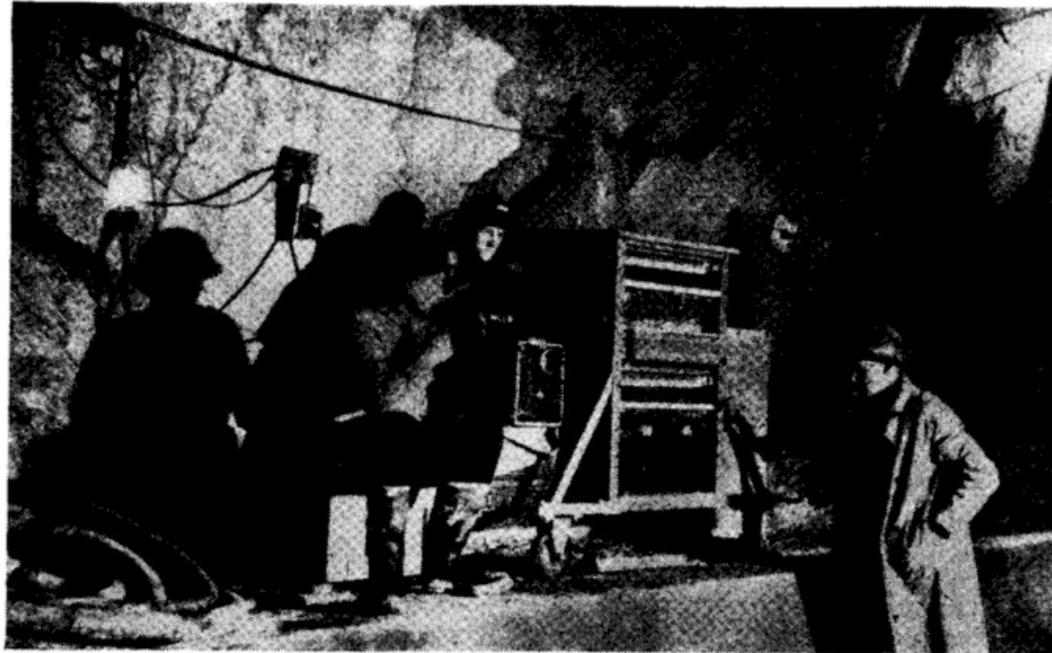
First known application of muography (1955)

Commonwealth Engineer, July 1, 1955

455

Cosmic Rays Measure Overburden of Tunnel

● Fig. 1—Geiger counter “telescope” in operation in the Guthega-Munyang tunnel. From left are Dr. George and his assistants, Mr. Lehane and Mr. O’Neill.

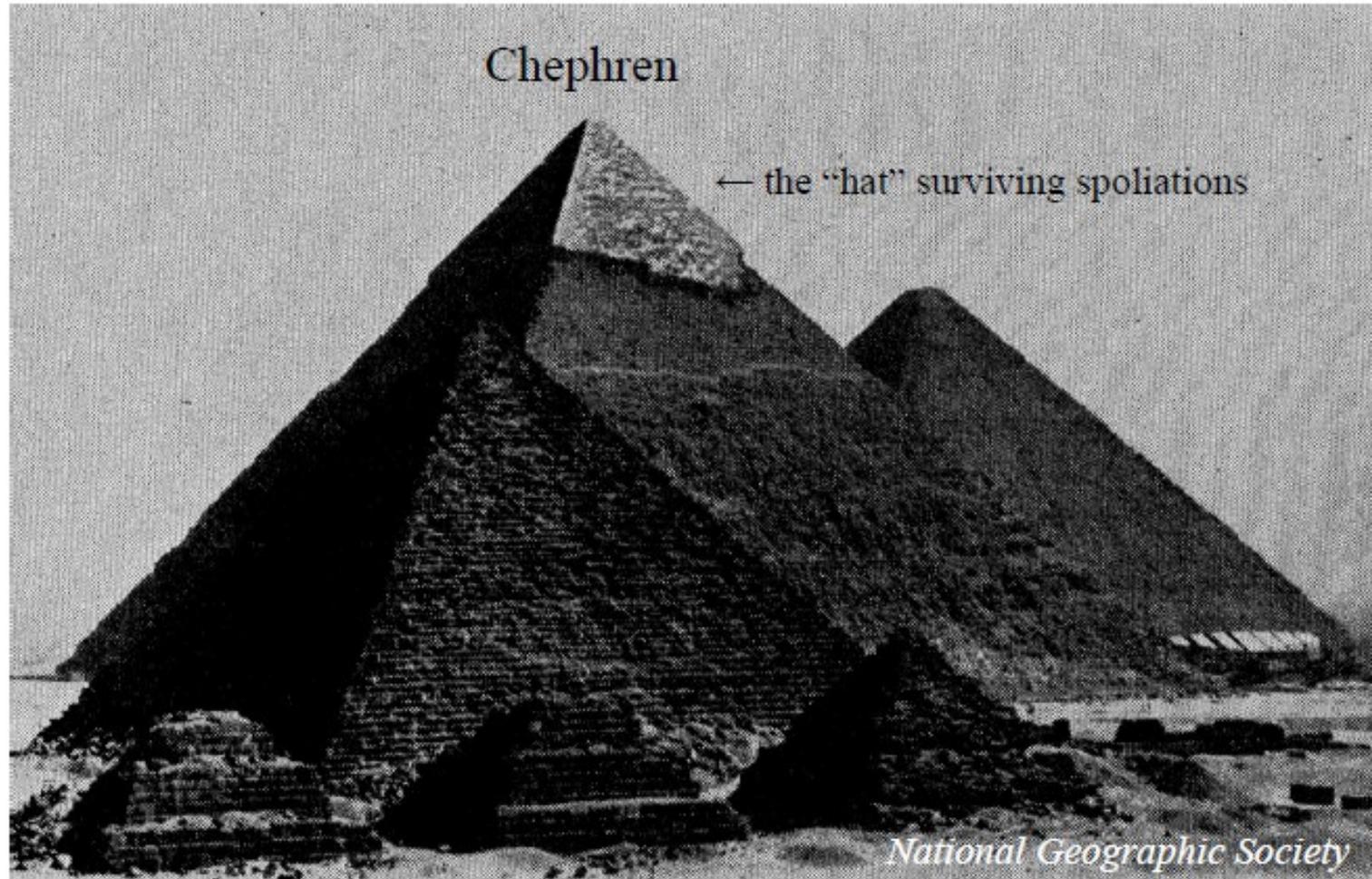


Geiger counter telescope used for mass determination at Guthega project of Snowy Scheme . . . Equipment described

By Dr. E. P. George[®]
University of Sydney, N.S.W.

- Muon flux used to measure ice thickness above a tunnel
- No directional information

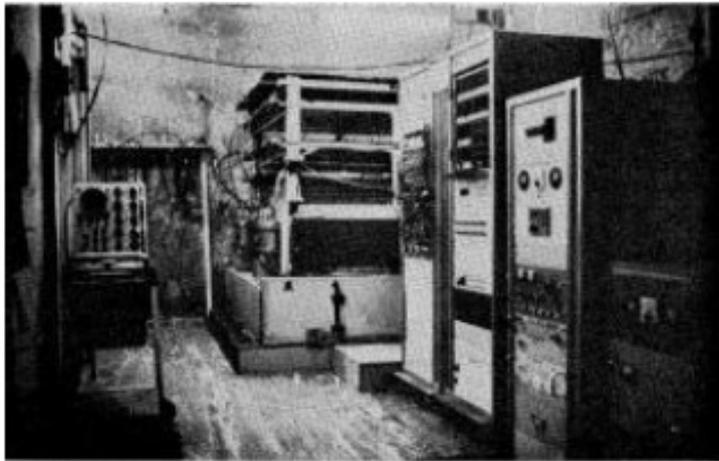
First application of muography to archaeology



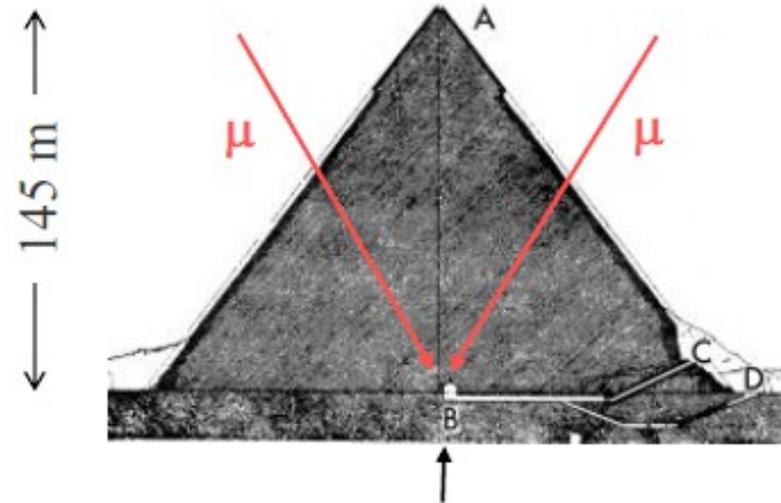
Search for hidden chambers in the Chephren's Pyramid

L.W. Alvarez et al. *Science* 167 (1970) 832

Alvarez's result: no hidden chamber

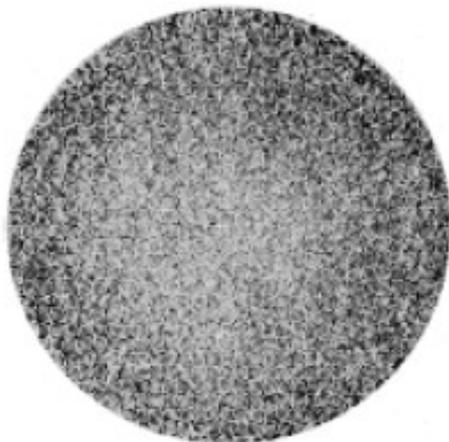


Spark chamber "muon telescope"

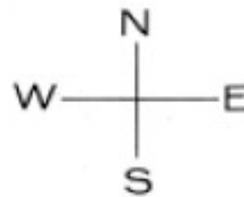


Telescope in Belzoni chamber

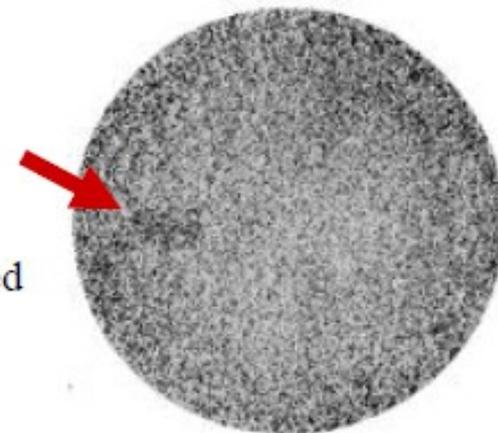
Data



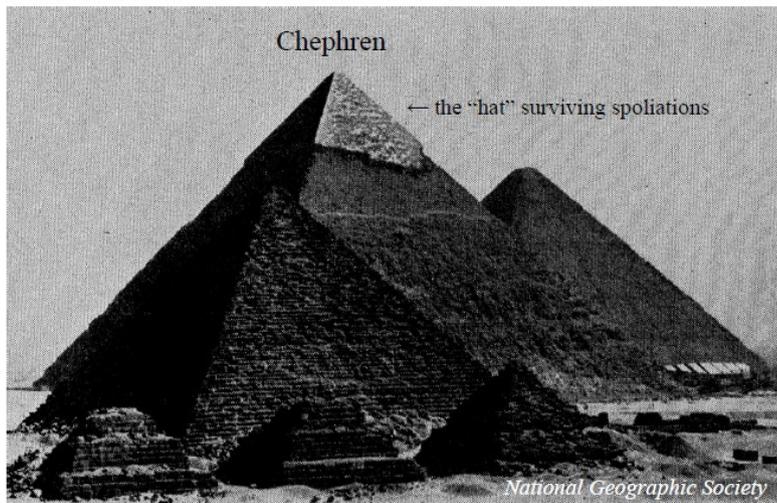
Simulation with hidden chamber



Data and simulation are corrected for pyramid structure and telescope acceptance

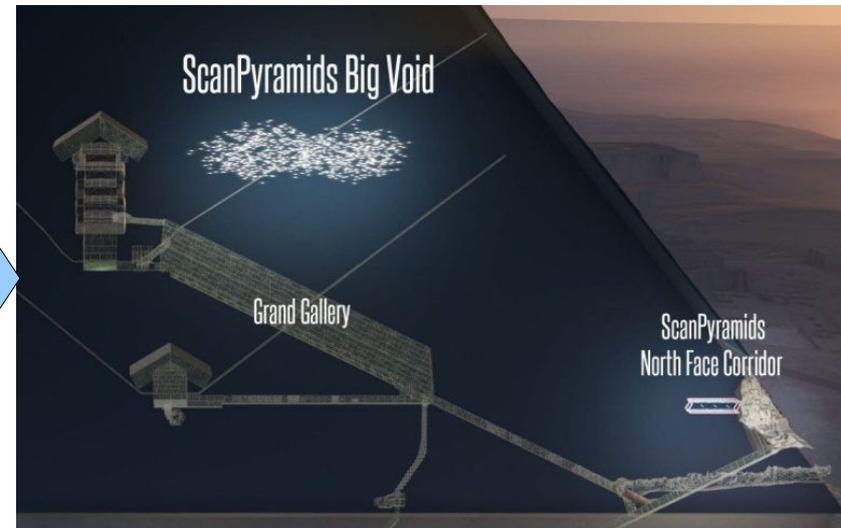
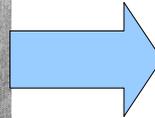


Fast-forward by ~50 years



Search for hidden chambers in the Chephren's Pyramid

L.W. Alvarez et al. *Science* 167 (1970) 832



Discovery of a big void in Khufu's Pyramid by observation of cosmic-ray muons
Morishima et al., *Nature* 552 (2017) 386

Alvarez chose the wrong pyramid...

(But would have he been able to spot this void?)

Outline

1. Some physics
2. Muography How-to
3. A few selected applications

Disclaimer:

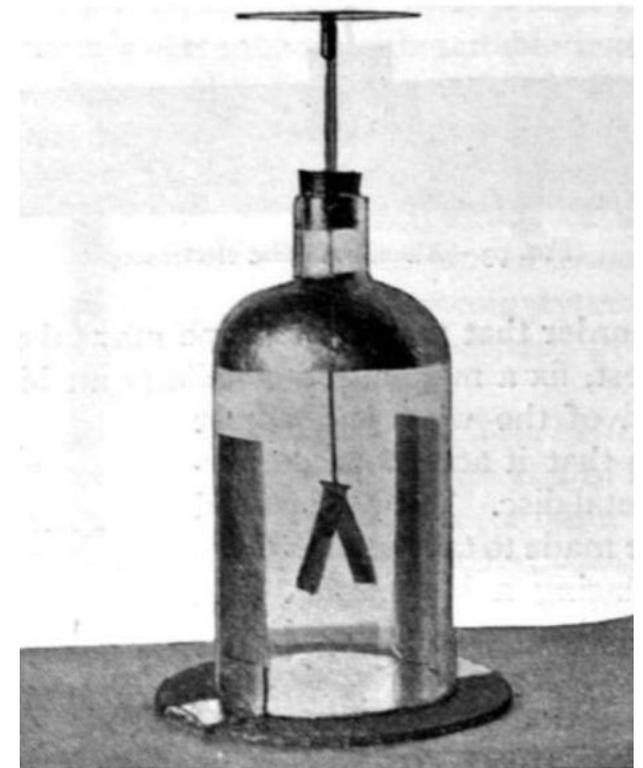
- Choice of sub-topics is very personal, not representative at all of the variety of activities in this area

1. Some physics

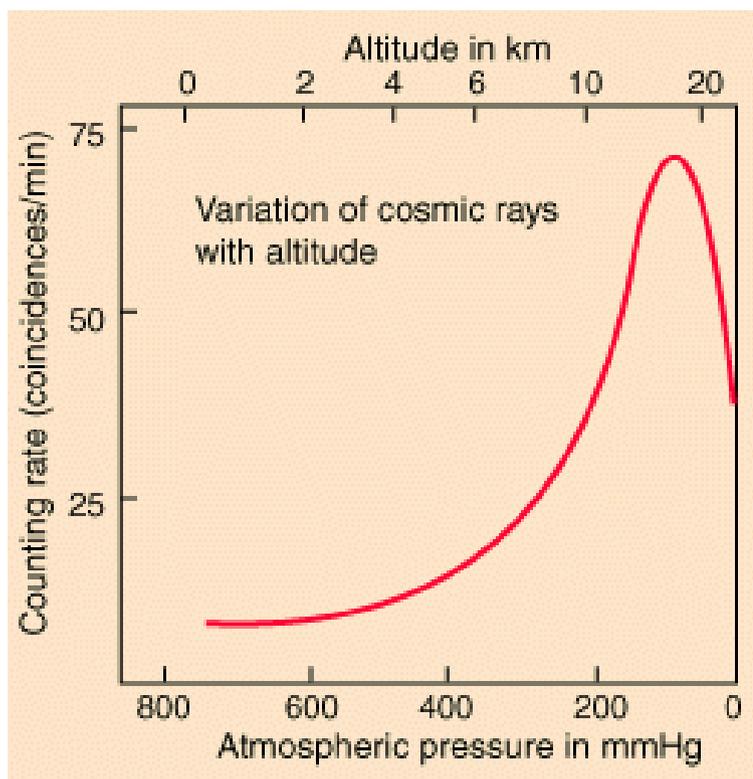
Discovery of cosmic rays

- Early 1900s: hypothesised that there is a natural background of ionizing radiation that discharges all the electroscopes
- People believed it was mostly due to radioactive rocks
- 1909: Theodor Wulf uses Tour Eiffel to measure this background at different heights; surprisingly, he reported that it increases with the altitude, but measurements were not so precise and he was met with skepticism
- 1911-12: Victor Hess improved the instrument and used a balloon to study the phenomenon between 1000 and 5000 m over sea level

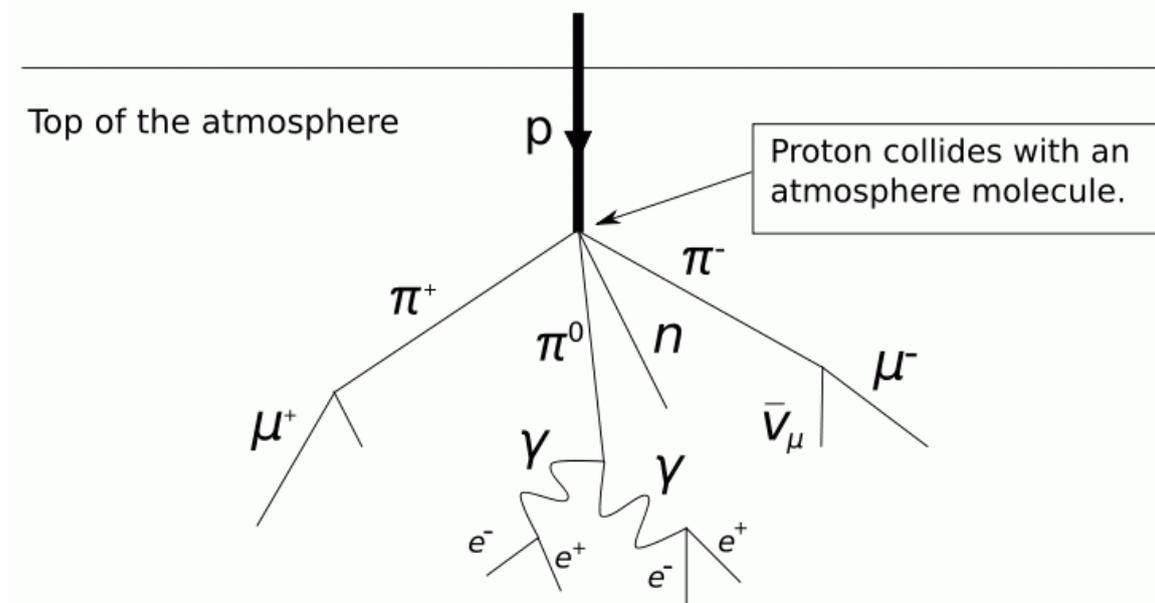
When the device is charged, the sheets move apart. Ionization of the gas leads to a discharge, and the sheets move towards each other.



Primary and secondary cosmic rays



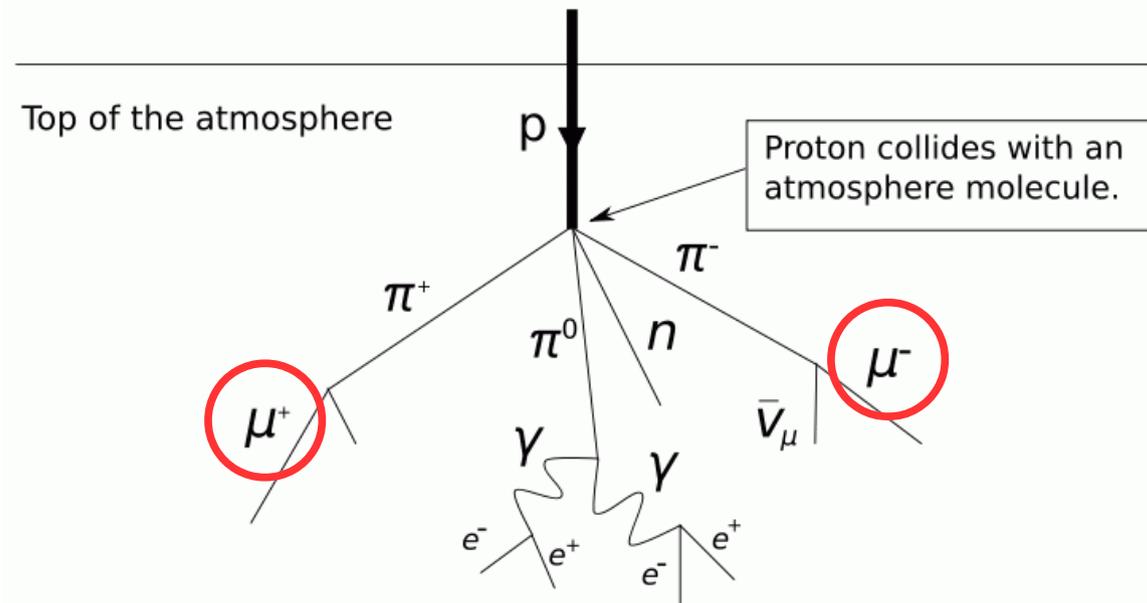
Picture from [here](#)



From wikipedia

The number of charged particles increases as the cascade progresses, but eventually most of them are absorbed

Secondary cosmic rays in the atmosphere



- Primary CRs (mostly protons) entering the atmosphere collide mostly with Oxygen and Nitrogen, producing a shower of particles
- Mostly x-rays, **muons**, protons, alphas, pions, electrons, neutrons
- They tend to stay within 1° of the direction of the primary CR
- Muon rate: $\sim 100 \text{ Hz/m}^2$ (~ 1 muon/second through your thumb)

An old mystery

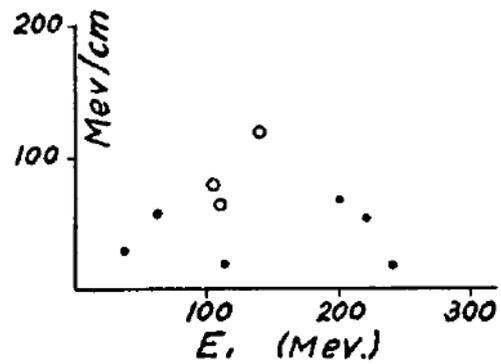
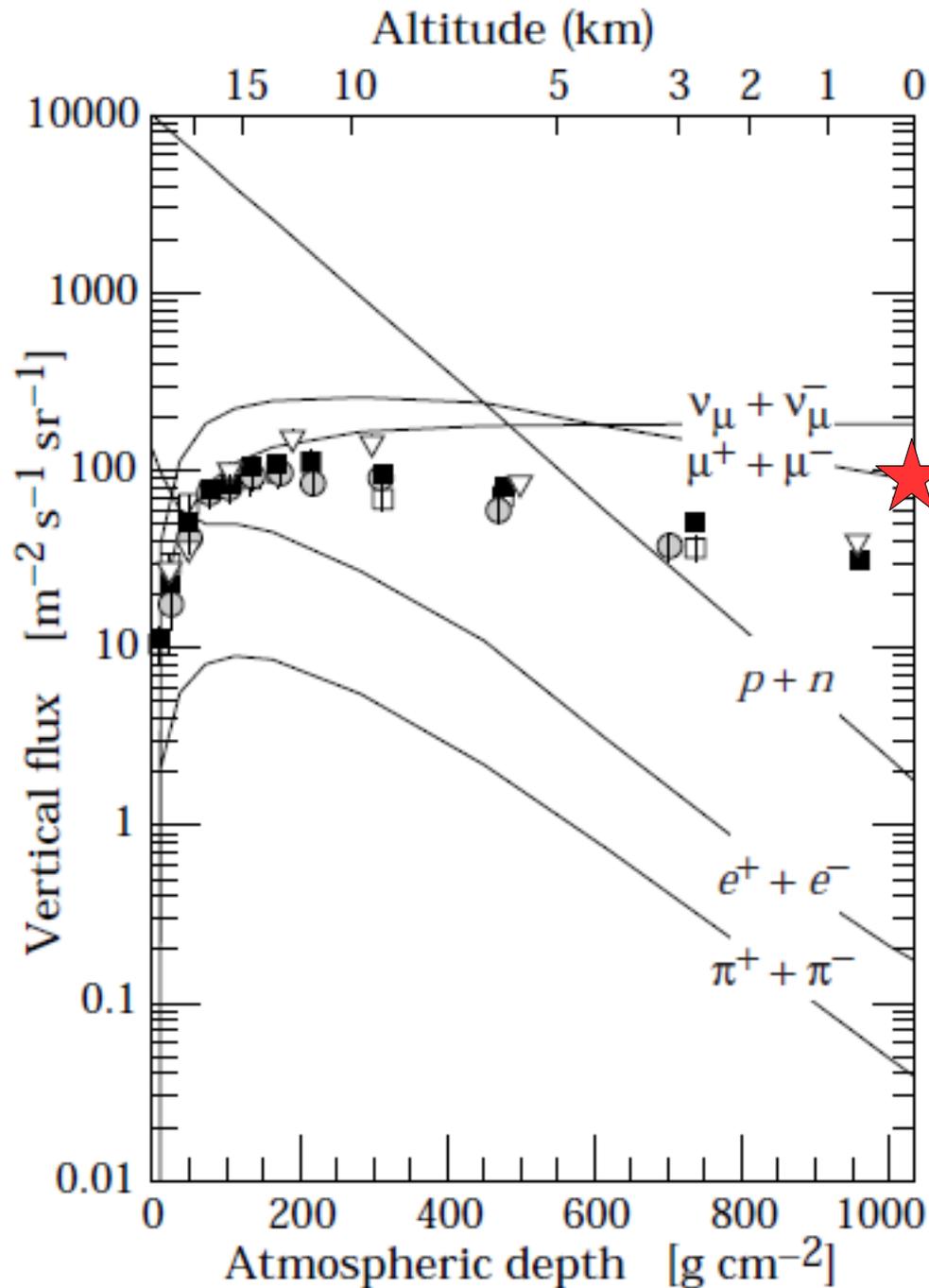


Figure 4: Early measurements of energy loss in 0.7-1.5 cm of Pb. Dots indicate single particles; circles, shower particles.

„Who ordered that?“ (I. Rabi)



At ground level, the visible flux is dominated by muons

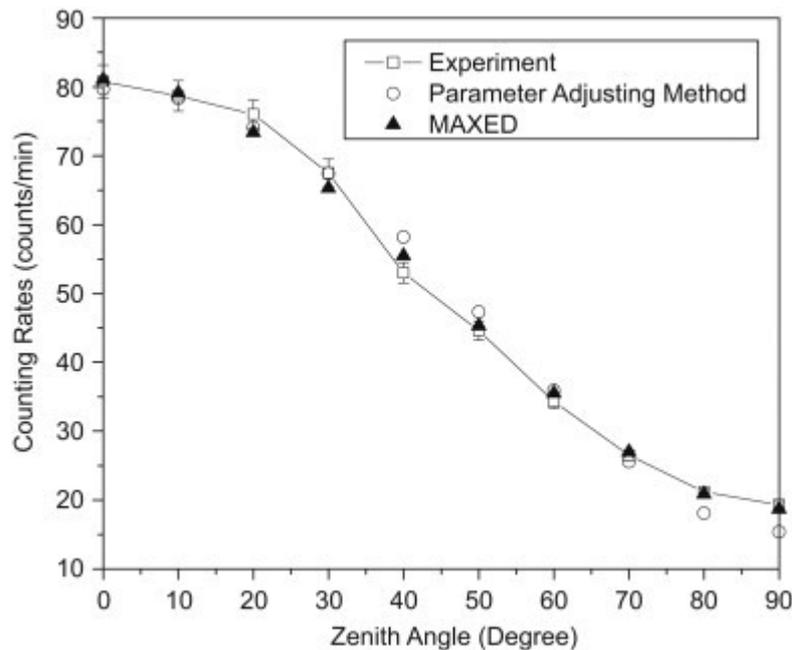
Source: Particle Data Group

All curves are for $E > 1 \text{ GeV}$; points are experimental measurements for negative muons

Angular distribution

$$I_{\theta} = I_0 \cos^n \theta$$

This is an approximation, and $n \sim 2$ works pretty well; but it depends on energy, latitude, altitude, depth, ...



From J.-W. Lin et al., *Measurement of angular distribution of cosmic-ray muon fluence rate*, NIM A 619 (2010) 24

⇒ Large difference in statistics between vertical and horizontal telescopes

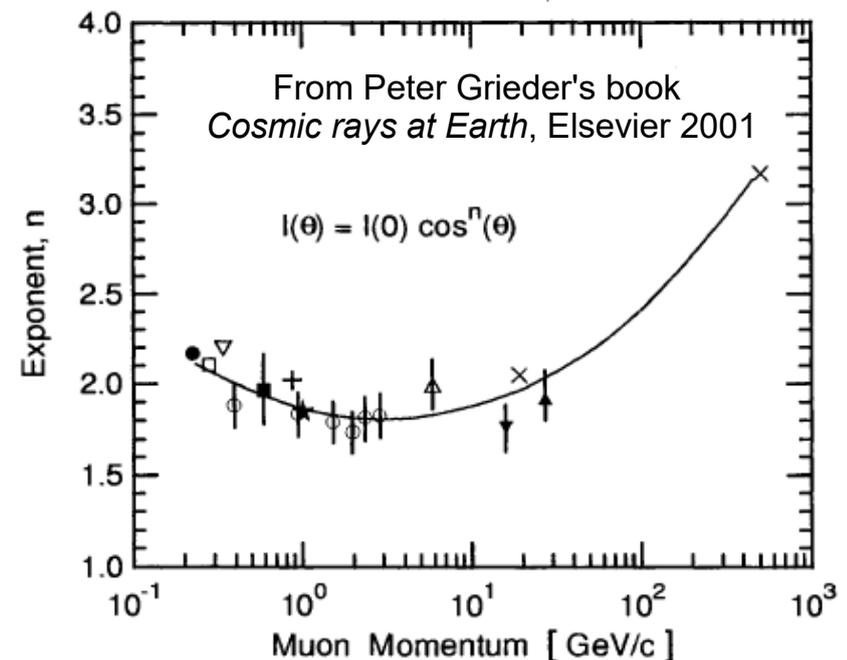
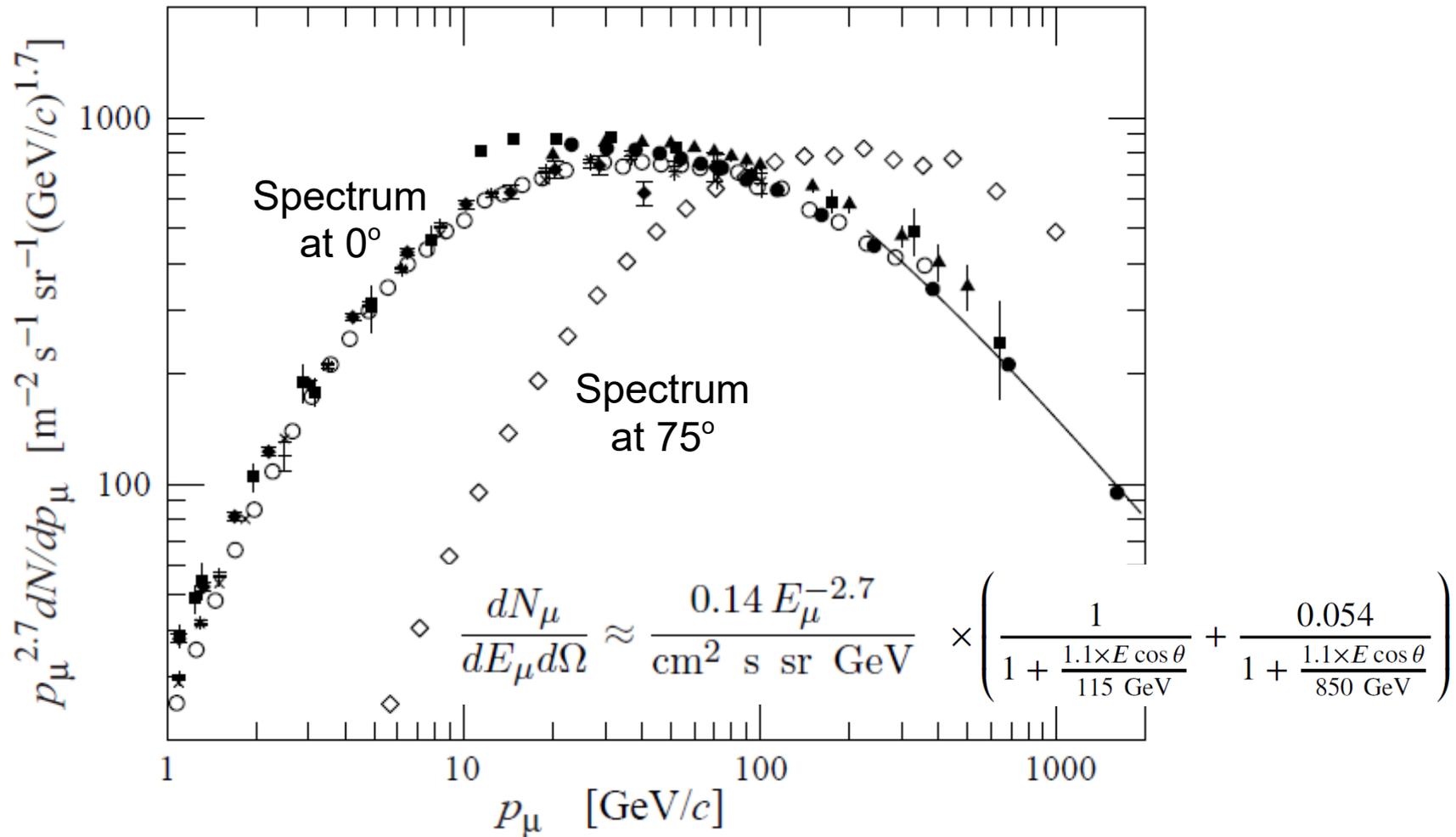


Figure 3.60: Momentum dependence of the exponent, n , of the zenith angular distribution of muons, $I(\theta, > p) = I(0^\circ, \geq p) \cos^n(\theta)$ at sea level (Bhattacharyya, 1974b).

Atmospheric muon spectrum



At large angle, more low-E muons decay before reaching the ground, and more high-E pions decay before interacting.

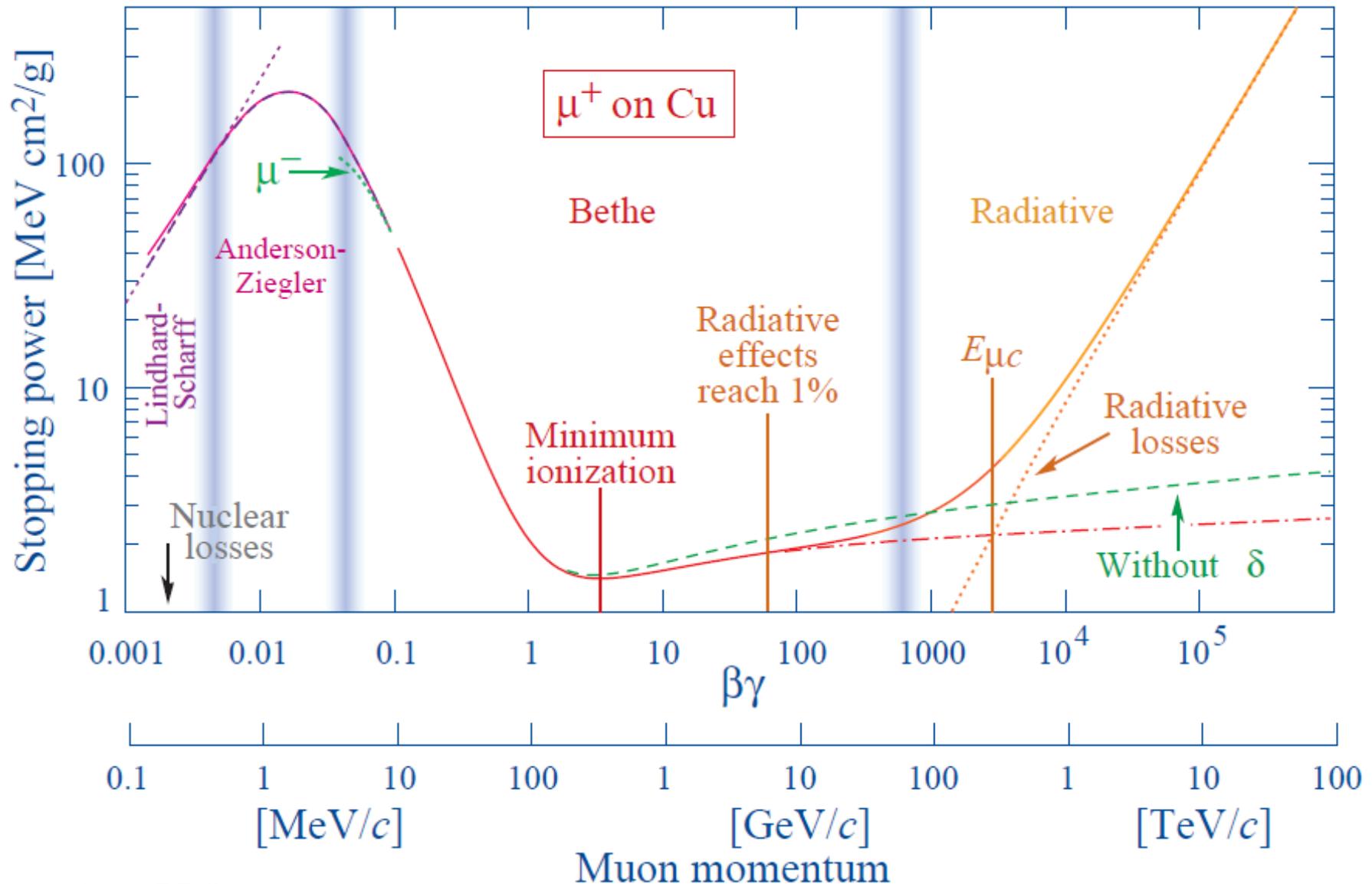
2. How-to

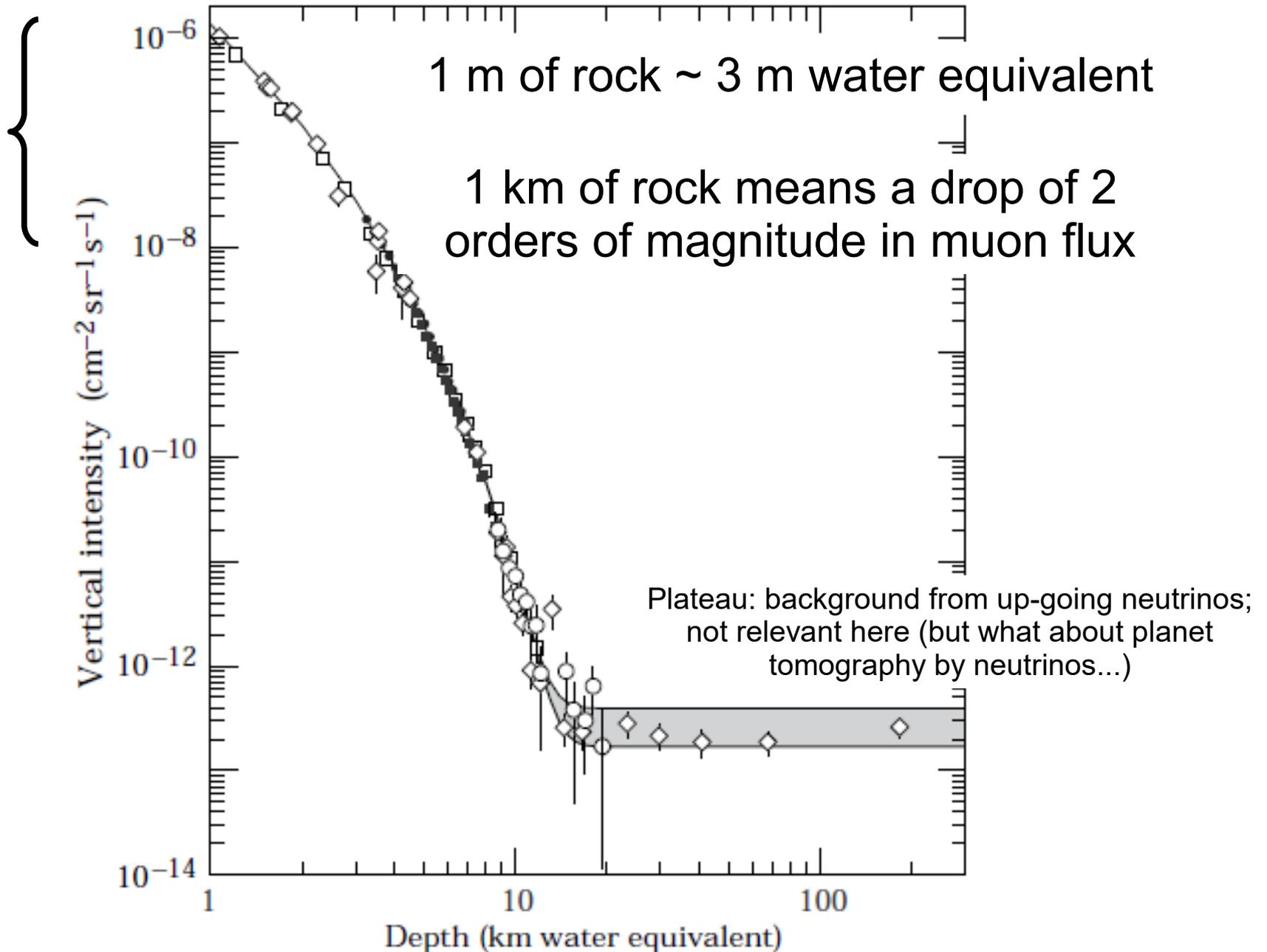
Absorption method



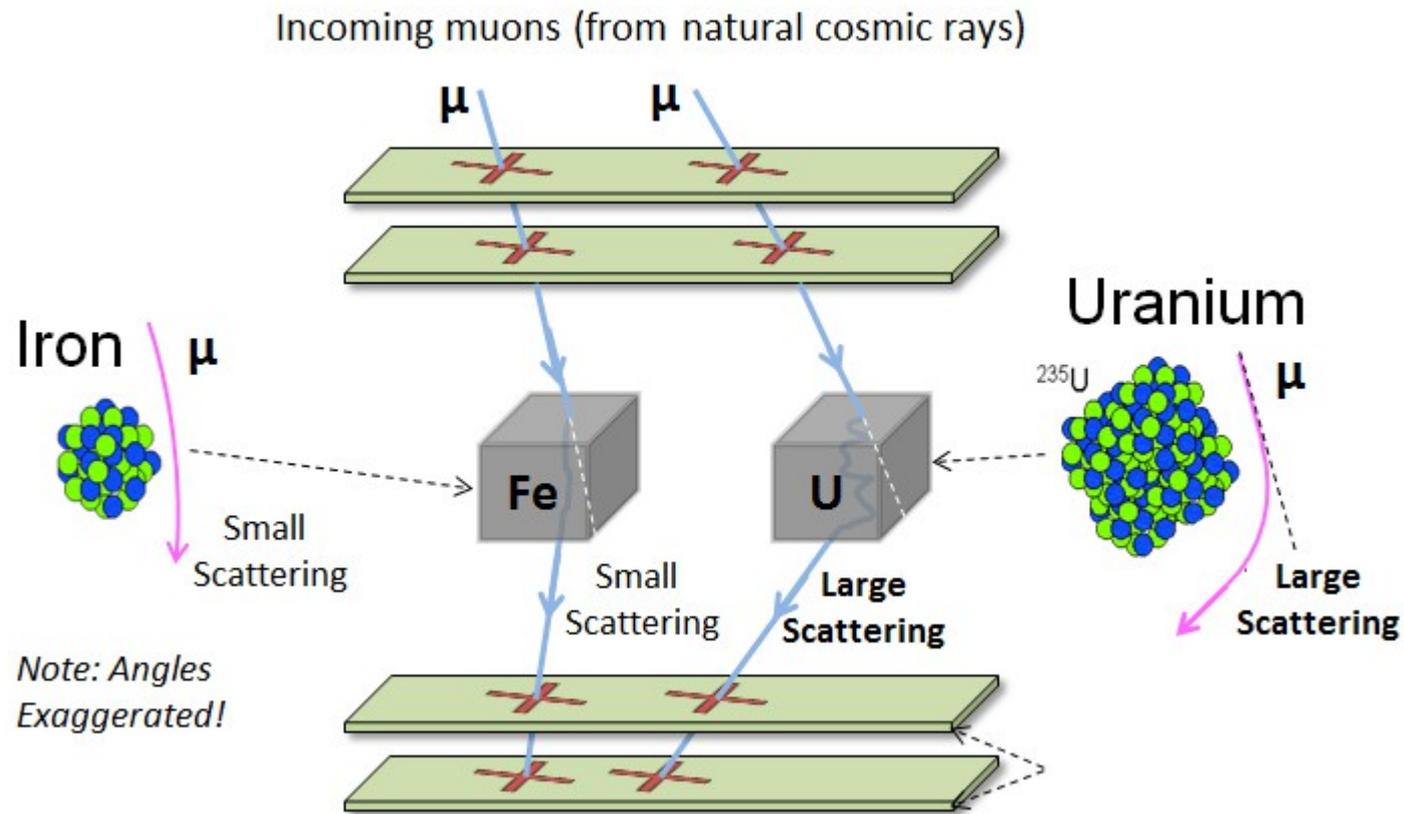
- Basic idea: just like normal radiography, with μ instead of X-rays

Stopping power

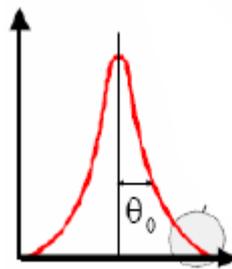
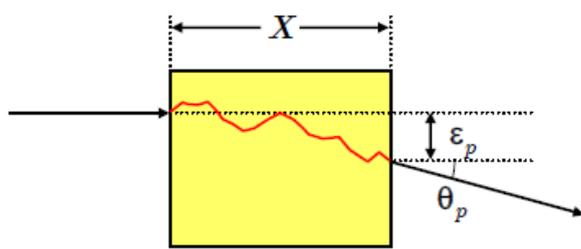




Scattering method



- Exploits multiple scattering in high-Z materials (e.g., spent nuclear reactor fuel, smuggled fissile material, etc.)



$$\frac{1}{\sin^4 \frac{\theta_p}{2}}$$

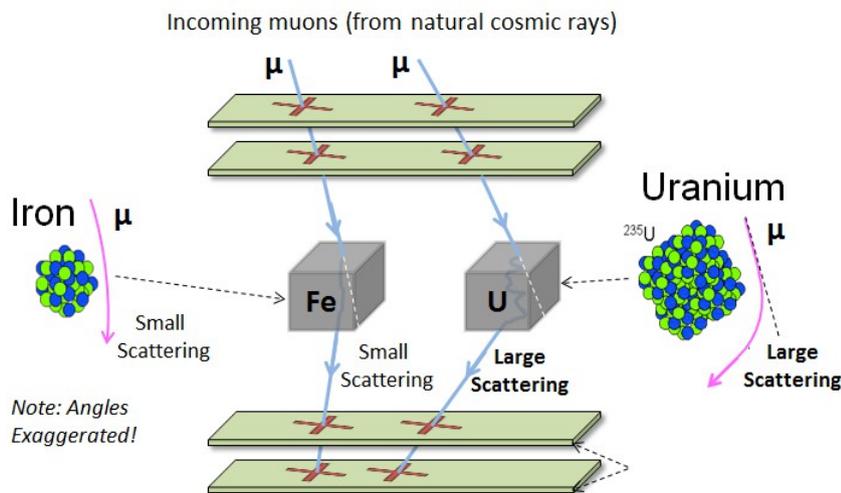
$$P(\theta_p) = \frac{1}{\sqrt{2\pi \langle \theta_p^2 \rangle}} \exp \left[-\frac{1}{2 \langle \theta_p^2 \rangle} \theta_p^2 \right]$$

- Deflection distribution follows Rutherford's law in the tails (single hard scattering) and is \sim Gaussian in the bulk (multiple scattering)

$$\langle \theta_p^2 \rangle = K \frac{X}{X_0}$$

$$K = z^2 \left(\frac{0.0136}{p\beta} \right)^2$$

- X_0 is the radiation length, and it depends on the atomic number

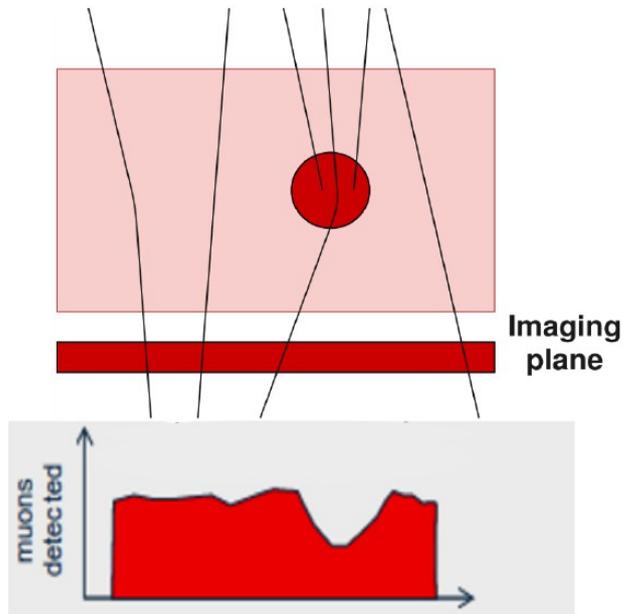


$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 [L_{\text{rad}} - f(Z)] + Z L'_{\text{rad}} \right\}$$

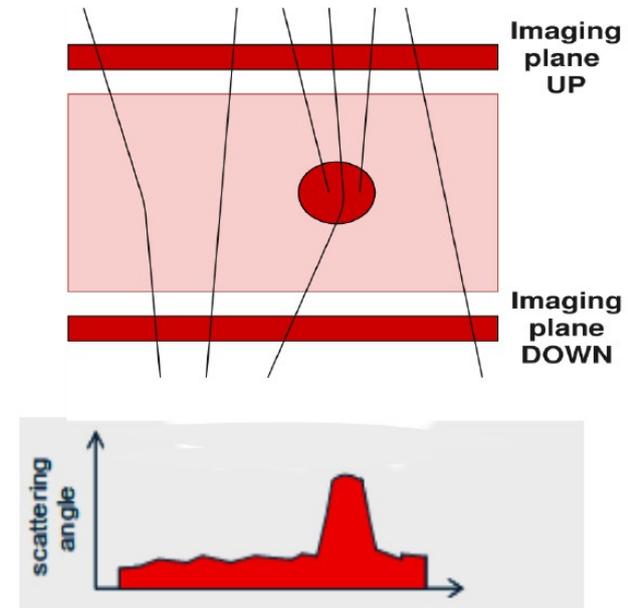
$$f(Z) = a^2 \left[(1 + a^2)^{-1} + 0.20206 - 0.0369 a^2 + 0.0083 a^4 - 0.002 a^6 \right],$$

where $a = \alpha Z$

Absorption vs scattering

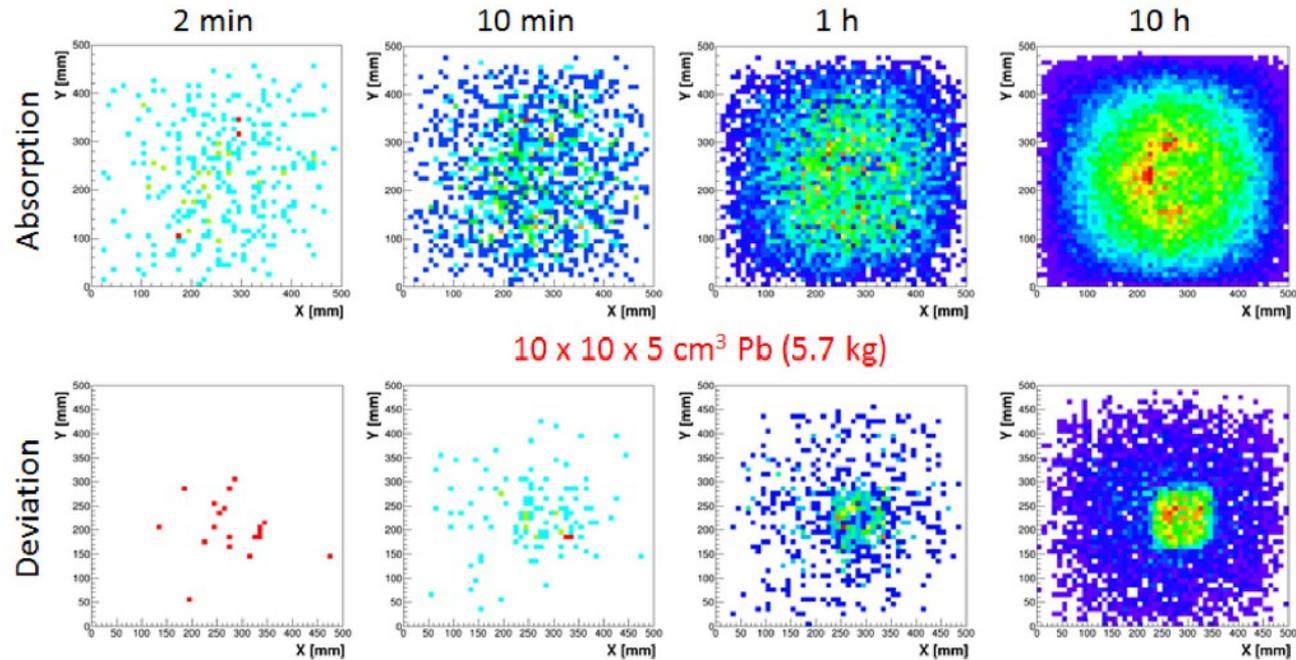


- Opacity measurement
- Sensitive to ρ
- Observable: deficit with respect to free sky
- Intrinsically 2D, can get 3D by using multiple points of view
- Slow

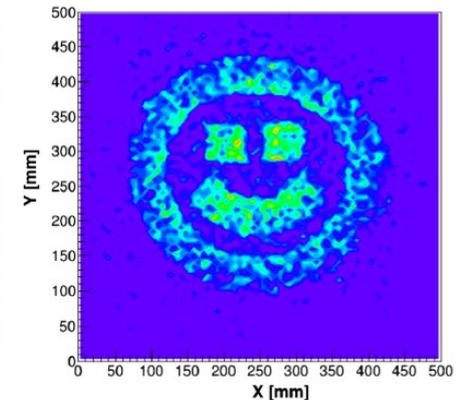
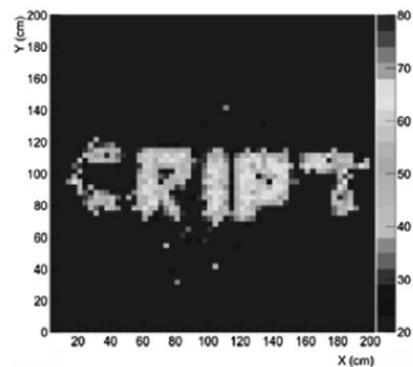
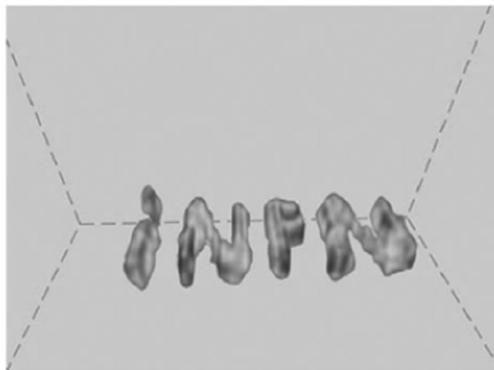


- Deflection measurement
- Sensitive to Z and ρ
- Observable: deflection
- (can be combined with absorption)
- Intrinsically 3D
- Fast

Absorption vs scattering

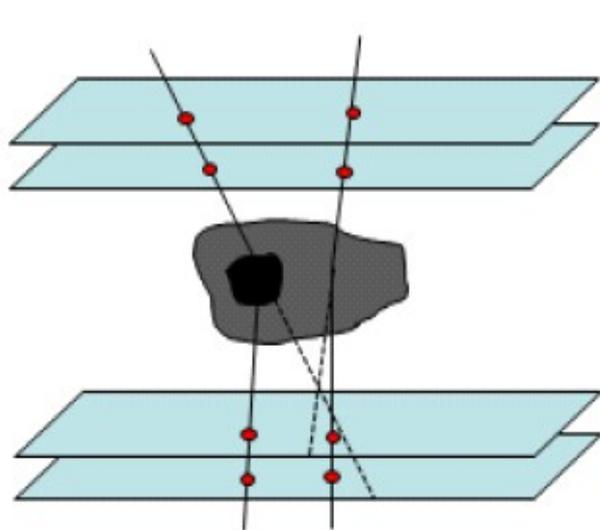


- Scattering method is much faster, as it uses more information
- Better definition too

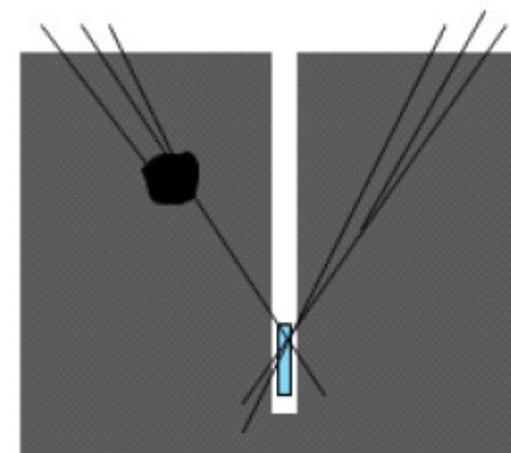
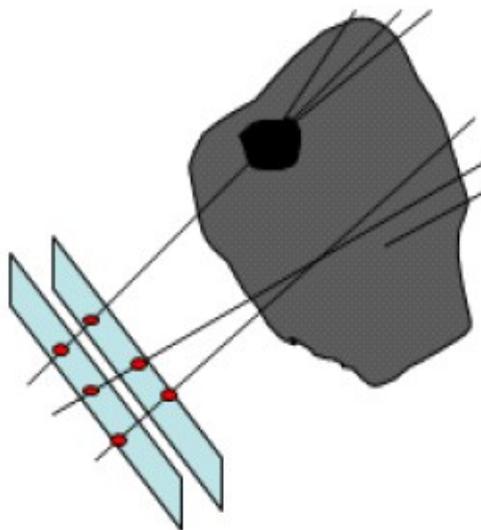


What to use for what

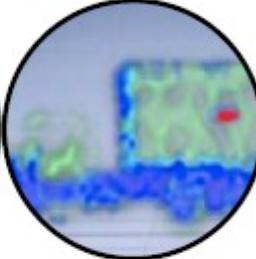
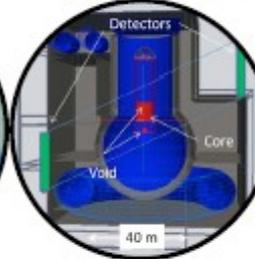
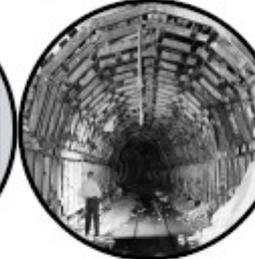
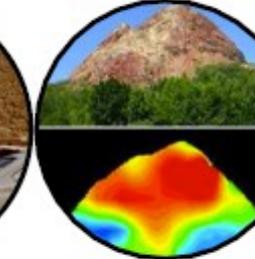
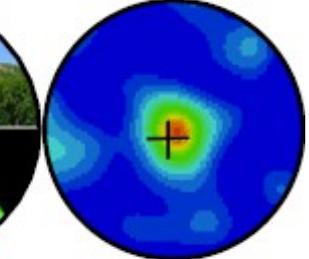
Material	Thickness	θ (°)	$P_{\text{absorption}}$
Air	100 m	0.094	0.78%
Lead	10 cm	1.01	2.9%
Water	1 m	0.35	4.2%
Ground	100 m		99%



Scattering



Absorption

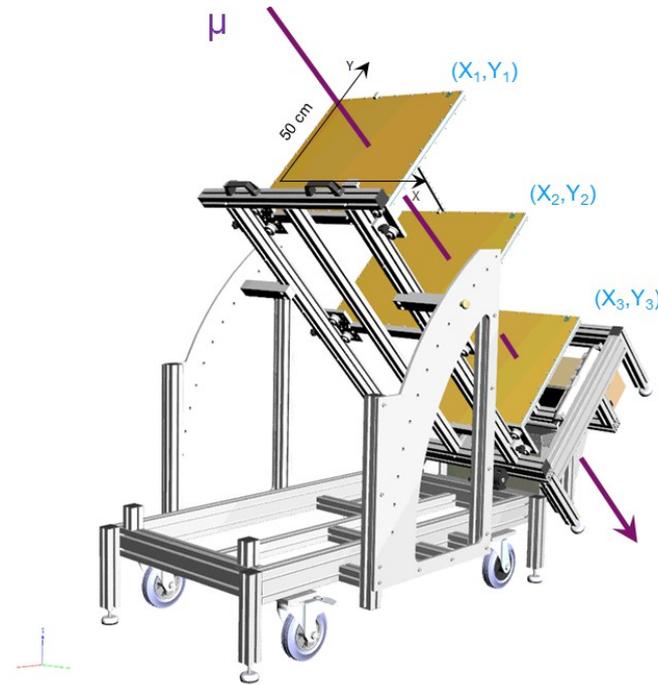
<p>Prototype detector system constructed in Glasgow for UK legacy nuclear waste assay</p>	<p>Los Alamos utilise Coulomb scattering of muons for the first time to identify nuclear contraband for border security</p>	<p>Simulation studies show the feasibility of identifying voids and movement of reactor core material at the stricken Fukushima-Daiichi plant</p>	<p>First reported use in radiography by E. P. George in Australia to measure ice thickness above a tunnel</p>	<p>Muon radiography used by Nobel Prize winner Luis W. Alvarez to look for hidden chambers in the Second Pyramid of Chephren in Egypt</p>	<p>Tanaka <i>et al.</i>, imaged the density profile of the 1944 Usa lava dome with cosmic-ray muon radiography</p>	<p>At the Soudan II detector in Minnesota (located 700m underground), the Moon shadow is imaged using muon flux</p>
<p>Clarkson <i>et al.</i>, Nucl. Instrum. Meth A 745, (2014) 138</p>	<p>Borozdin <i>et al.</i>, Nature 422 (2003) 277</p>	<p>Borozdin <i>et al.</i>, Phys. Rev. Lett. 109 (2012) 152501</p>	<p>George, Commonwealth Engineer July 1 (1955) 455</p>	<p>Alvarez <i>et al.</i>, Science 176 (1970) 832</p>	<p>Tanaka <i>et al.</i>, Geo. Res. Lett. 34 (2005) 22311</p>	<p>Cobb <i>et al.</i>, Phys. Rev. D 61 (1999) 092002</p>
						

Small volumes with large density variations favours scattering

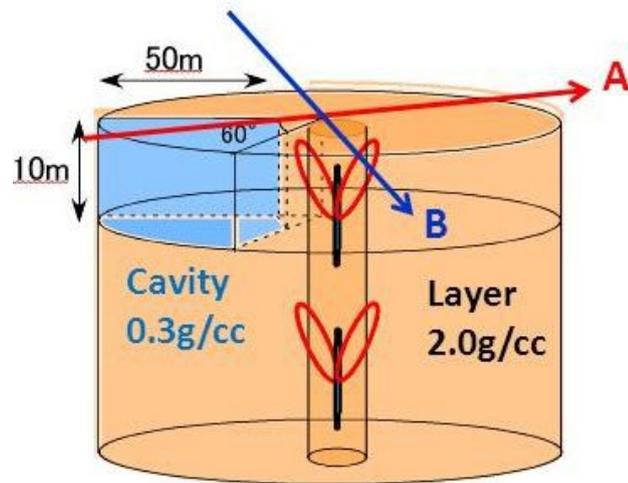
Large volumes with small density variations favours energy loss

Detector geometry

- Typically, a "telescope"
 - Trade-off between angular precision (better if long) and acceptance (better if short)
- Deep underground applications (e.g., for mining exploration) need a borehole
 - More acceptance to the sides than to the top

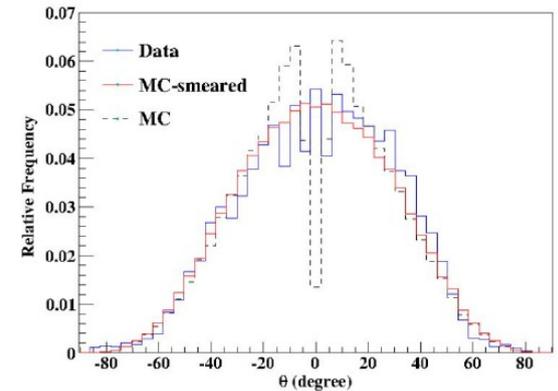
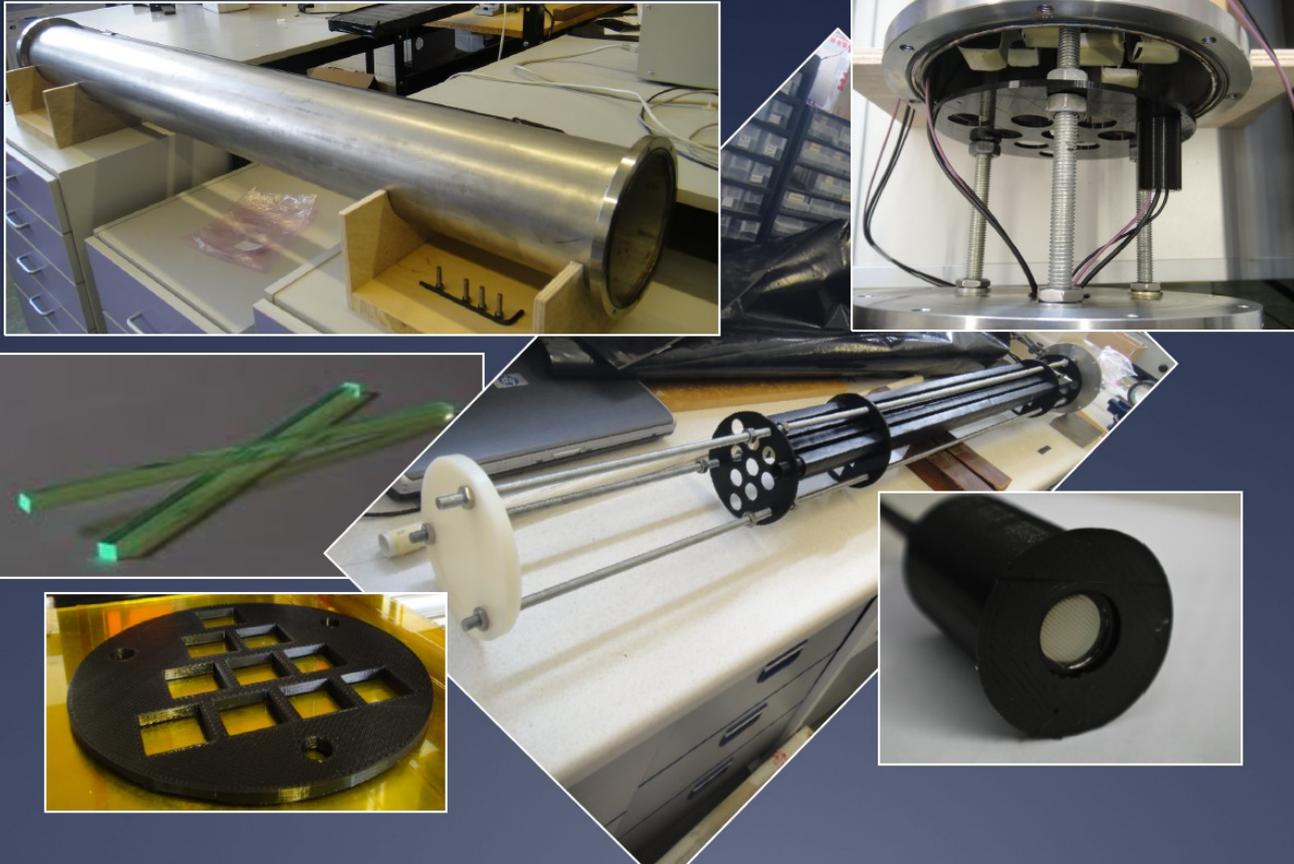


CEA telescope for ScanPyramids



TOMUVOL apparatus

Borehole Detector Prototype



- Representative example from Project Deep Carbon (application to monitoring of Carbon Capture & Storage sites), slides by Lee Thompson

Scintillators

- Solid plastic scintillators, coupled to photomultipliers
- Strengths:
 - ✓ Cheap
 - ✓ Robust
 - ✓ Quick signal → can use time-of-flight to reject backgrounds
- Weaknesses:
 - × Poor space resolution
 - × Photomultipliers response may depend on temperature (issue if operating outdoors for months)

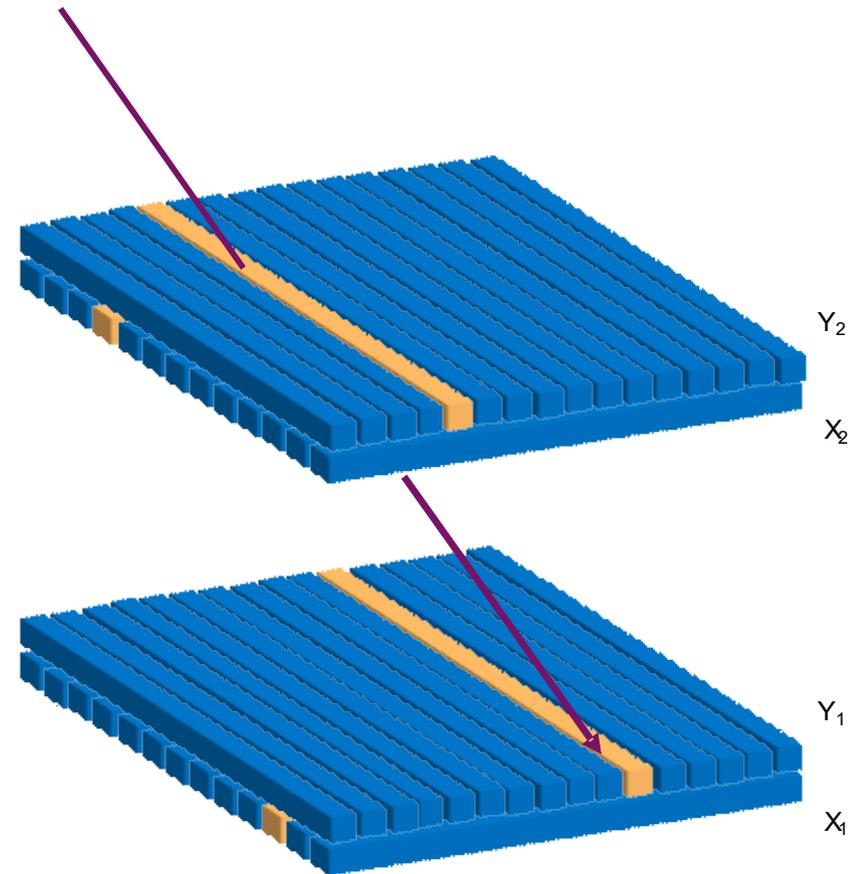
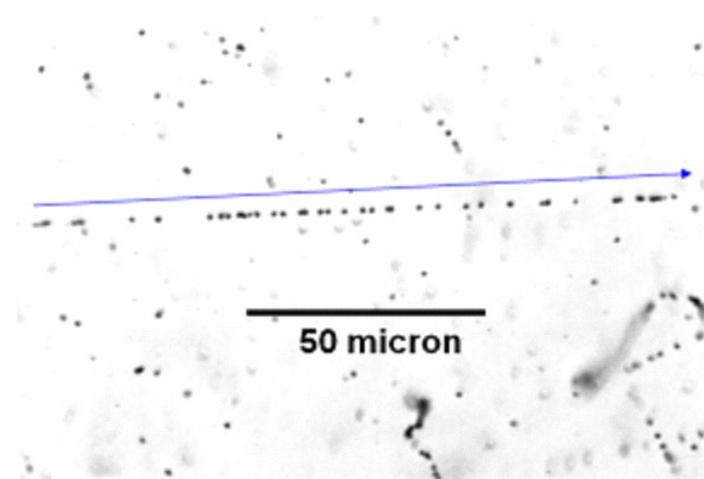
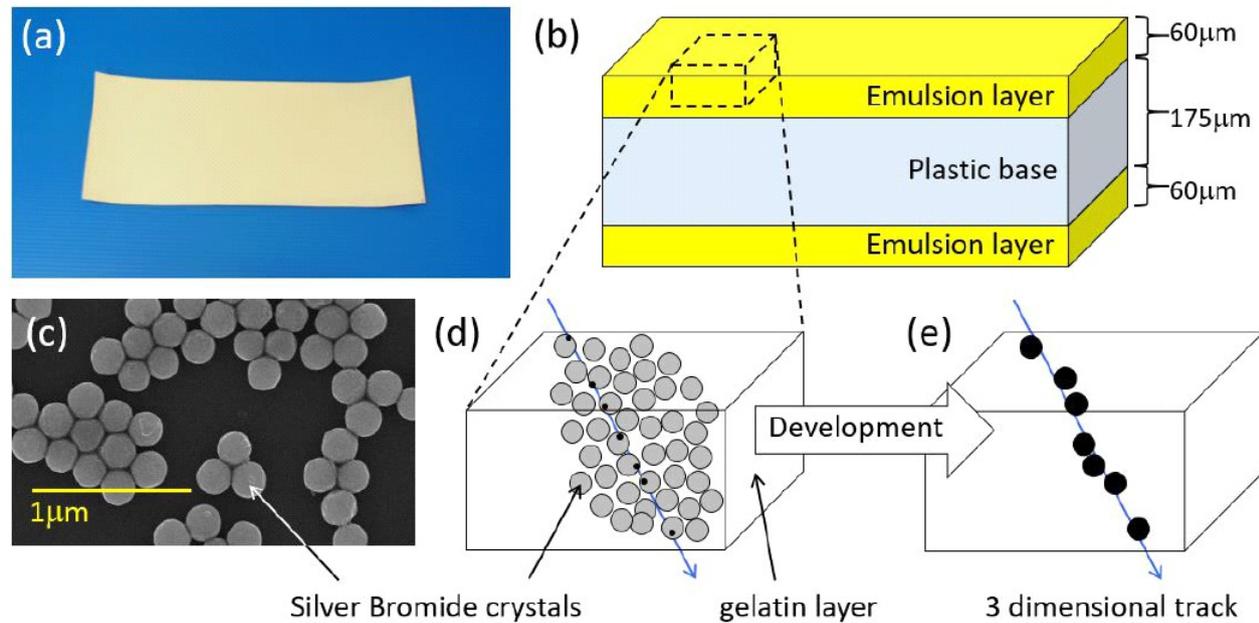


Illustration by S.Procureur

Nuclear emulsions

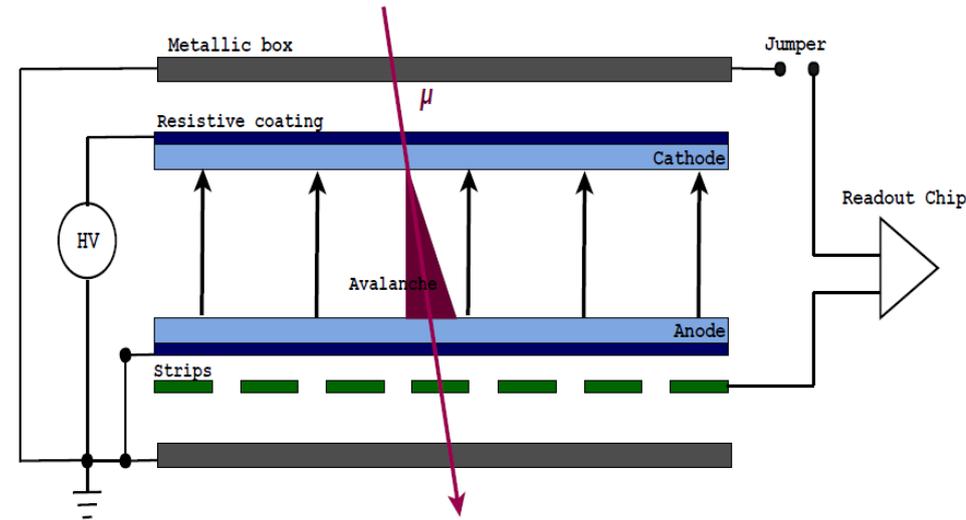
- Photographic plates
- Well known technique in neutrino experiments
- Strengths:
 - ✓ Excellent resolution
 - ✓ No need for power supply
- Weaknesses:
 - ✗ Fragile
 - ✗ No real-time information
 - ✗ No background rejection
 - ✗ Dedicated analysis infrastructure (scanners)



Popular among the muography teams that are spin-offs of OPERA or other ν experiments that used this technique (e.g.: Bern, Salerno, Nagoya)

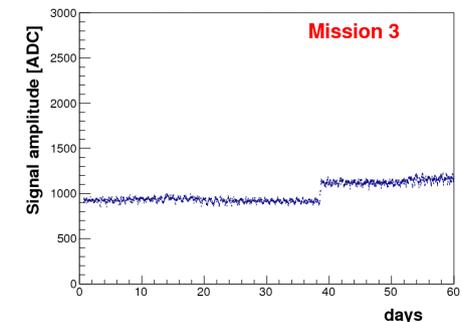
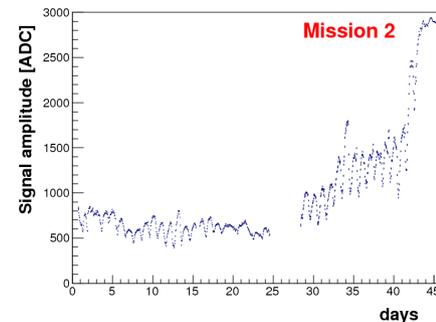
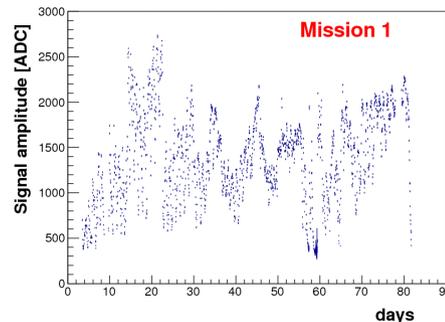
Gaseous detectors

- Huge variety of techniques are in use in muography (drift tubes, RPC, MWPC, MicroMegs, ...), with very different complexity, cost, robustness
- General strengths:
 - ✓ Very good space resolution
 - ✓ Quick signal → can use time-of-flight to reject backgrounds
- General weaknesses:
 - ✗ Logistics (gas bottles), leakages, security issues
 - ✗ Stability



Example: RPC, illustration by Sophie Wuyckens

Gain variations of CEA/ScanPyramid MicroMegs detector, with increasingly complex gain corrections:



More exotic choices

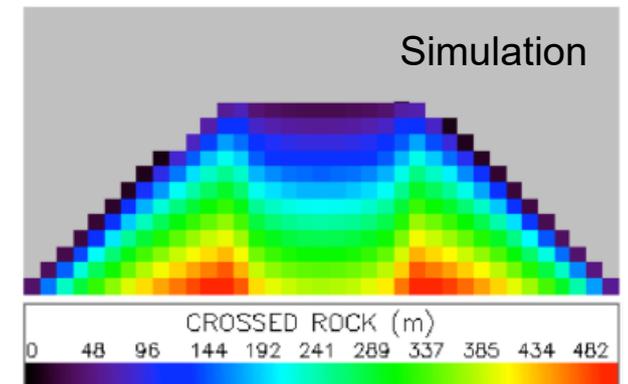
- Silicon detectors

- Lot of expertise in HEP with Si pixel and microstrip detectors; $<100 \mu\text{m}$ resolution
- Problem: very expensive (CMS microstrips: ~ 1000 euros per module)
- Being considered for space applications (compact payload, and rad-hard)

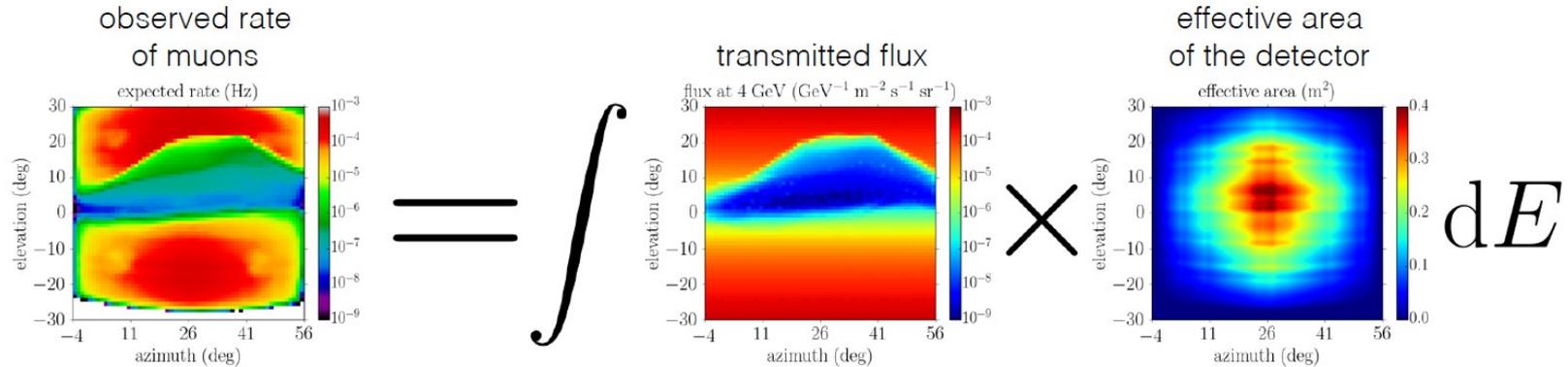


- Cherenkov detectors

- ASTRI, an INAF prototype for the CTA, located at Serra La Nave, on Mt. Etna's slopes
- Cosmic muons used for calibration; "parasitic" usage for muography; but location not optimal (5 km from target) and definitely not portable
- INAF-PA team is proposing the development of a cheaper and portable version of ASTRI
- Momentum threshold (20 GeV) limits statistics
- Bonus: Cherenkov ring radius gives E_{μ}

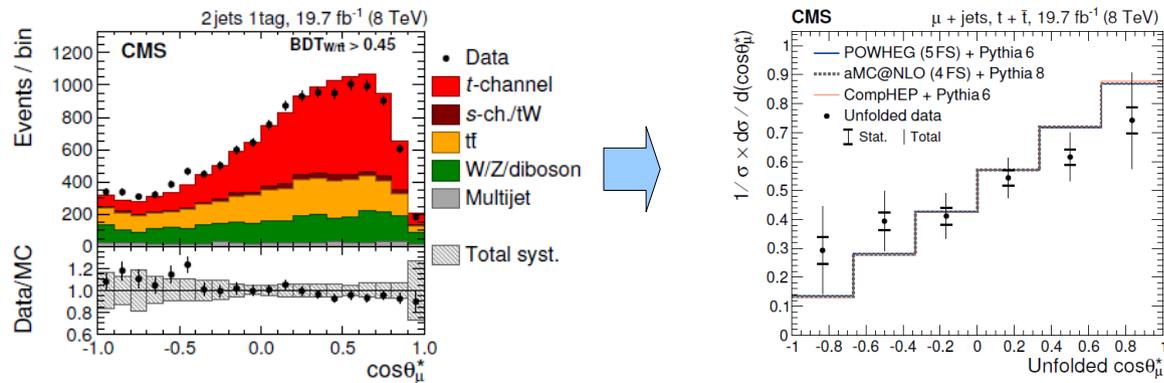


From raw data to density map



From Anne Barnoud, with TOMUVOL data on Puy de Dôme

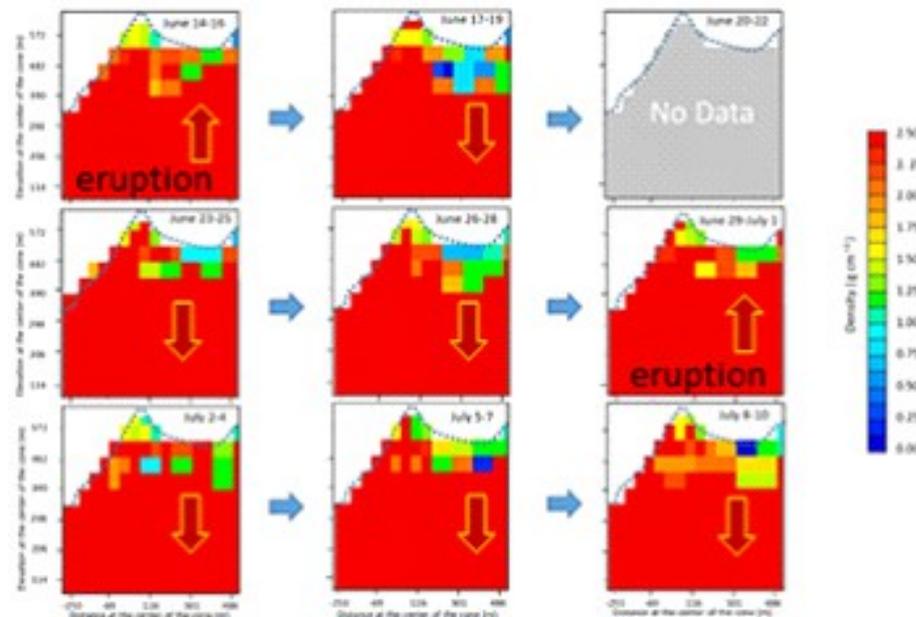
Not so different from:



3. applications

Mountains and volcanoes

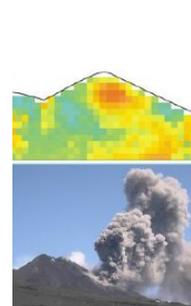
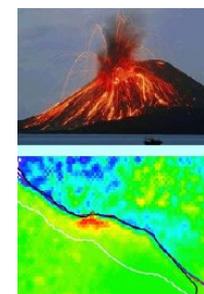
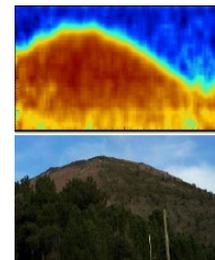
- Pioneered since the 90's by Nagamine's team in Japan, intense activity since early 00's in Japan, Italy, France
- Both "static" and "time-series" studies are potentially useful for volcanology, the latter also for civil protection
- Intrinsically "academic" activity, trend towards collaborations



Satsuma-Iwojima volcano, Japan, 2013 eruption
H.Tanaka, T.Kusagaya, H.Shinohara, Nature Comm.5 (2014) 3381

Some activities in Italy

- Vesuvius:
 - MU-RAY / MURAVES collaboration (INFN Naples & Florence + INGV)
 - Plastic scintillators + SiPM
- Stromboli:
 - Salerno's HEP&Geo + Naples HEP
 - Nuclear emulsions (OPERA spin-off)
- Etna:
 - INAF + INGV
 - Cherenkov telescope (CTA spin-off)
 - (In the past, also DIAPHANE collaboration from France, with plastic scintillators)



Flux normalization, Backgrounds

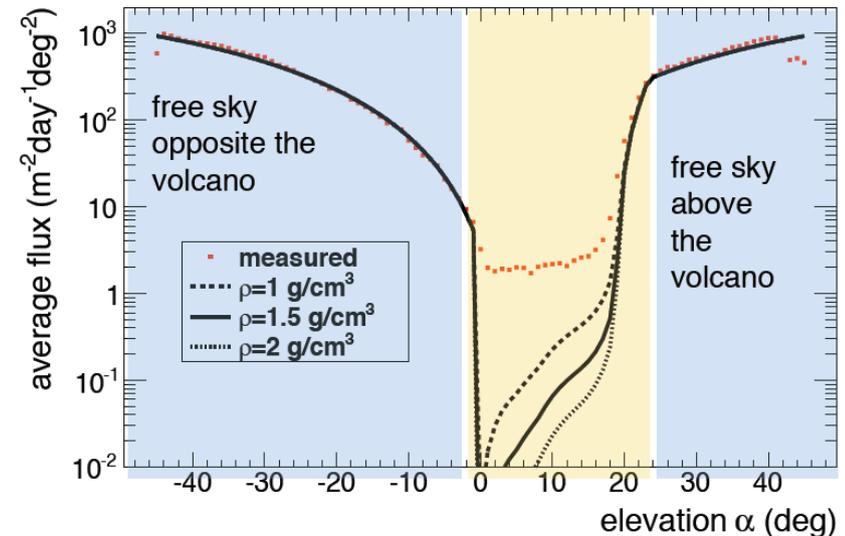
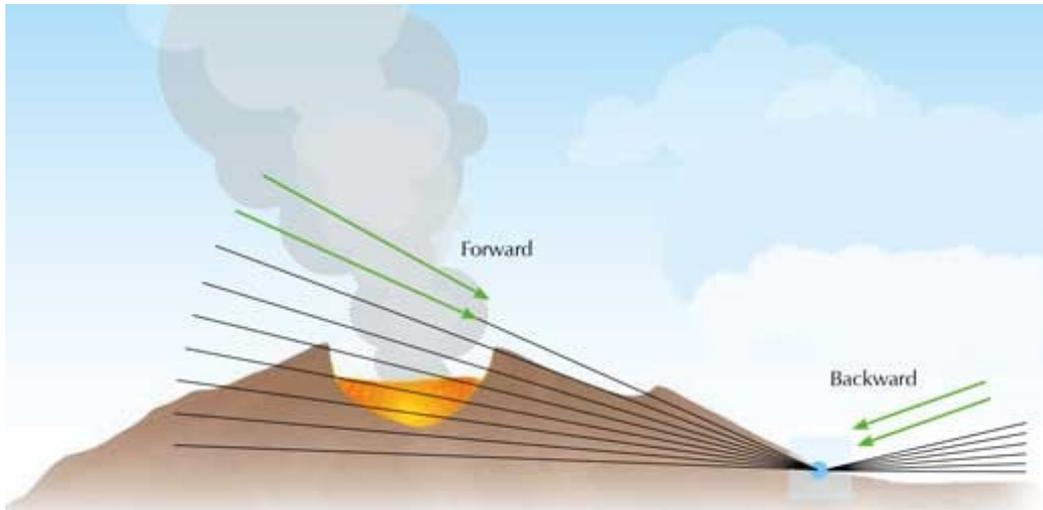
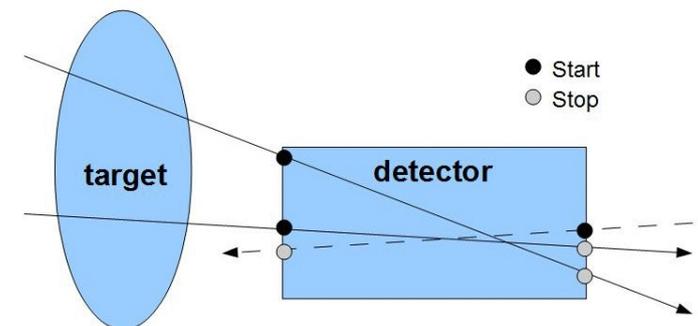


Image from <http://www.scienceinschool.org/2013/issue27/muons>, adapted from H.Tanaka et al., Earth Plan. Sci. Lett. 263 (2007) 104

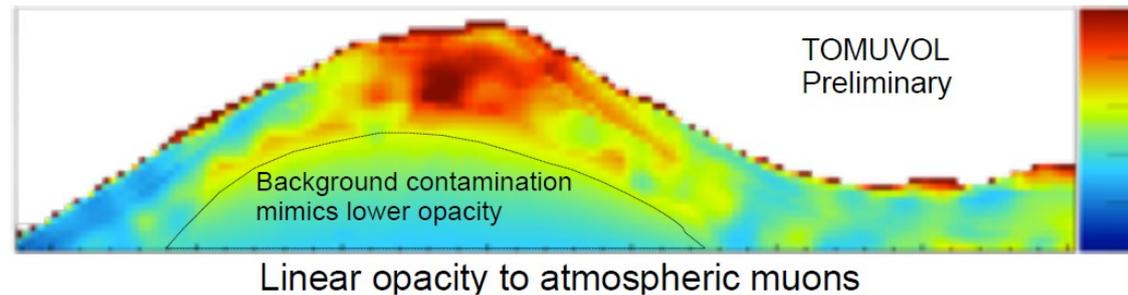
Ambrosino et al. (TOMUVOL+MU-RAY coll.), J.Geophys.Res.Solid Earth 120 (2015), 7290

Muons from outside the target (including backward) are a help and a nuisance:

- In-situ flux normalization from the free sky
- Background (true muons uncorrelated with the target) due to large-angle scattering
- Time-of-flight helps



The most interesting region is the most difficult

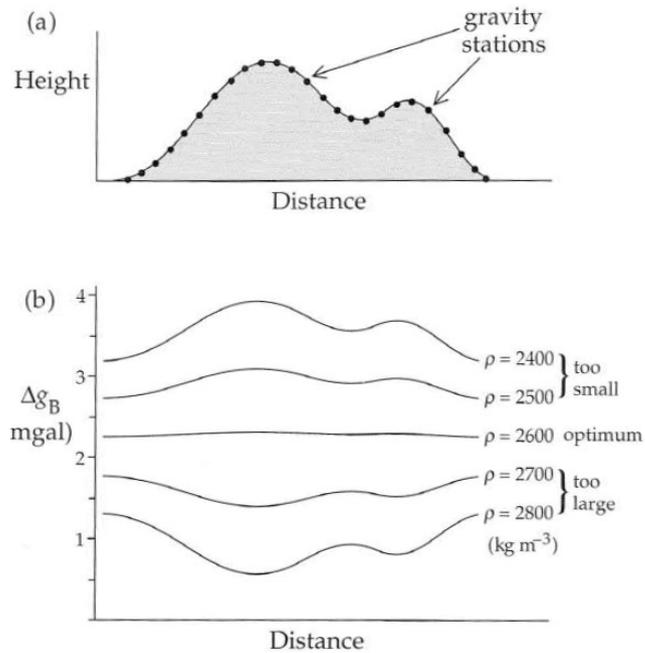


Two months of data with TOMUVOL detector
on Puy de Dôme, dormant volcano in France

From Carloganu & Saracino, Physics Today dec.2012

Combination with "standard methods"

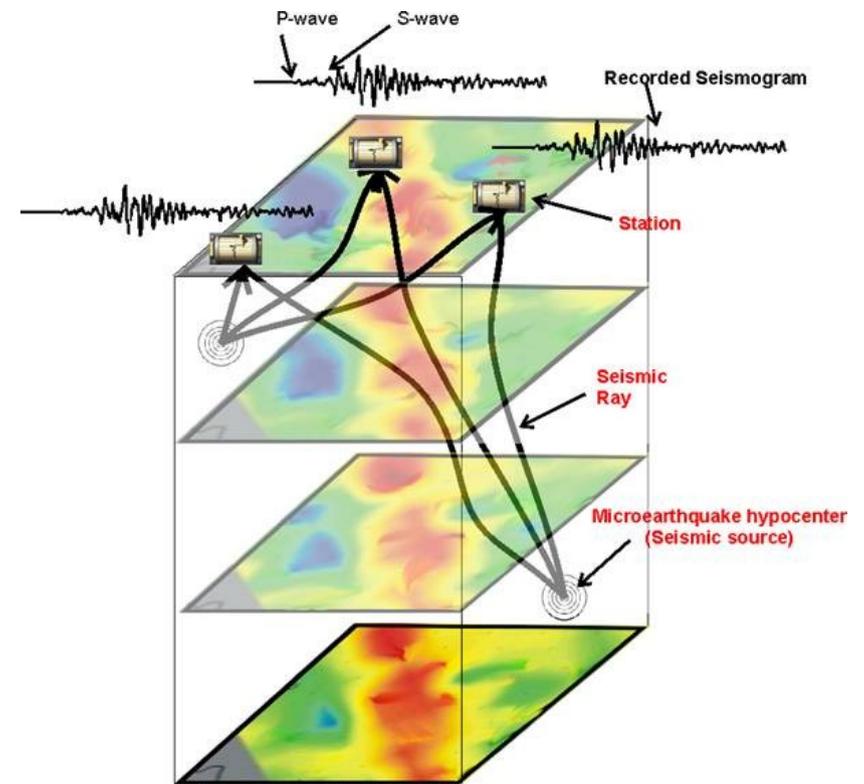
Gravimetry:



Observable: Bouguer anomalies

$$\Delta g_{BP} = 2\pi G \rho h$$

Seismic tomography:



Combination with "standard methods"

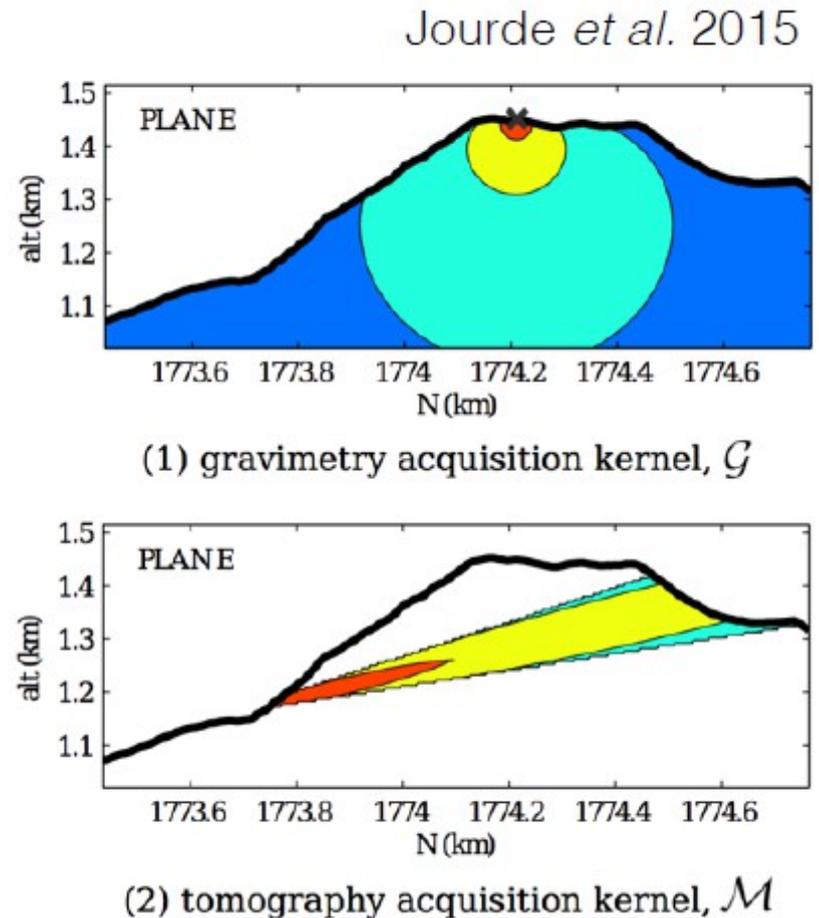
$$\begin{matrix}
 \mathbf{A} & \boldsymbol{\rho} & = & \mathbf{d} \\
 \left[\begin{matrix} \mathbf{G} \\ \mathbf{M} \end{matrix} \right] & \left[\boldsymbol{\rho} \right] & = & \left[\begin{matrix} \mathbf{g} \\ \mathbf{q} \end{matrix} \right]
 \end{matrix}$$

←

←

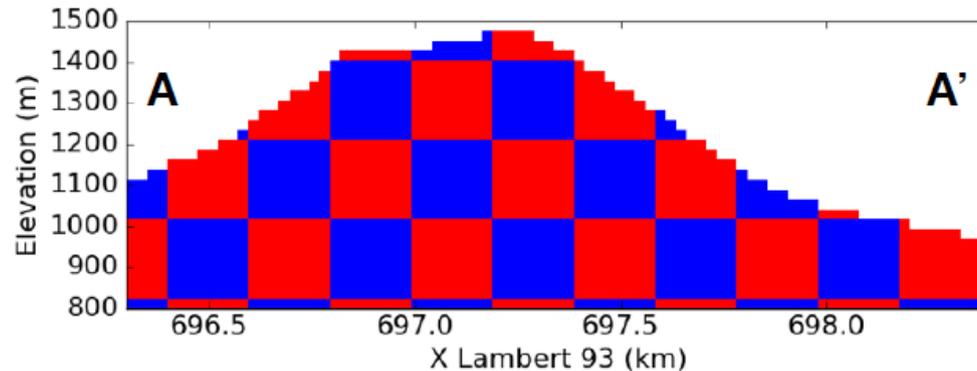
Most geopropecting methods are non-linear inversion problems: solutions wildly degenerate, need strong constraints to converge, different assumptions lead to qualitatively different results

Muography: highly directional, breaks degeneracy of the other methods

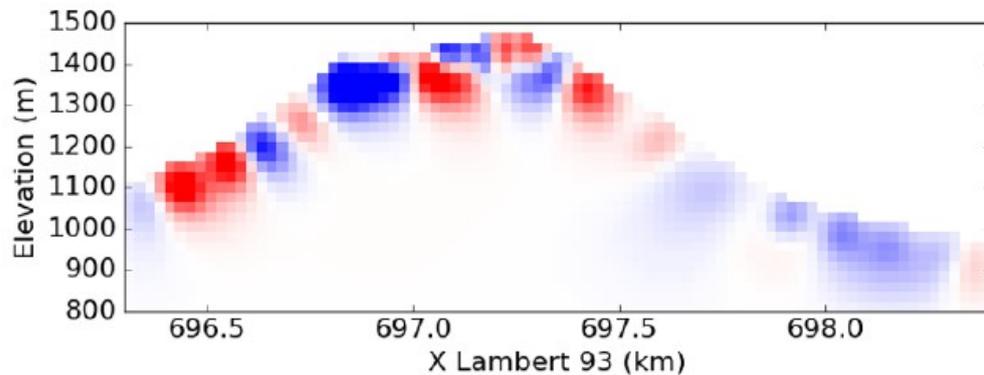


Checkerboard test

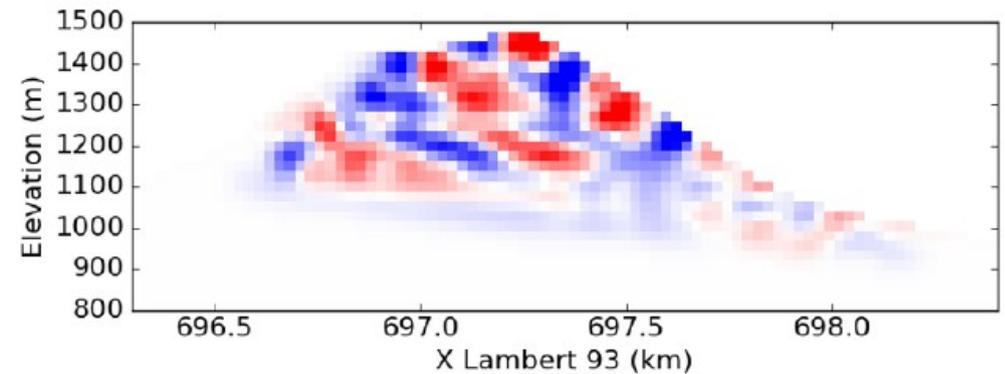
Simulated density pattern:



Red: high density
Blue: low density



Seen from gravimetric inversion



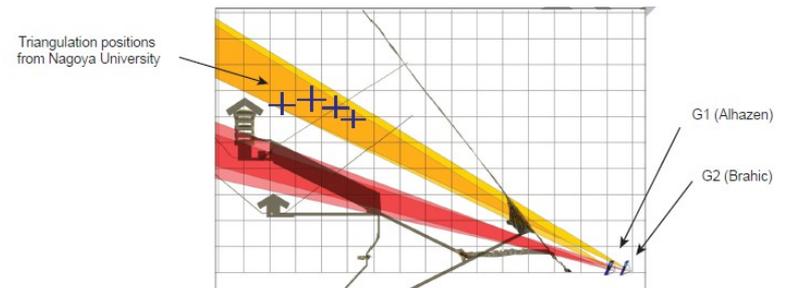
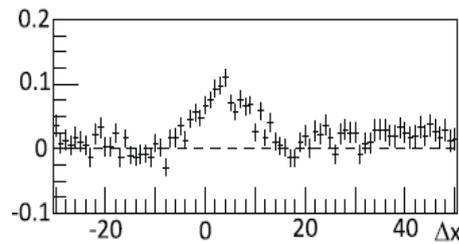
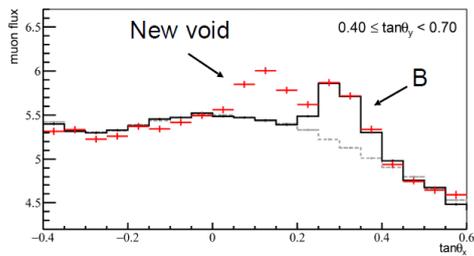
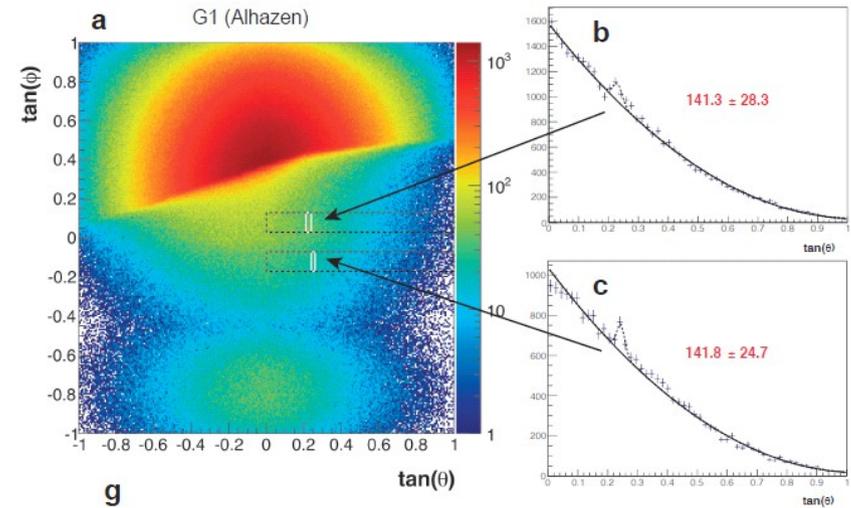
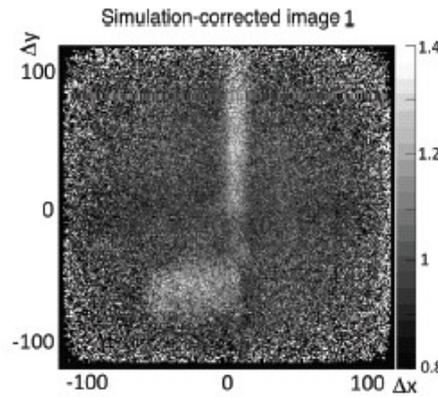
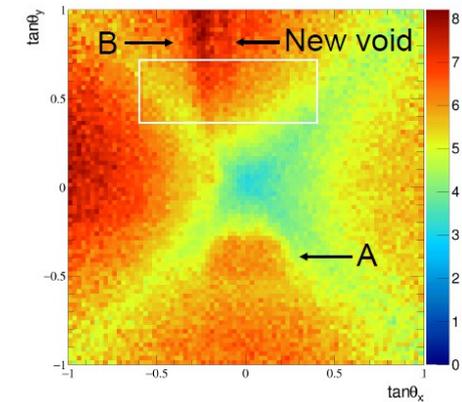
Seen from muographic inversion

Khufu's (Cheops) Great Pyramid (ScanPyramids mission)

Nagoya
(emulsions,
indoors)

KEK
(scintillators,
indoors)

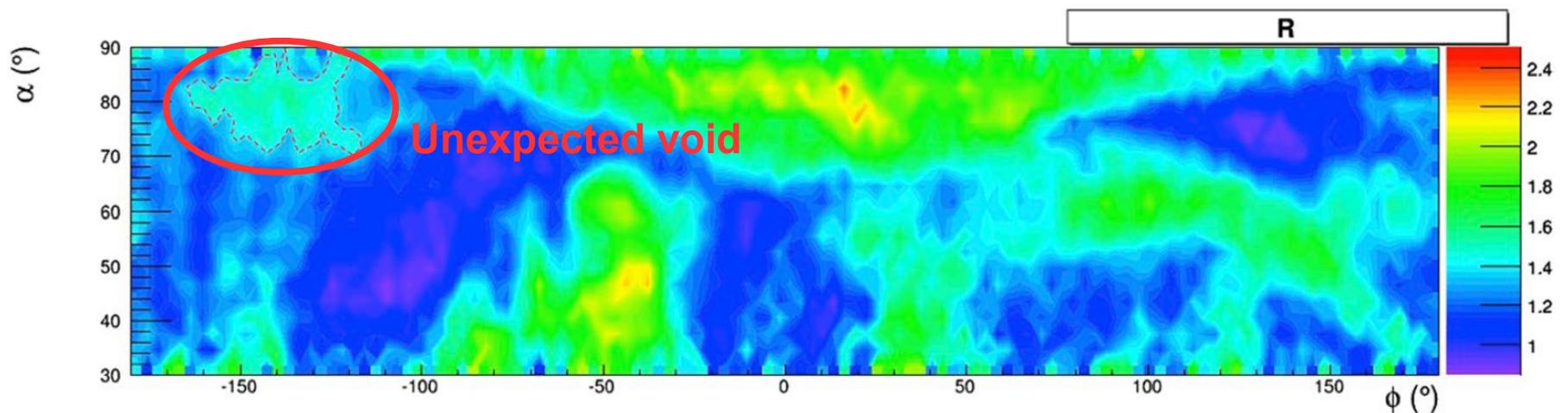
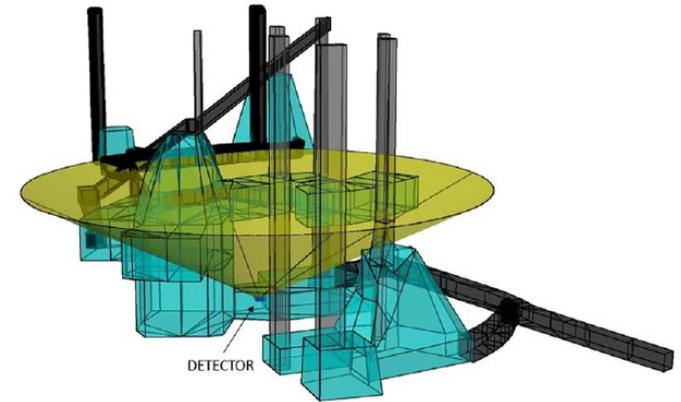
CEA
(MicroMegas,
outdoors)



Coherent conclusions from 3 detectors

Galleria Borbonica, Naples

Detector: MURAVES (same as used on Vesuvius), arranged vertically and with small spacing to increase acceptance



Homeland security

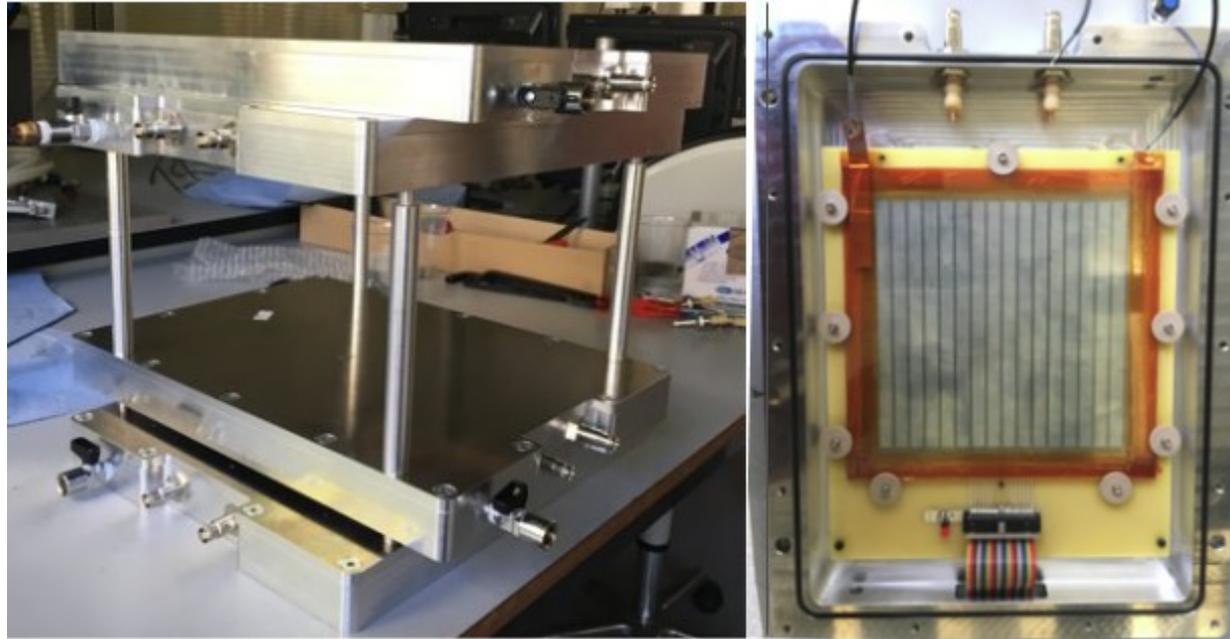


CMS Drift Tubes spin-off at INFN LNL / Padova (P.Checchia et al)

- Currently done with X rays
- Idea: secondary inspection with muon rays to clear/confirm alarms
- Activities also in USA (LANL spin-off)

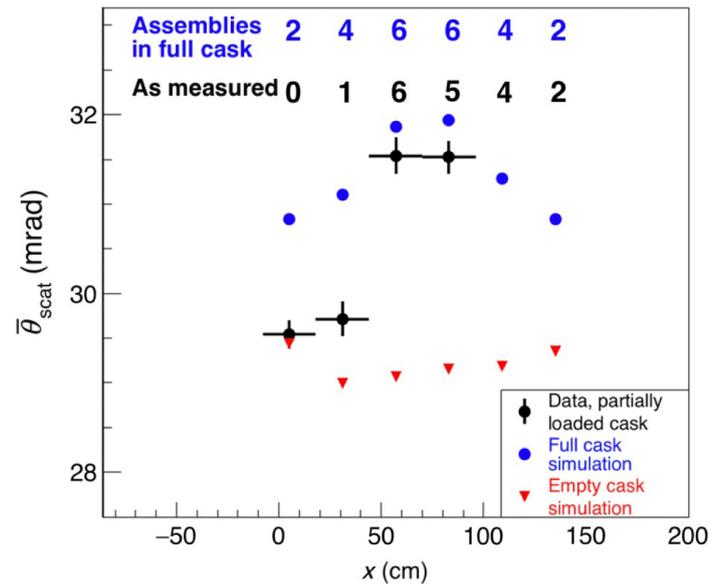
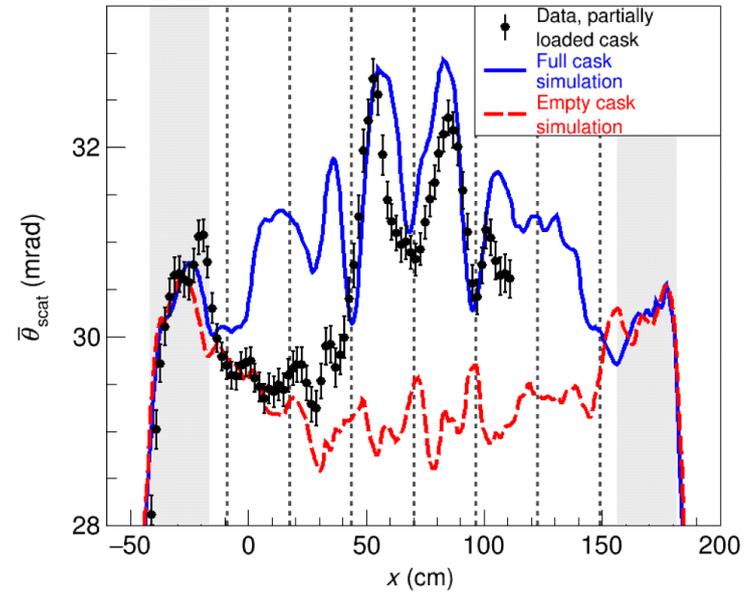
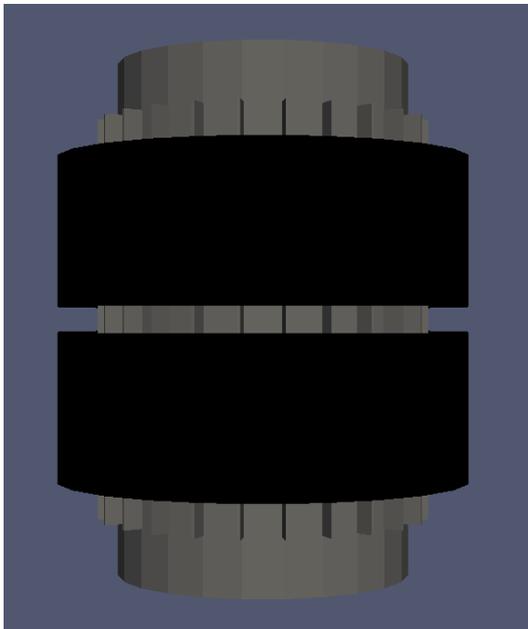
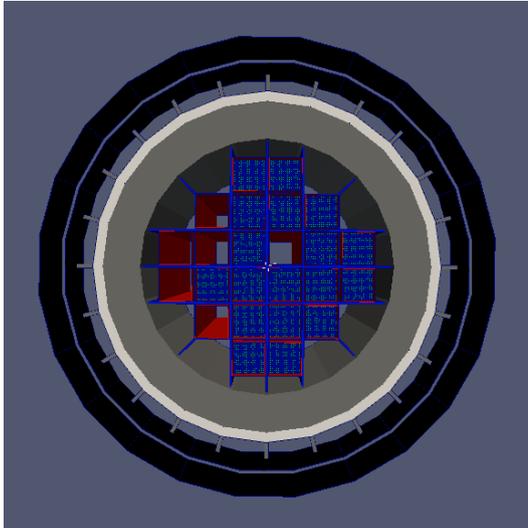
From <http://cms.cern/content/security-and-environmental-protection>

Another CMS spin-off



- Same RPC as CMS experiment, but smaller (16x16 cm²)
 - Also same gas mixture and same front-end electronics
- First prototype, just to gain experience
 - Second one will have x4 or x10 strip density
 - ...hence will require different electronics, or/and smart multiplexing
- Design principle: must be portable
 - Particular care in making gas-tight layers (10⁻⁹ mbar l/s)
 - Total weight including the electronics: ~50 kg
 - Robust: went to the Utah Desert and back

Nuclear waste monitoring



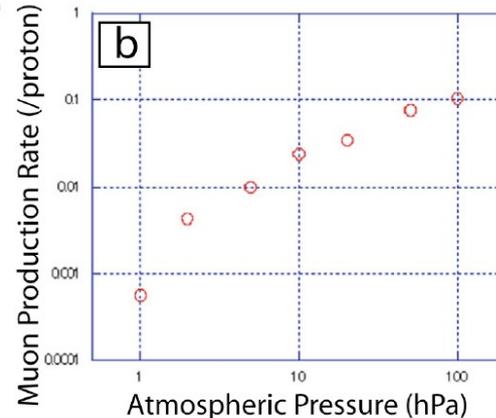
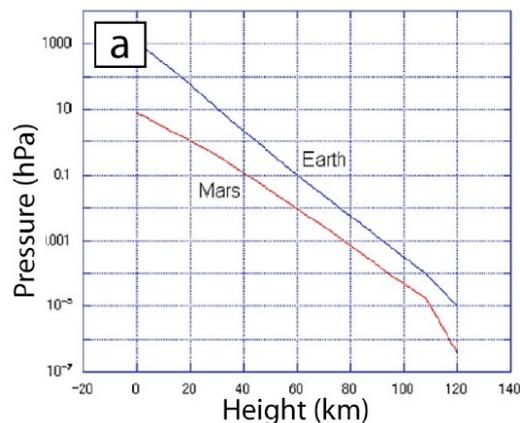
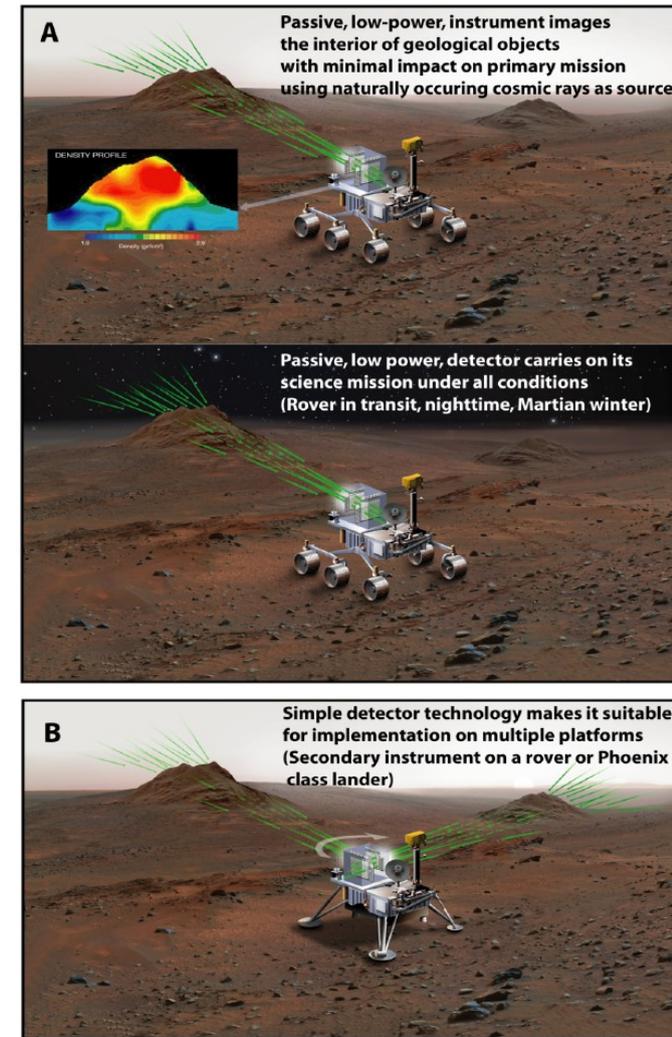
Muography and the private sector



- Usually in HEP the role of the private sector is to provide equipment, facilities, material to the publicly funded institutes
- In muography, a few companies try to valorize the outcomes of academic research
 - Some of them are actually university spin-offs
 - Still in infancy; fragile/hypothetical market
- Examples:
 - Geophysical applications: IRIS Instruments (France), TECNO IN (Italy), NEC (Japan), Lingacom (Israel)
 - Nuclear waste monitoring: Lynkeos Technology (UK)
 - Mining exploration: CRM GeoTomography Technologies (Canada)
 - Pipeline x-section measurement: Muon Systems (Spain)
 - Homeland security: Decision Systems (USA)

Planetary exploration

- The usual requirements for muography instruments on Earth are even more important for space missions
 - Compact size, low weight
 - Very robust! Must survive landing!
 - Low power consumption
 - Temperature variations
- Test with a „rover“ on Mt.Omuro (Japan) in 2012
- Additional challenges from the thin atmosphere:
 - Smaller flux of muons
 - (on the other hand, horizontal muons are less suppressed and their flux is actually larger than vertical)
 - Larger hadronic background (primary cosmic!)
 - **Passive, low-power, instrument images the interior of geological objects with minimal impact on primary mission using naturally occurring cosmic rays as source**



Small solar system bodies

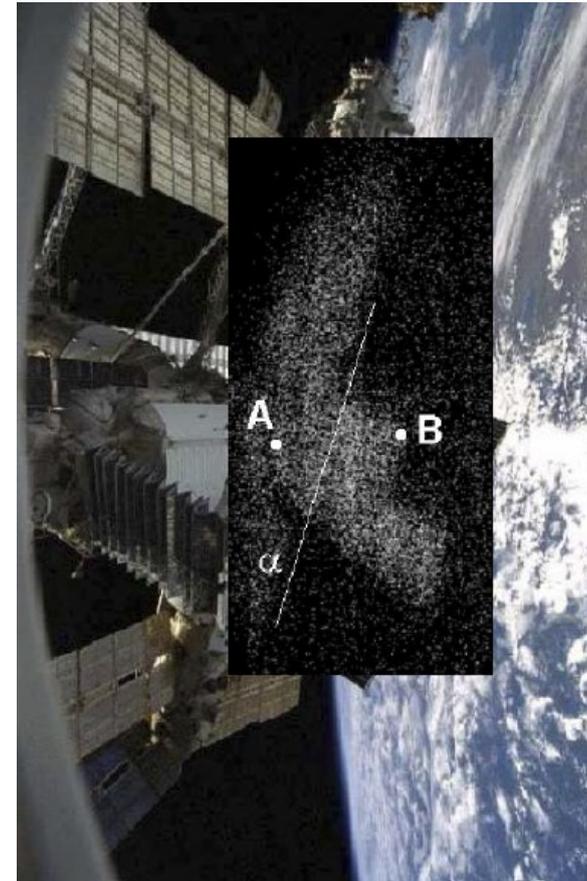
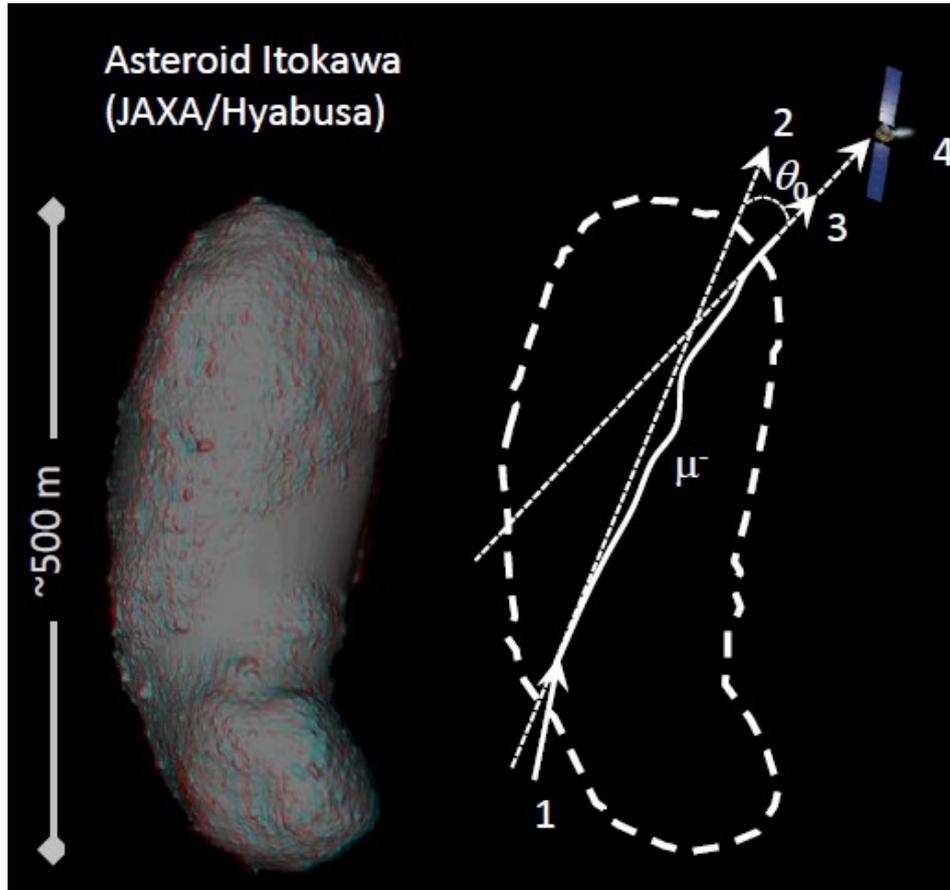


Image from AMS-1 of the MIR space station using secondary π and μ

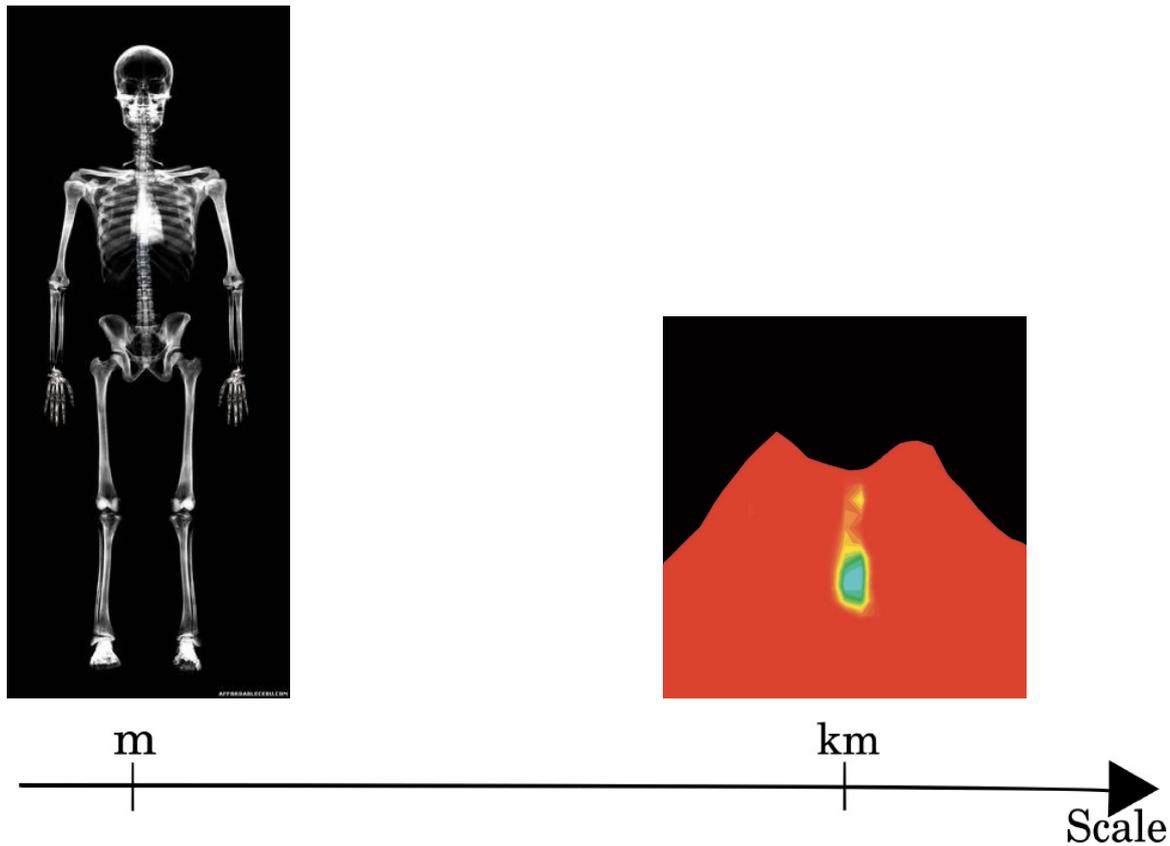
Thanks for your attention!



Acknowledgements: Sophie Wuyckens, Sebastien Procureur, Lorenzo Bonechi, Chris Morris, David Mahon, Anne Barnoud, Lee Thompson,

Compare attenuation length of X-rays and muons

For X-rays of O(1-10 keV) it ranges between 10^{-6} and 10^{-1} meters depending on material (Z) and energy



Detector requirements

- **Angular resolution:** from sub-mrad to ~ 10 mrad
- **Large acceptance**
- **Large detection area:** trade-off between the cost, transportability and required statistics
- **Robustness**
- **Autonomy and access**
- **Time resolution:** depends on the application
- **Low cost**

Topology of background events

Background events are **grazing downward going muons** of $\sim 1-10$ GeV deflected close to the detector and possibly on the volcano flank.

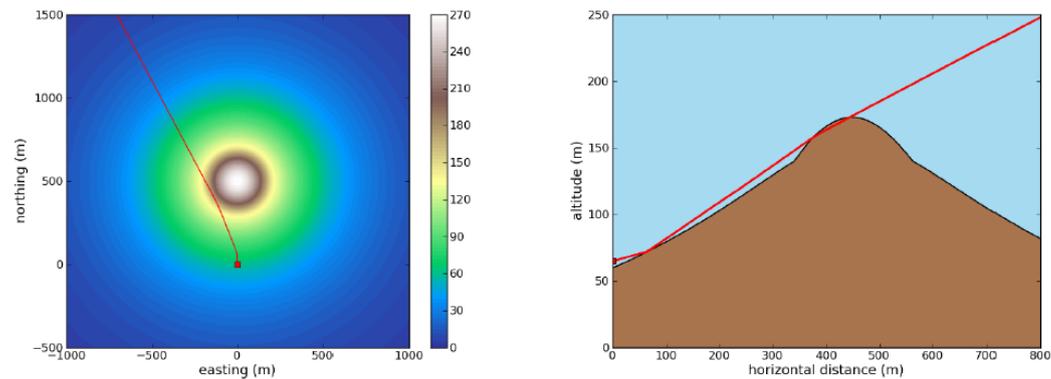


Figure 8 : 100 MeV background muon with an elevation of 5 deg. Left: top projection, right: side projection. The red line stands for the muon track.

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Slide from Valentin Niess

Primary cosmic rays

- Stable ($>10^6$ years) charged particles and nuclei
 - Protons: 87%; alphas: 12%; other nuclei: 1%; electrons: 2%; also photons, neutrinos, antimatter

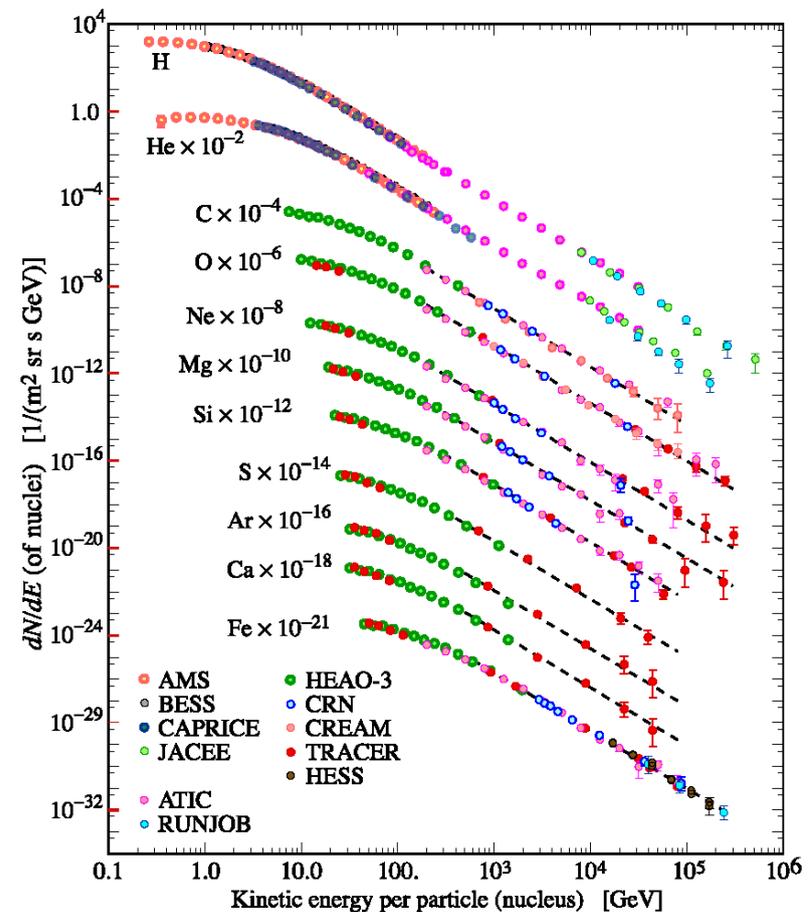
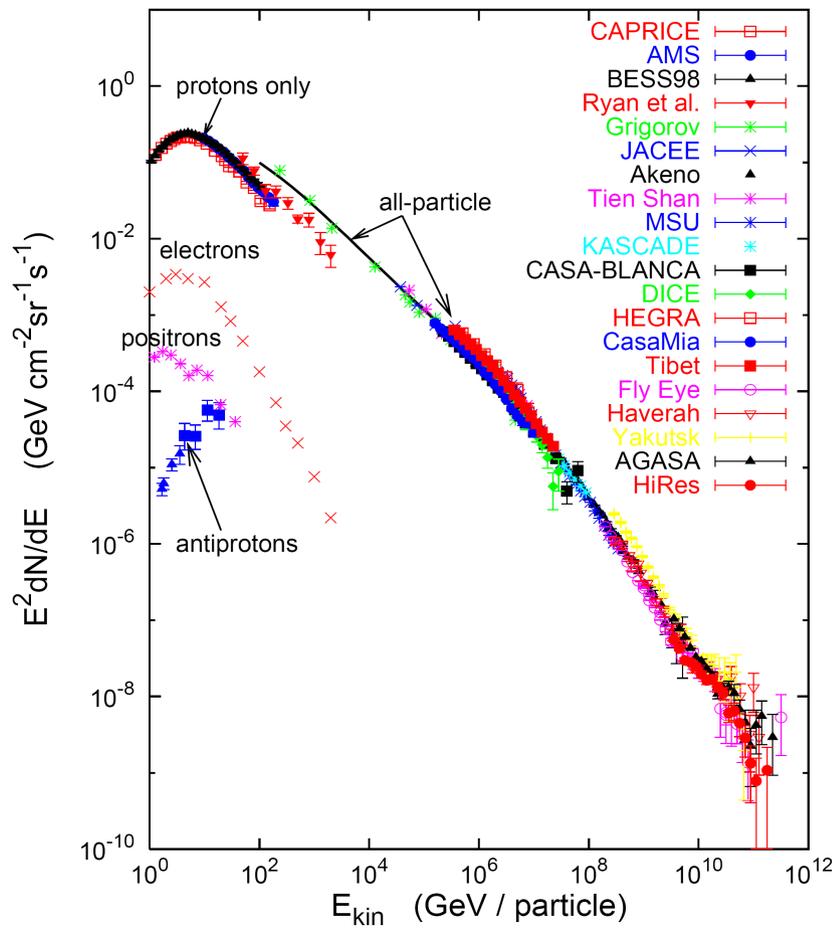
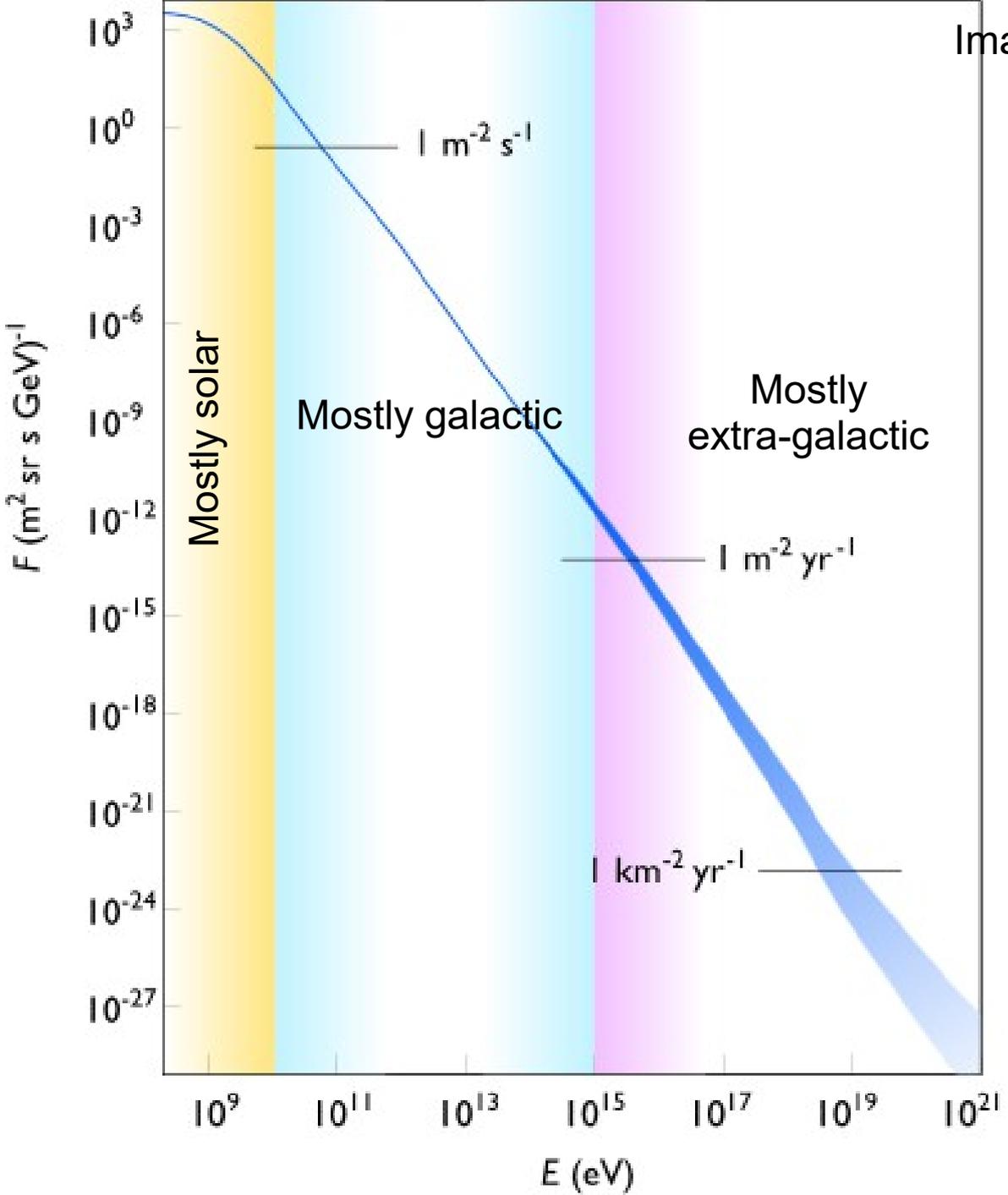


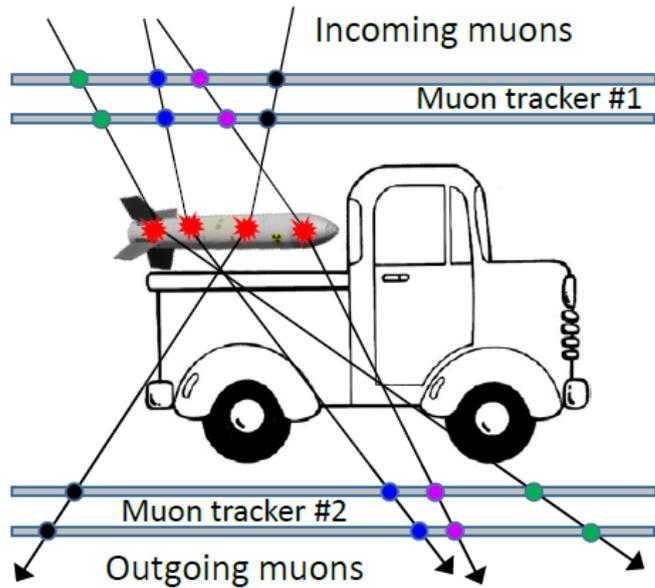
Image from wikipedia



Sources of galactic cosmic rays

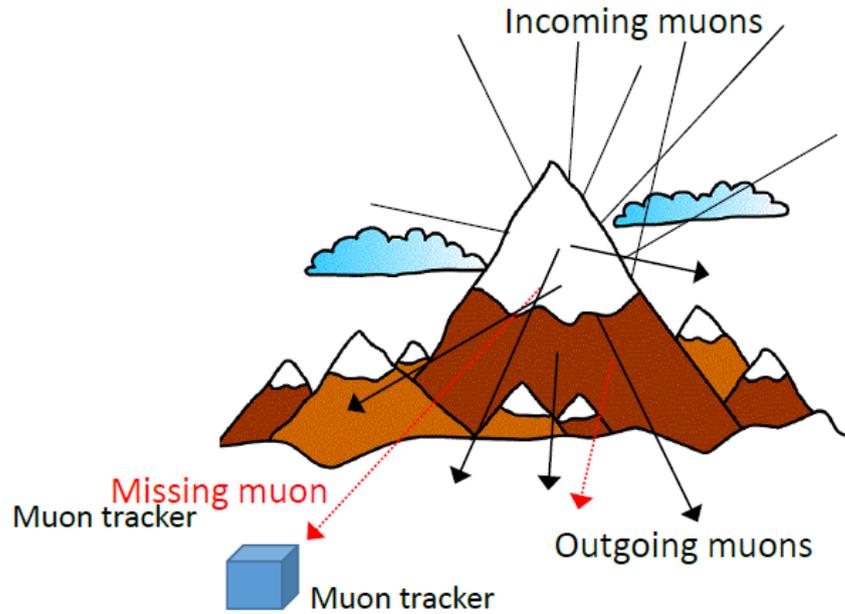
- We are not sure
 - Because they are deflected several times by galactic magnetic field, they reach Earth from random directions
- Abundance of heavy elements suggests an origin from supernovae, for most but maybe not all of them
 - Remember: ~ 1 SN every 50 years in the Milky Way
- Cosmic rays are probably accelerated by the shock waves in the interstellar gas due to supernovae
- Energy of typical SN explosion: $\sim 10^{51}$ ergs
 - It would be sufficient that O(%) of this energy is converted in kinetic energy of the cosmic rays; the exact way is unknown
 - Data from Fermi satellite recently proved that SN are sources of CR, with $\sim 10^{49} - 10^{50}$ ergs of CR kinetic energy

Muon tomography



Based on muon scattering.
Appropriate for relatively small volumes.

Muon radiography



Based on muon absorption.
Appropriate also for very large volumes.

Slide by Lorenzo Bonechi

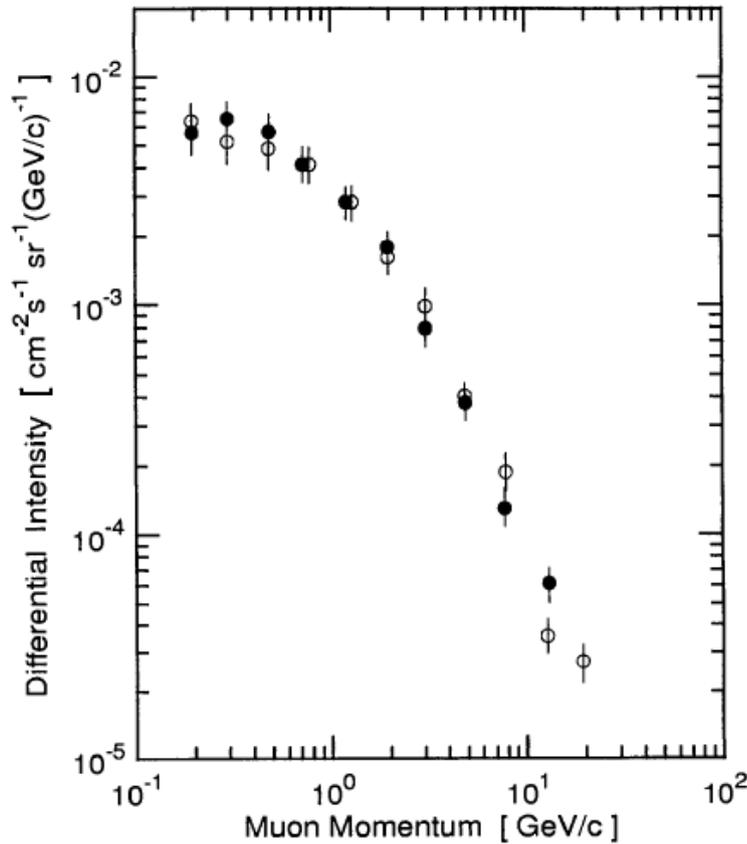


Figure 2.138: Differential momentum spectra of positive and negative muons combined at a zenith angle of 37.5° in eastern and western directions, at an altitude of 2960 m (Zugspitze, Germany) (Allkofer and Trümper, 1964). The data are not corrected for proton and electron contamination.

● 37.5° east | ○ 37.5° west

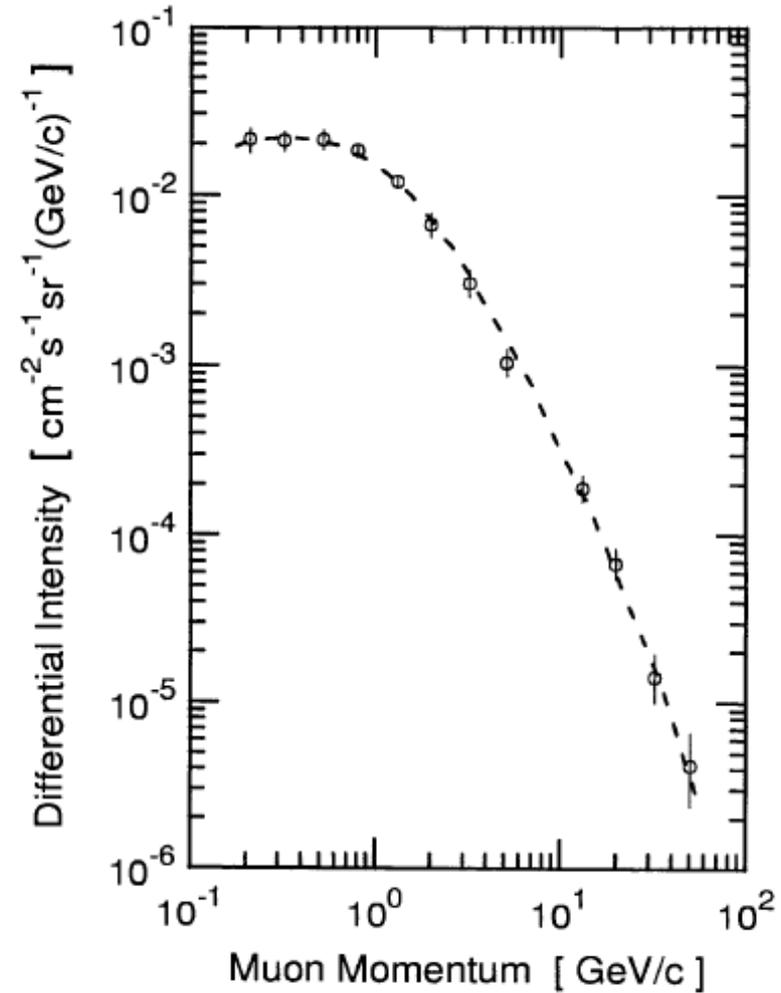
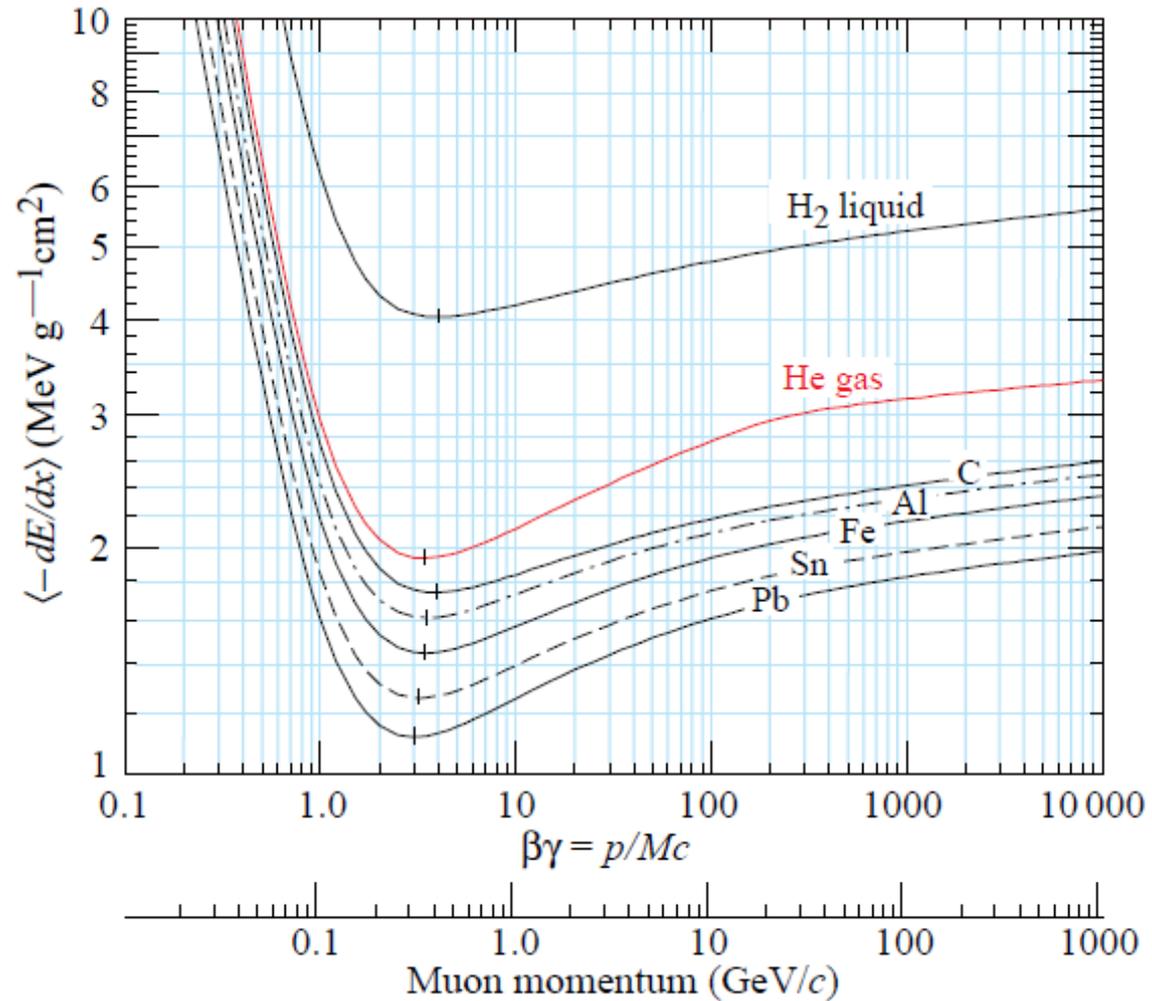


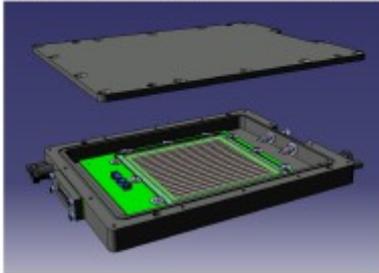
Figure 2.141: Vertical differential momentum spectrum of muons at an altitude of 5260 m (Mt. Chacaltaya, Bolivia) (Allkofer and Kraft, 1965).

Stopping power

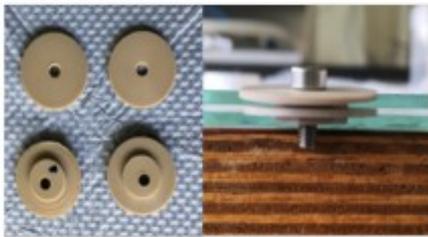


UCL's mini-gRPCs

Mechanical design @Nicolas Szilazi



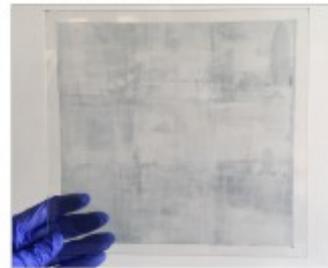
Spacers



Aluminum box



Resistive coating



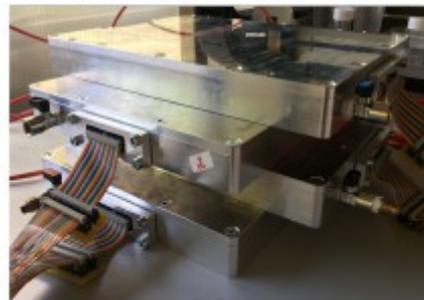
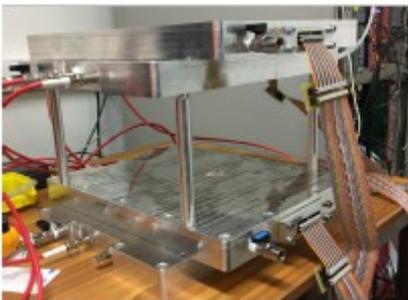
Vacuum tests



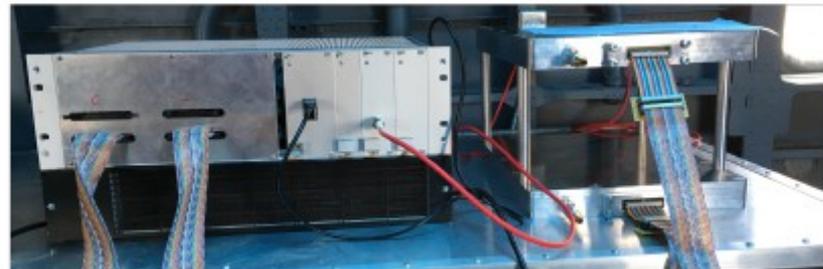
Inside chamber



Telescope configurations



Detectors assembled with readout and high voltage electronics system



A few months of progress

Purity evolution of events

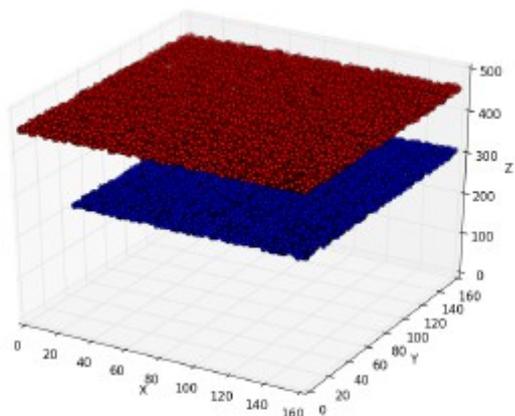
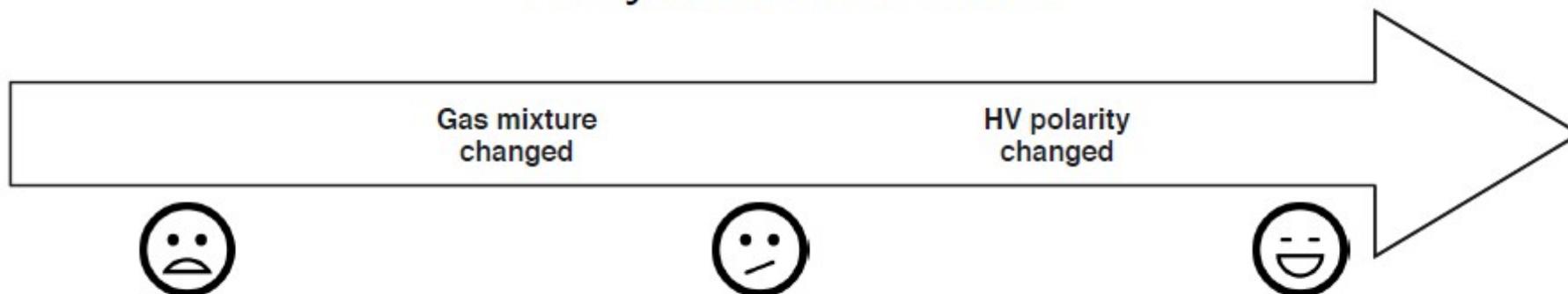


Figure: @MDRS - 4.5kV
& th 100

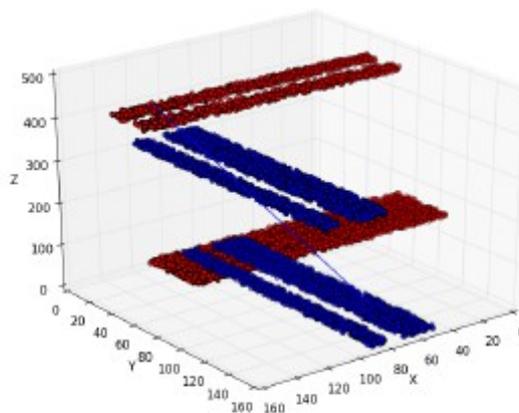


Figure: @UCL with negative
HV - 6.8 kV & th 100

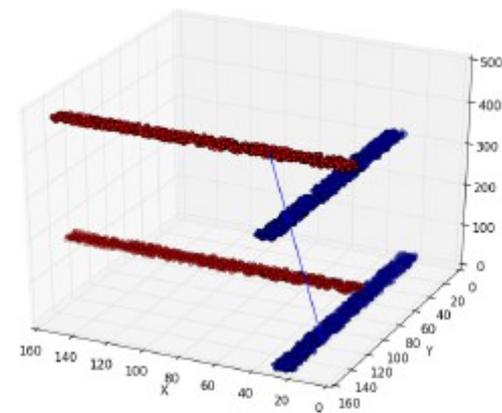
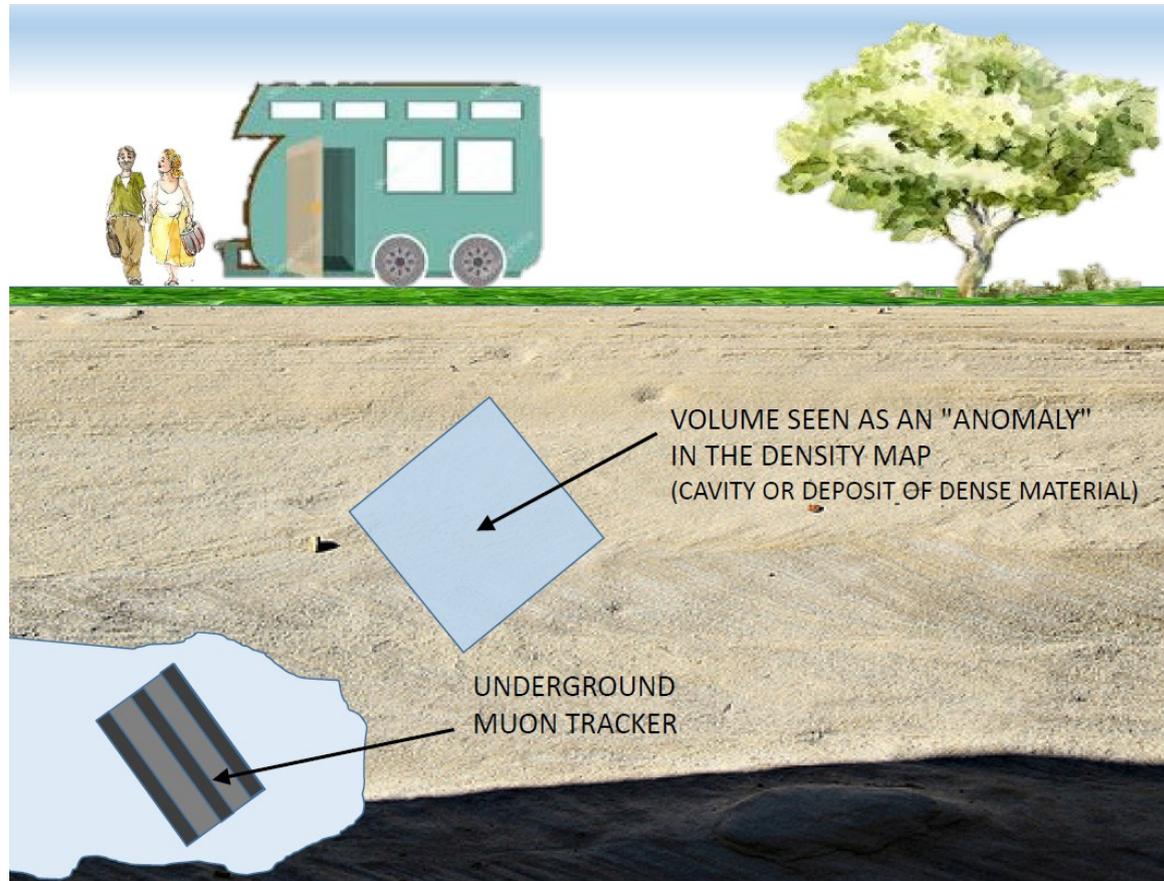
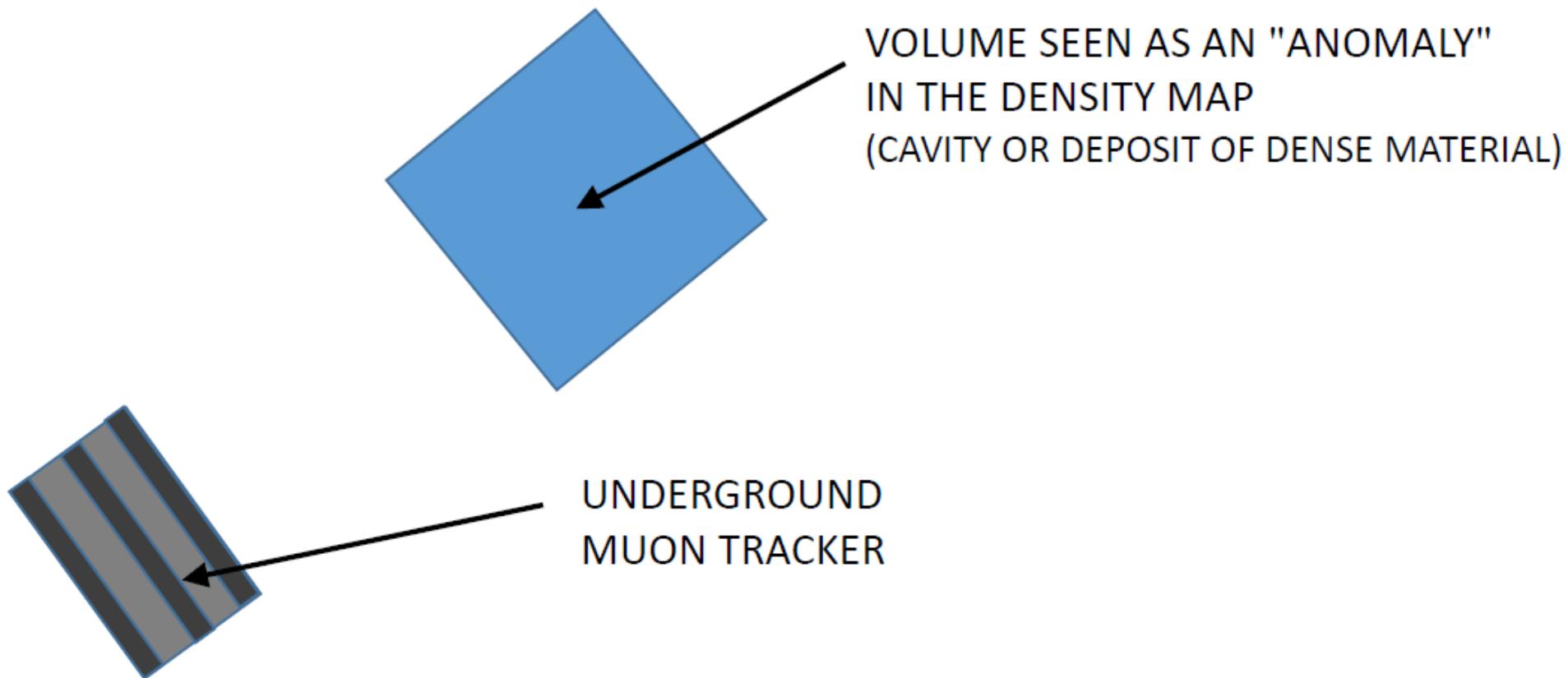


Figure: @UCL with positive
HV - 6.6 kV & th 105

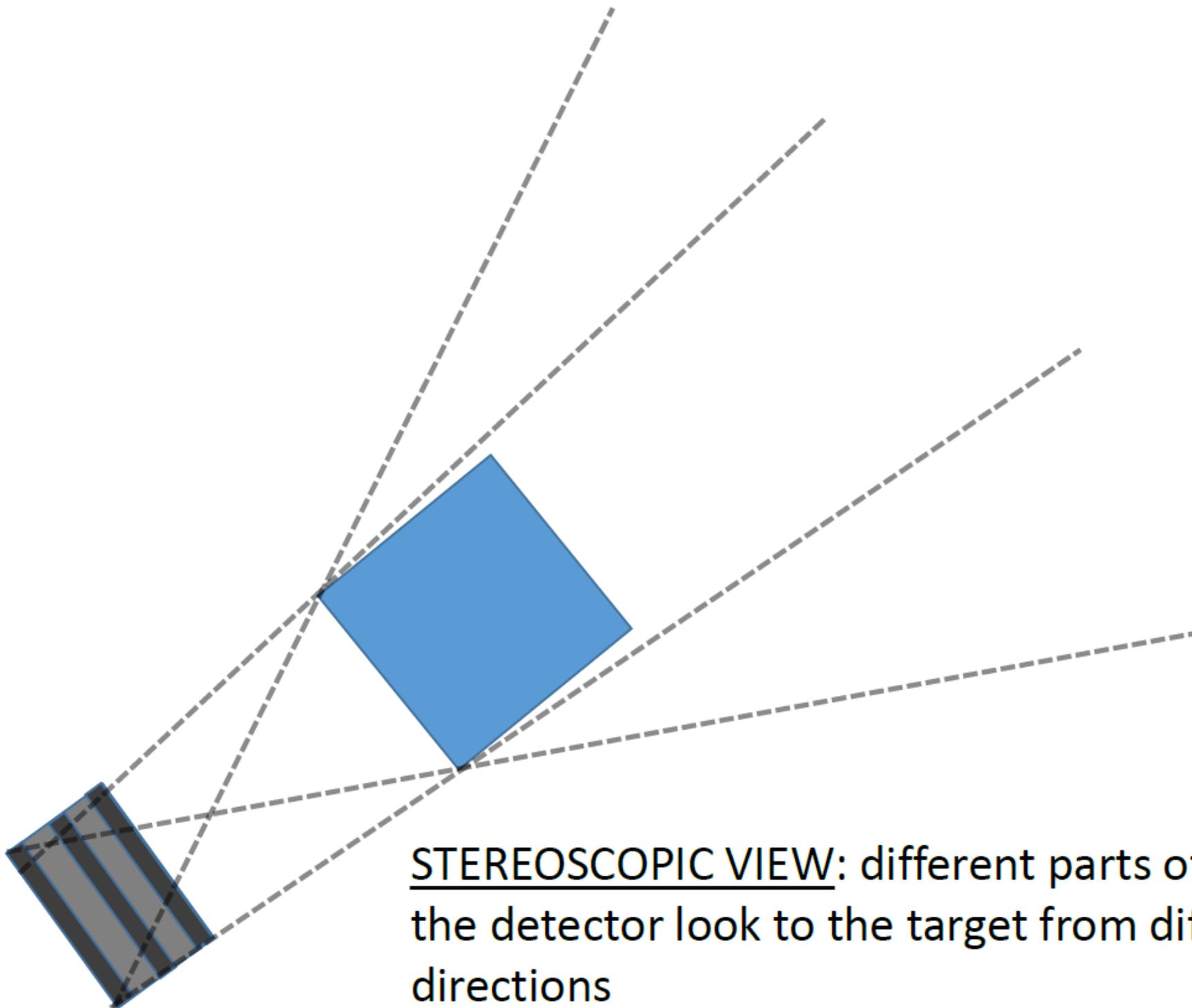
Underground cavities: Backprojection method

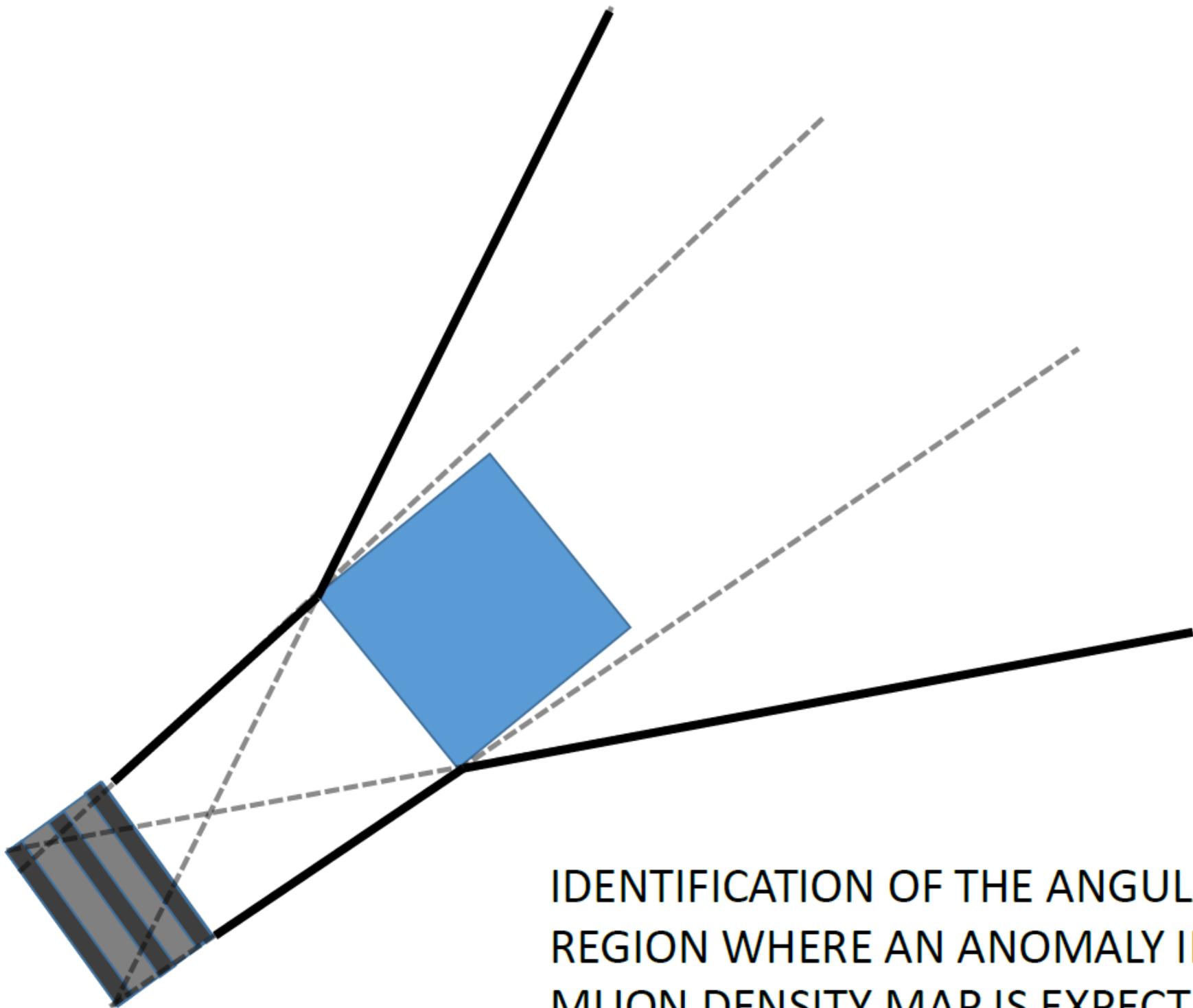


This and the next cartoons are
courtesy of L. Bonechi

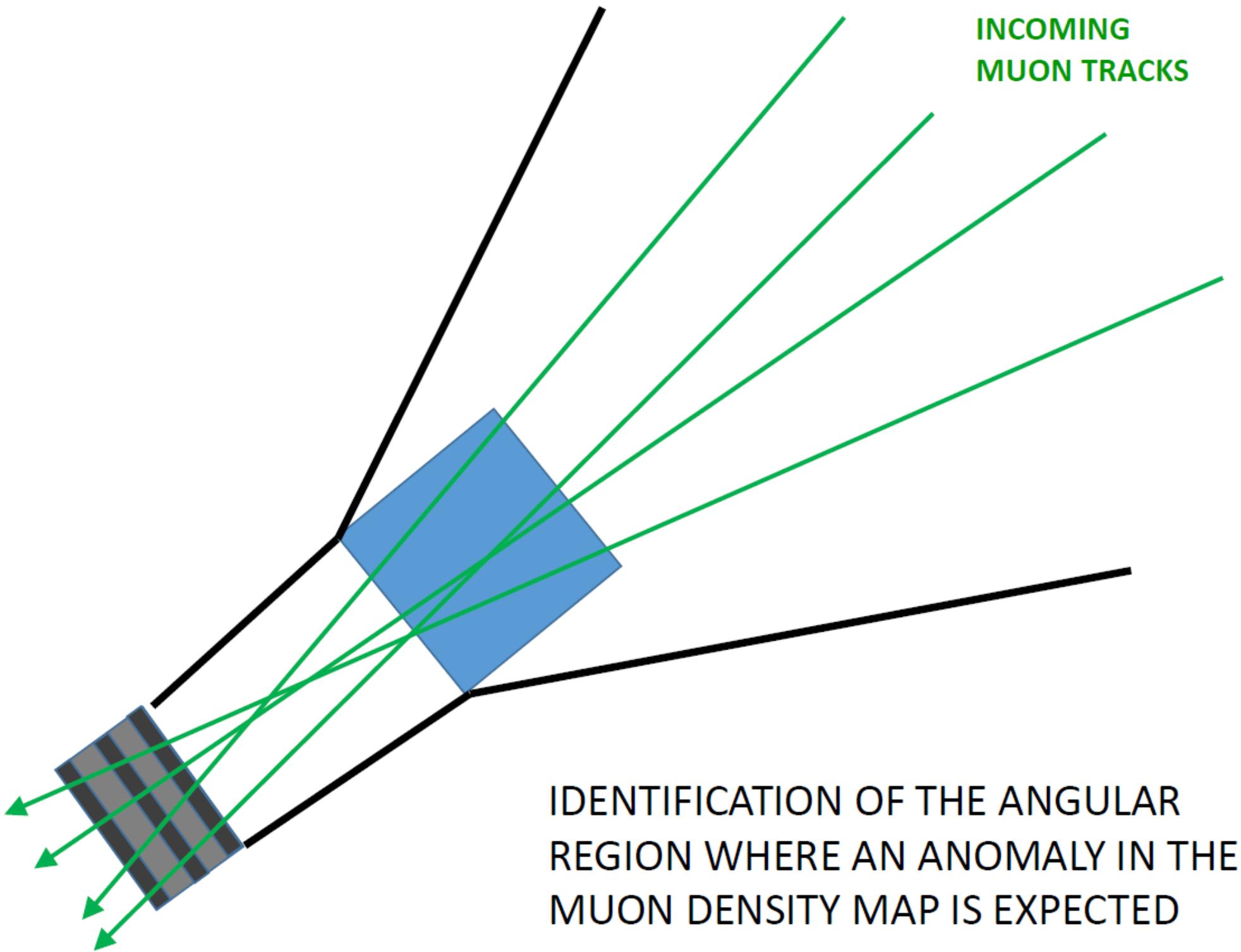


Assumptions: size of the target, and distance from the detector, are not much larger than the detector





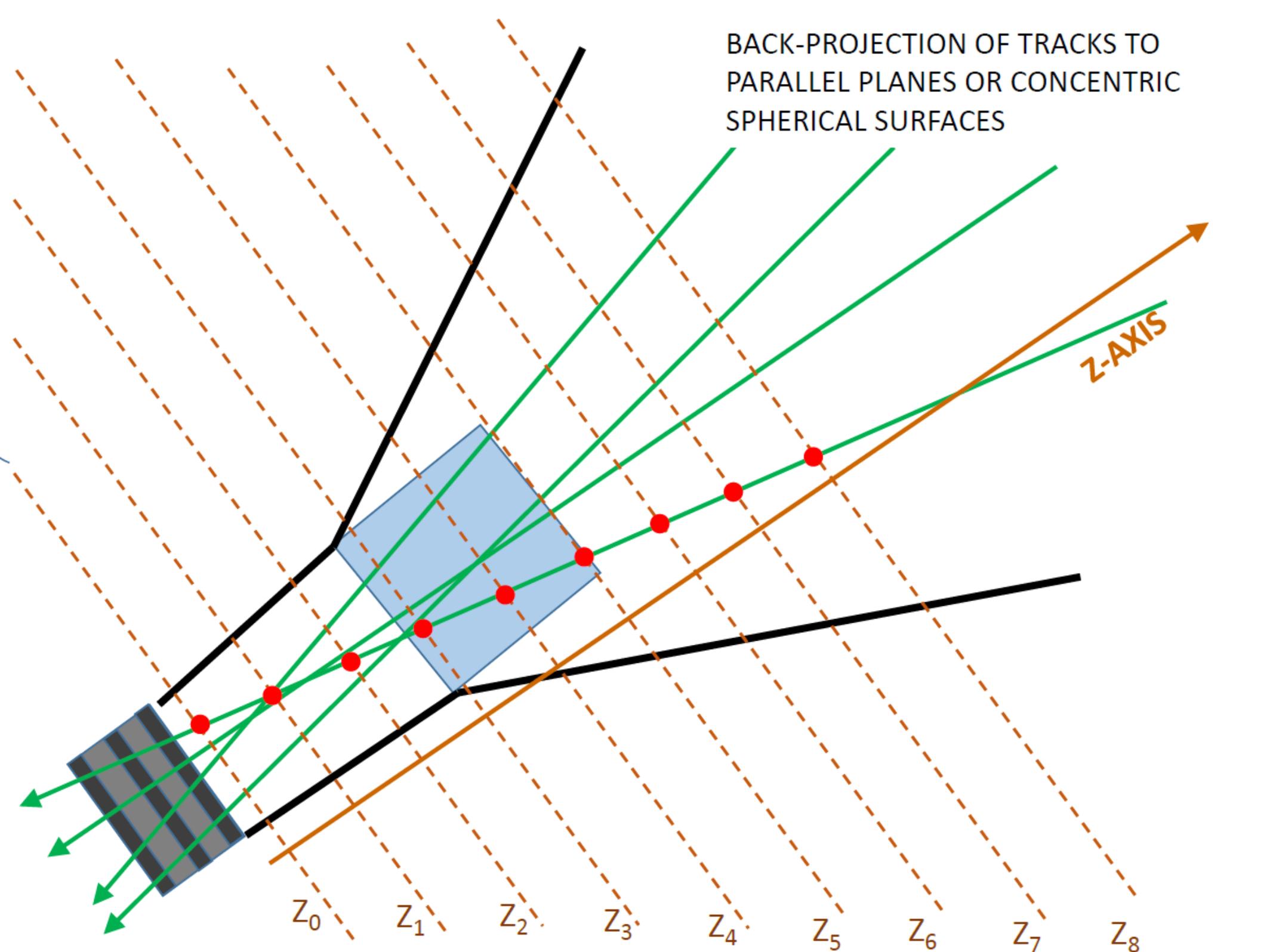
IDENTIFICATION OF THE ANGULAR
REGION WHERE AN ANOMALY IN THE
MUON DENSITY MAP IS EXPECTED



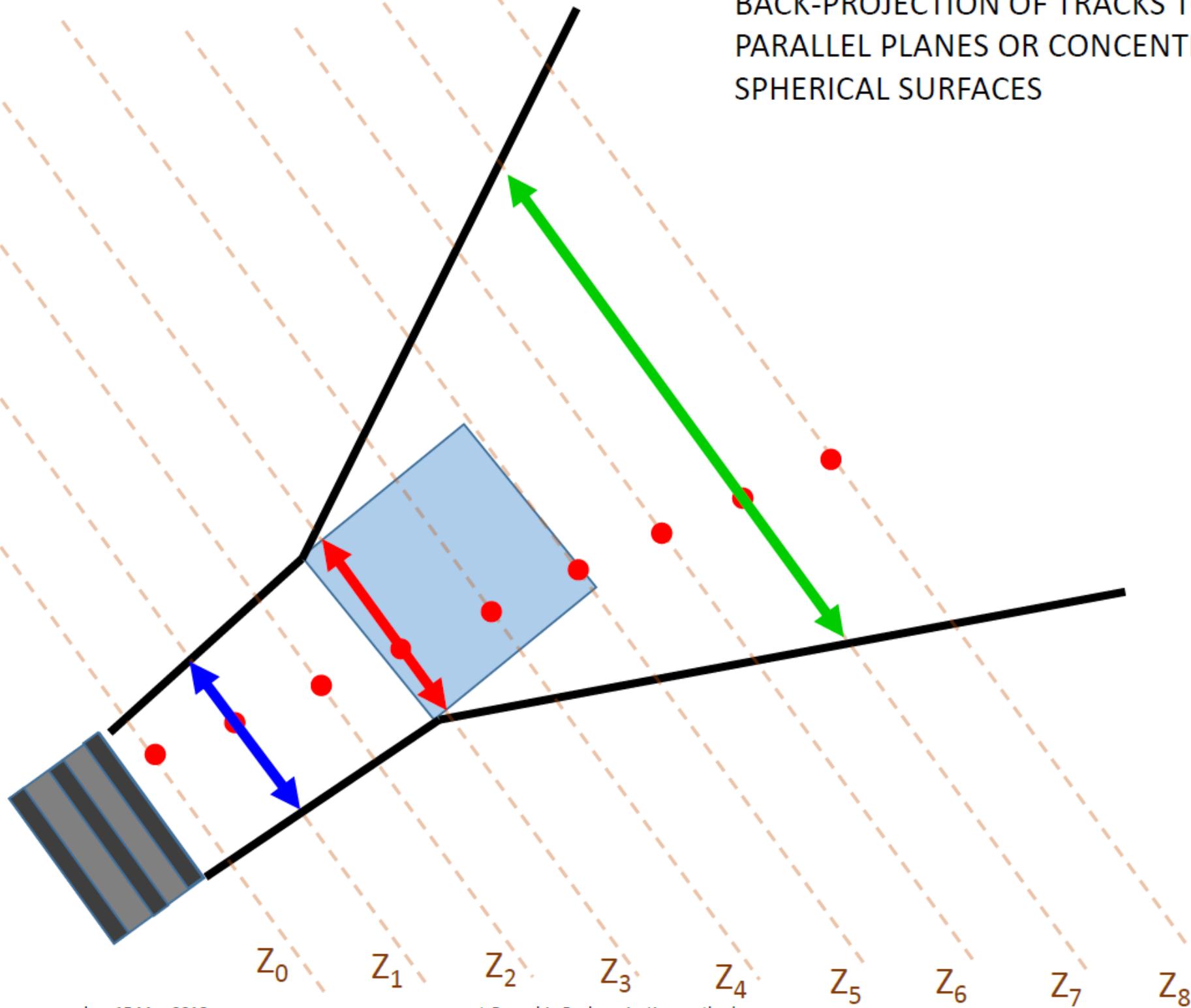
INCOMING
MUON TRACKS

IDENTIFICATION OF THE ANGULAR
REGION WHERE AN ANOMALY IN THE
MUON DENSITY MAP IS EXPECTED

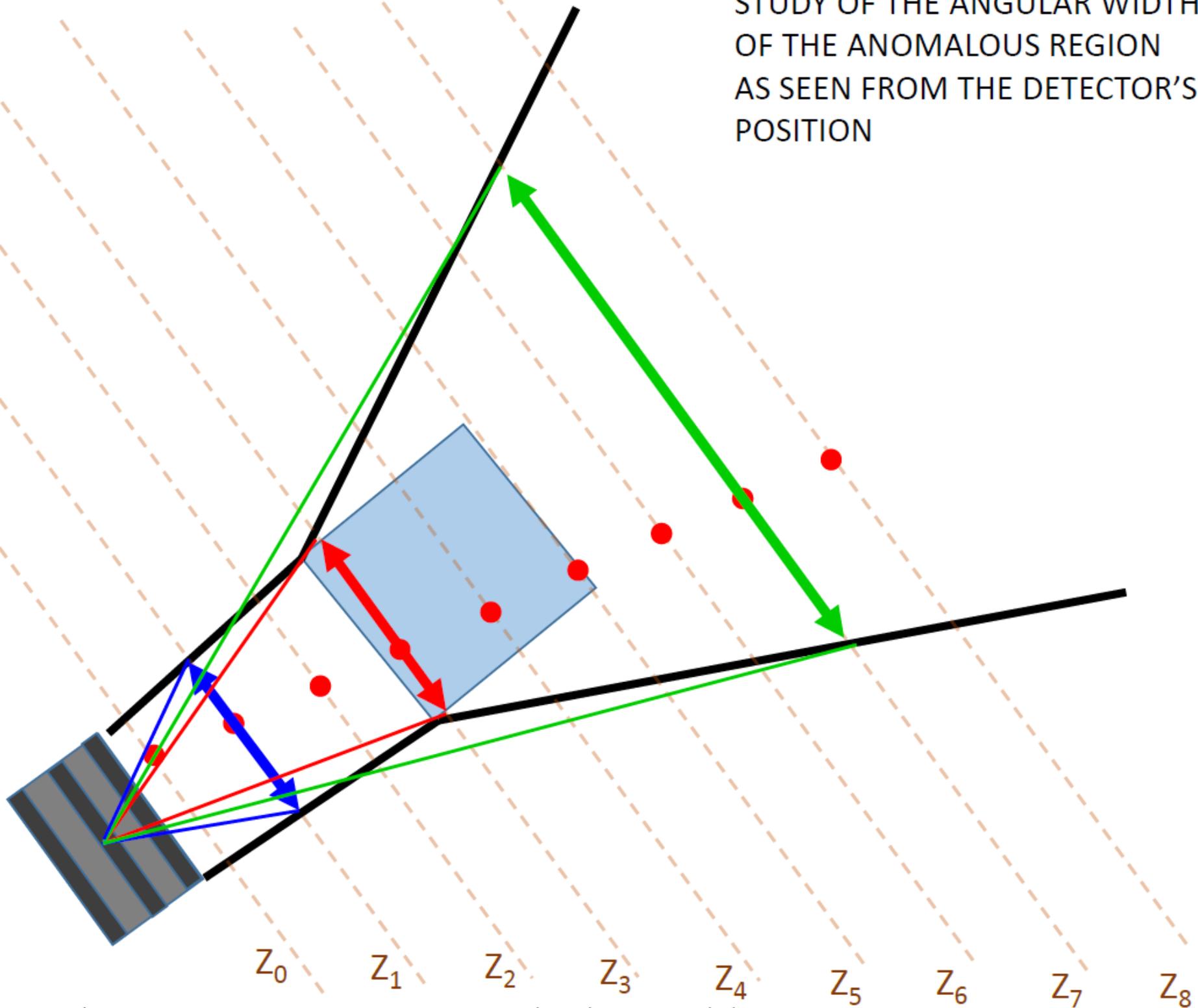
BACK-PROJECTION OF TRACKS TO
PARALLEL PLANES OR CONCENTRIC
SPHERICAL SURFACES



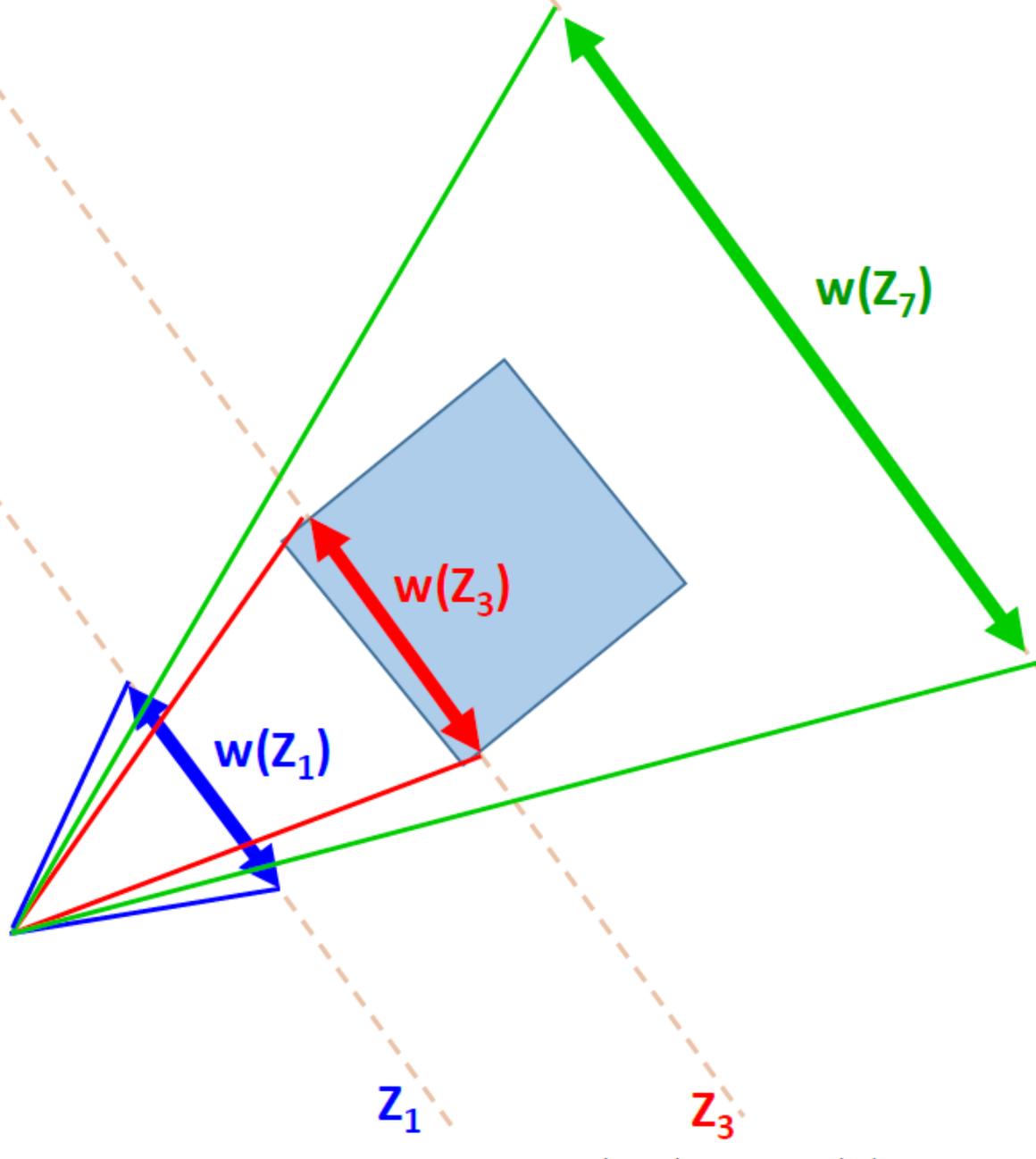
BACK-PROJECTION OF TRACKS TO
PARALLEL PLANES OR CONCENTRIC
SPHERICAL SURFACES



STUDY OF THE ANGULAR WIDTH
OF THE ANOMALOUS REGION
AS SEEN FROM THE DETECTOR'S
POSITION



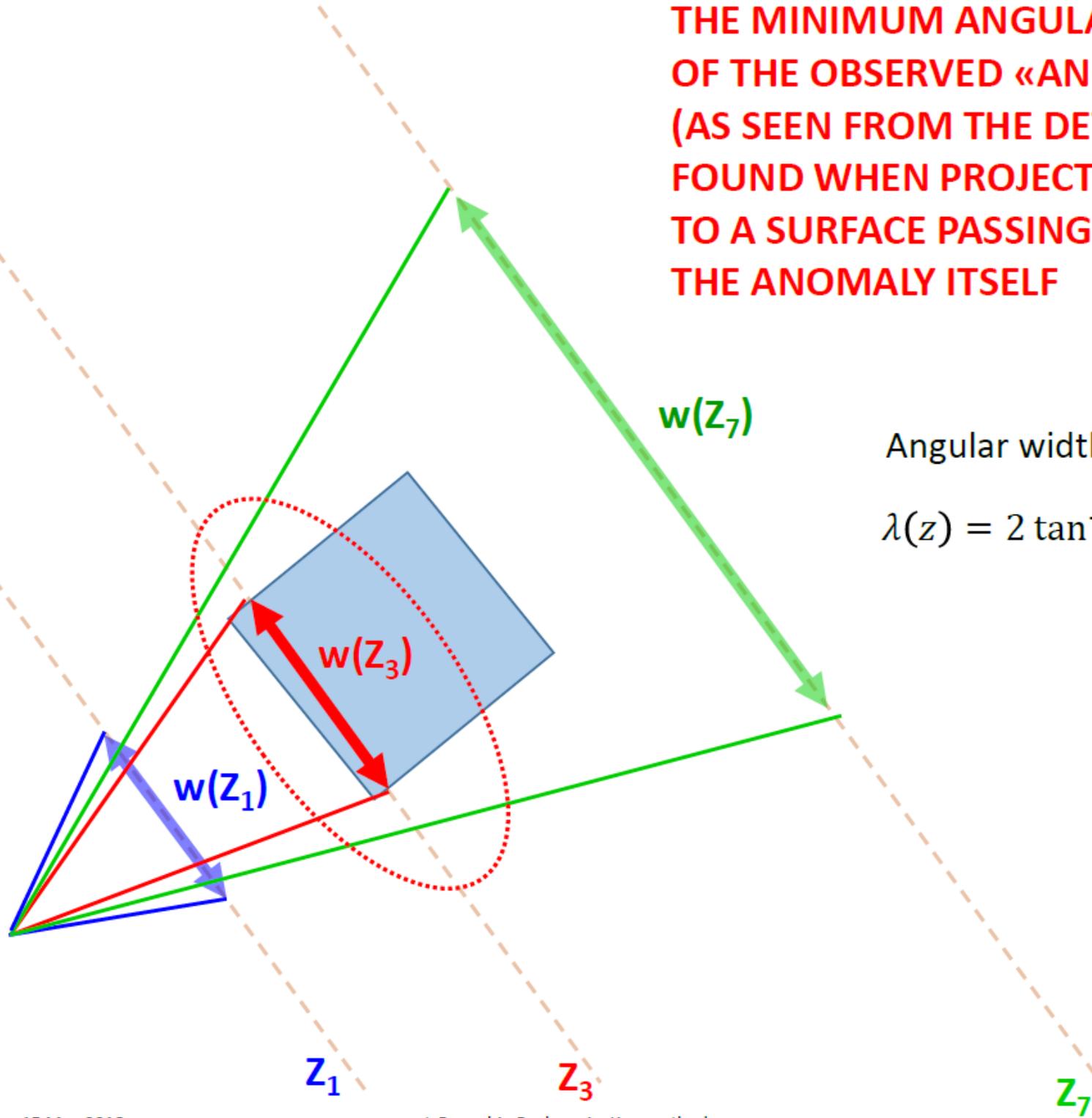
STUDY OF THE ANGULAR WIDTH
OF THE ANOMALOUS REGION
AS SEEN FROM THE DETECTOR'S
POSITION



Angular width:

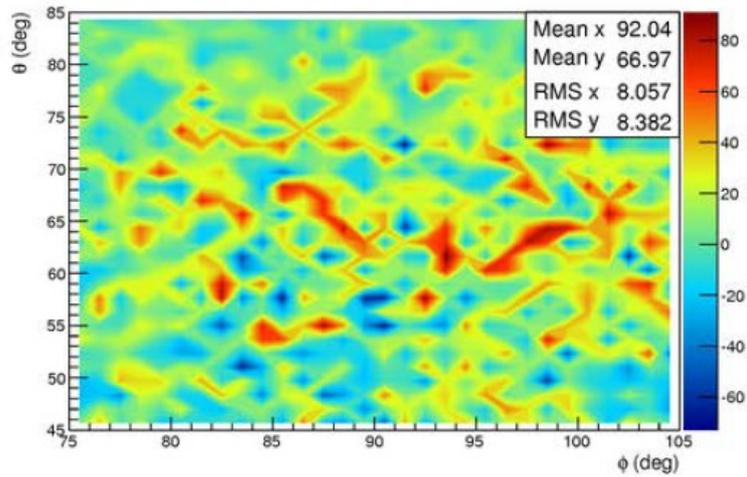
$$\lambda(z) = 2 \tan^{-1} \left(\frac{w(z)}{2z} \right)$$

THE MINIMUM ANGULAR WIDTH OF THE OBSERVED «ANOMALY» (AS SEEN FROM THE DETECTOR) IS FOUND WHEN PROJECTING TRACKS TO A SURFACE PASSING THROUGH THE ANOMALY ITSELF

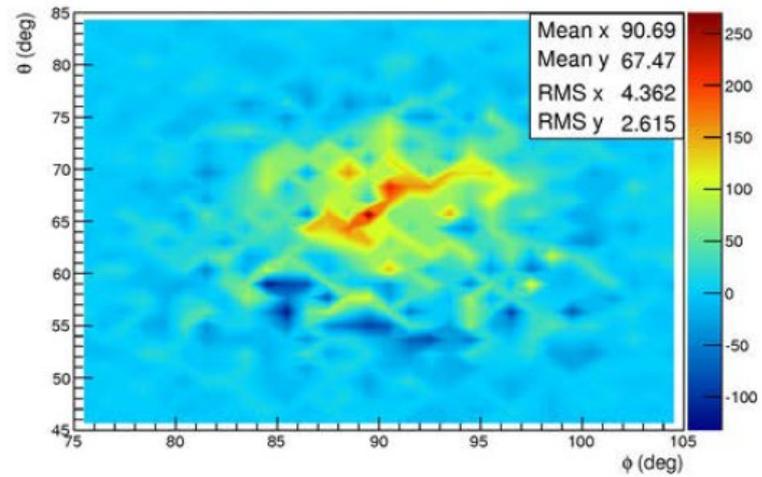


Angular width:

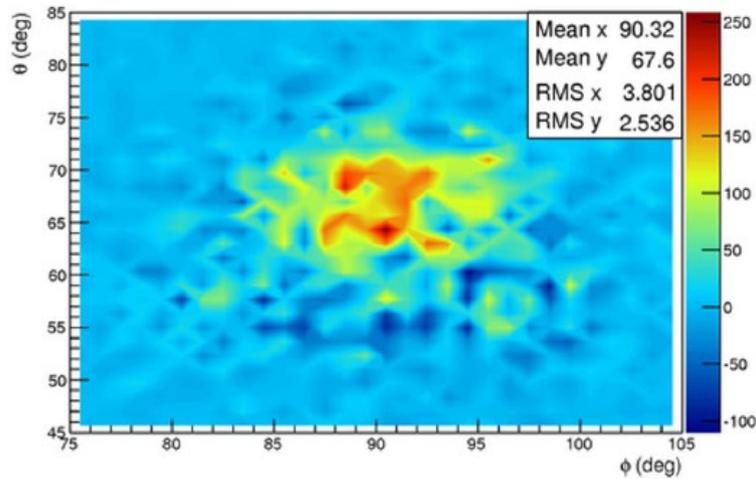
$$\lambda(z) = 2 \tan^{-1} \left(\frac{w(z)}{2z} \right)$$



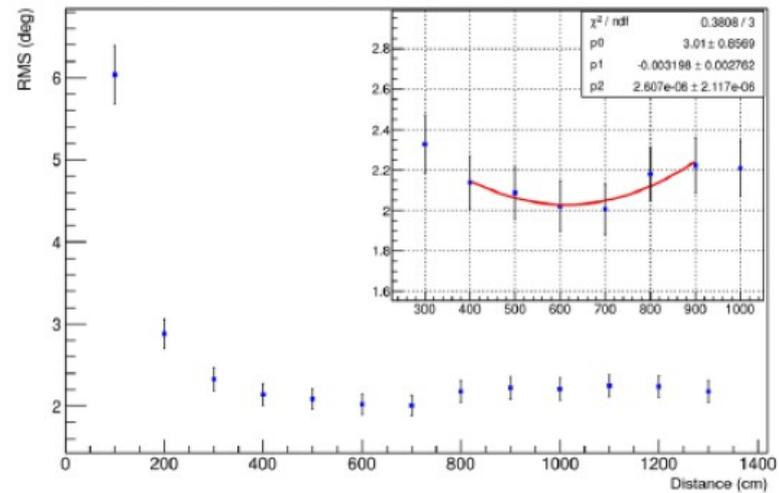
(a) Projection on a 1 m spherical surface



(b) Projection on a 4 m spherical surface



(c) Projection on a 7 m spherical surface



Thesis by dr. Lorenzo Viliani Univ. of Florence

Simulation study with GEANT4