Kolmogorov meets Turing

Workshop on probabilistic methods for the analysis of stochastic processes and randomized algorithms

March 19th, 2015

Abstract

The goal of this workshop is offering a multidisciplinary perspective over recent research directions in different areas, including applied probability theory, discrete mathematics and algorithmics, whose common denominator is the application of probabilistic tools to the analysis of complex stochastic processes and algorithms.

Where Dipartimento di Ingegneria Informatica, Automatics e Gestionale A. Ruberti, via Ariosto 25, Roma. Aula Magna, I floor

When Thursday March 19th, 2015

Schedule

- 9.30 - 10.15. Artur Czumaj (University of Warwick): Testing Cluster Structure of Graphs
- 10.15 - 10.45. Fabio Martinelli (University Roma Tre): Reversible Markov chains with kinetic constraints
- 10.45 - 11.15: Coffee break
- 11.15 - 11.45. Andrea Clementi (University of Rome "Tor Vergata"): Information spreading in dynamic graphs
- 11.45 - 12.15. Pietro Caputo (University Roma Tre): Empirical neighborhood distribution in sparse random graphs: some large deviations estimate

12.15 - 13.45: Lunch break

- 14.00 - 14.45. Alexandre Stauffer (University of Bath): Rumor spreading on dynamic graphs

- 14.45 - 15.15. Luca Becchetti (Sapienza University of Rome): Plurality Consensus in the Gossip Model
- 15.15 - 15.45: Coffee break
- 15.45 - 16.15. Fabio Toninelli (CNRS, University of Lyon 1): Random tilings and Glauber dynamics
- 16.15 - 16.45. Marek Adamczyk (Sapienza University of Rome): Stochastic probing

Organizers:

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List of abstracts

Marek Adamczyk: *Stochastic probing.* In a *stochastic probing* problem we are given a universe E, where each element $e \in E$ is *active* independently with probability $p_e \in [0, 1]$, and only a *probe* of e can tell us whether it is active or not. On this universe we execute a process that one by one probes elements — if a probed element is active, then we have to include it in the solution, which we gradually construct. Throughout the process we need to obey *inner* constraints on the set of elements taken into the solution, and *outer* constraints on the set of all probed elements. This abstract model was presented by Gupta and Nagarajan (IPCO 2013), and provides a unified view of a number of problems. Thus far all the results in this general framework pertain only to the case in which we are maximizing a linear objective function of the successfully probed elements. In this paper we generalize the stochastic probing problem by considering a monotone submodular objective function. For any $T \in (0, 1]$, we give a $(1 - e^{-T})/(T(k^{in} + k^{out}) + 1)$-approximation for the case in which we are given $k^{in} \geq 0$ matroids as inner constraints and $k^{out} \geq 1$ matroids as outer constraints. In the case that $k = k^{in} + k^{out} > 1$, we show that the optimal value for T is given by $T = -1 - \frac{1}{k} - W_{-1}(-e^{-1-\frac{1}{k}}) \approx \sqrt{\frac{2}{k}} - \frac{1}{3k}$, where W is the Lambert W function.

There are two main ingredients behind this result. First is a previously unpublished stronger bound on the continuous greedy algorithm due to Vondrak. Second is a rounding procedure that also allows us to obtain an improved $1/(k^{in} + k^{out})$-approximation for linear objective functions.
Joint work with Maxim Sviridenko and Justin Ward.

Luca Becchetti: *Plurality Consensus in the Gossip Model.* Reaching *Plurality Consensus* is a fundamental task in distributed computing. In

this problem, each node of a network initially supports an opinion or color. The goal is designing a simple, local and decentralized protocol or *dynamics*, a copy of which is run at every node of the network, so that network eventually reaches a monochromatic configuration reflecting the initial plurality. We consider the extra-state protocol (or third-state dynamics) introduced but only analyzed in the binary case (i.e. $k = 2$). The interest for this dynamics involves areas beyond computer science: it appears to play a major role in important biological processes modelled as so-called chemical reaction networks [Cardelli et al. 2012]. We analyse the extra-state protocol using a technique that strongly departs from past work and that allows us to address the plurality consensus problem in the general setting. Our analysis achieves almost-tight bounds on convergence time.
Joint work with Andrea Clementi, Emanuele Natale, Francesco Pasquale and Riccardo Silvestri.

Pietro Caputo: ***Empirical neighborhood distribution in sparse random graphs: some large deviations estimates.*** We introduce a generalized configuration model for the analysis of graphs with prescribed tree-like neighborhood distributions. The combinatorial analysis of this model allows us to compute a suitable entropy functional that governs the large deviations of the empirical neighborhood distribution of sparse Erdos-Renyi random graphs.
Joint work with Charles Bordenave.

Andrea Clementi: ***Information spreading in dynamic graphs.*** We give a short overview of the main rigorous results about the speed of Information Spreading in some specific Markovian models of Evolving Graphs. Then, we describe a recent approach to bound this speed in a rather general class of such Evolving Graphs.
Joint work with Riccardo Silvestri and Luca Trevisan.

Artur Czumaj: ***Testing Cluster Structure of Graphs.*** Cluster analysis is a fundamental task in data analysis that aims at partitioning a set of objects into a disjoint collection of objects with similar characteristics. In this talk, we will use the concept of conductance to measure the quality of cluster structure and will focus on a question of approximately recognizing cluster structure of a graph in sublinear time in the framework of property testing in the bounded degree model. We show how to test in $O(\sqrt{n})$ time whether a graph with nodes can be partitioned into no more than k parts (clusters) such that the outer-conductance of each cluster is ϕ_o at most and the inner-conductance of the induced subgraph on each cluster is at least ϕ_i, for a large spectrum of parameters k, ϕ_o, ϕ_i. By the lower bound of $\Omega(\sqrt{n})$ for testing graph expansion, which corresponds to the case when $k = 1$ in

our problem, our algorithm is asymptotically optimal up to polylogarithmic factors.
This is joint work with Pan Peng and Christian Sohler.

Fabio Martinelli: ***Reversible Markov chains with kinetic constraints.*** We consider the following model. Every vertex of a finite connected graph has a 0/1 valued variable. At each time unit a vertex is chosen uniformly at random and a new value is proposed for the variable at that vertex by a coin flip. If the neighboring values satisfy a certain constraint (e.g. they are not all equal to 1) then the new value is accepted, otherwise it is rejected.

Markov chains of this type have been introduced by physicists as models for glassy dynamics. Despite the apparent simplicity, their behavior can be very rich, with very large relaxation times. Their study involves bootstrap percolation, combinatorial analysis, coercive inequalities, random walks on matrices as well as other techniques related to convergence to equilibrium of reversible Markov chains.

After a general introduction I will describe some specific results for the so called East model where the graph is a line interval and the constraint requires that the variable to the left of the vertex to be updated equals 0.

Alexandre Stauffer: ***Rumor spreading on dynamic graphs.*** I will discuss a classical randomized algorithm (known as push-pull) to distribute a rumor among the vertices of a graph. In this algorithm, at each step, each vertex contacts a uniformly random neighbor to either transmit the rumor or get to know the rumor.

I will focus on the case where the graph changes while the rumor is being spread. I will address the following questions: i) Can dynamics speed up the spread of the rumor? ii) Can dynamics slow down the spread of the rumor? iii) Can we derive mild conditions to guarantee an upper bound on the spreading time?

Fabio Toninelli: ***Random tilings and Glauber dynamics.*** Glauber dynamics is a natural Markov chain that can be used for instance to sample uniformly a random element in a large combinatorial set or, more generally, to sample a certain Boltzmann-Gibbs distribution. Of particular interest is the question of how quickly the chain approaches its equilibrium measure.

We will consider in particular the case of random tilings of (a large domain of) the plane, and will discuss conjectures and recent results.
Joint collaboration with P. Caputo, B. Laslier, F. Martinelli.